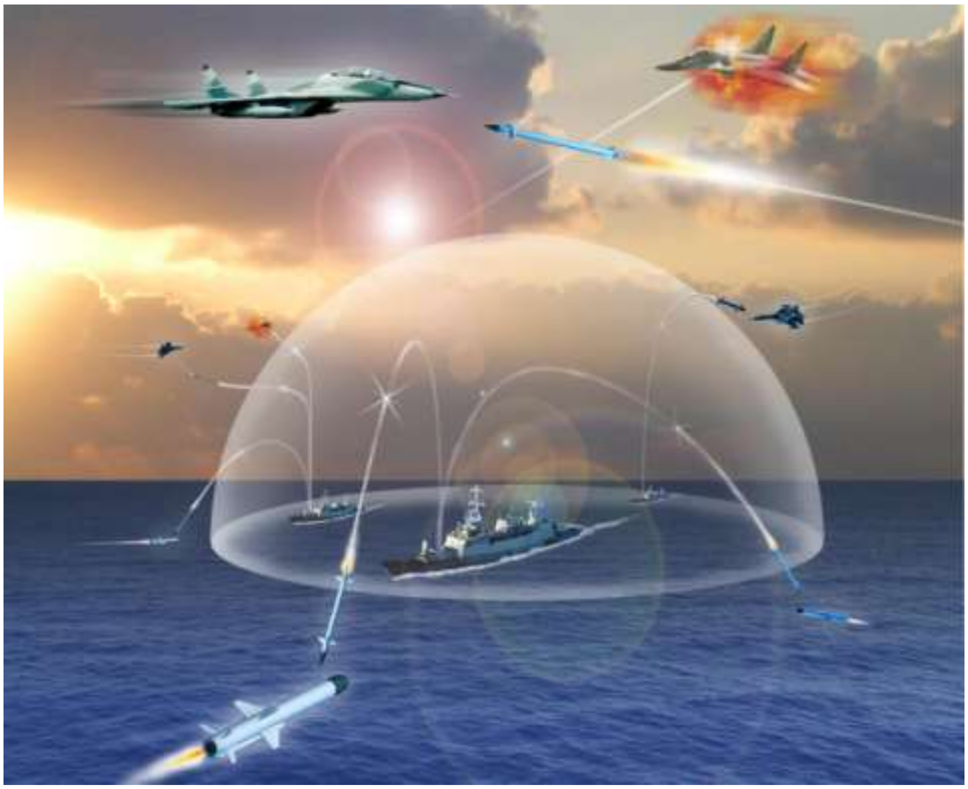
ETM
ENGINEERING & TECHNOLOGY MANAGEMENT

A Weapon Optimization Tool for Modern Air Defense Capabilities of Naval Platforms



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Authors:	:	Hakan Kutgun Chris Chase Tim Valentine

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ABSTRACT

Air defense is of vital importance for naval missions, as airborne attacks represent one of the most dangerous threats to maritime surface platforms. Considering the variety of modern airborne threat types and air defense weapons,, air defense munitions selection is already a challenge to start with. The addition of physical and budgetary constraints creates an even more complex challenge for anyone involved in the selection process. This study aims to provide ship captains and higher echelon with a systematic and transparent tool for preliminary selection of air defense munitions for modern naval surface platforms, considering the constraints in perspective with mission specific requirements.

1. Introduction

Air defense is of vital importance for naval missions as airborne attacks represent one of the most dangerous threats to maritime surface platforms. Due to the variety of modern airborne threats, there is no one-size-fits-all type of air defense weapon. Therefore, a variety of air defense weapons should be present on boards naval platforms, which makes air defense munitions selection a very important element of any naval mission. It is ship commander’s duty to plan for the defensive capabilities of the mission and make its case to the higher echelon for weapons allocation. This project aims to provide ship captains with a generic computational tool to ease their duty of air defense munitions selection using information regarding the ship, mission, threats and orders of the higher echelon.

2. Problem Definition

Although commander of the ship would like to stock more than enough of each type of defensive munitions onboard, it is not possible due to 2 important reasons. The first one is the payload capacity of the naval platform itself. Air defense systems tend to be very heavy and occupy significant space, both of which are very valuable resources for any ship. The second one is budgetary allocation. Usually air defense munitions is considered as a shared resource within a battalion, thus higher echelon would oversee the proper distribution of all available munitions to the vessels under their command. Additionally there may also be a number limitation to a number of weapons assigned to a single threat as it would be economically and strategically infeasible to engage all weapons on a single threat.

While the number of munitions gets constrained by the properties of the ship and orders from the higher echelon, the types of air defense munitions required is dependent on the types of threats to be expected. The commander has an idea about the type, importance and number of threats he may encounter, specific to each mission. In real life this information usually comes from mission intelligence reports and presented to the commander. Although it may be considered in a wider variety, type of weapons and threats will be limited tothe list below for the sake of this project.

There are 5 types of weapons:

- Short range AD missiles (SHORAD)
- Medium range AD missiles (MERAD)
- Long range AD missiles, (LORAD)
- Anti ballistic missiles (A-BM)
- Barrel type AD weapons (BAD)

And 7 types of threats:

- Unmanned Aerial Vehicle
- Fighter Jet
- Bomber Jet
- Light Aircraft
- Ballistic Missile
- Subsonic Cruise Missile

- Supersonic Cruise Missile

Aside from the type and number threats expected, the importance of each threat to a spesific mission may be different. For example enemy bombers may have the highest importance for a ship's mission to defend mainland from aerial bombardment, whereas UAV's may get the prime importance for an anti-reconnaissance mission.

The decision to be made is how many of each type of air defense weapons to take on board, taking into account mission specific data with payload and cost constraints.

3. Literature Review

The weapon target assignment problem (WTA) is an intrinsic aspect of military strategy and operations research within defense based applications[1] and it is an important subject of the field called military industrial engineering [2]. In the classic WTA problem there is a set number of resources/weapons available as well as targets to assign them to. Our project is new in this aspect as we are going to use WTA to determine the number of weapons to be assigned. WTA problem can either be static or dynamic. In the static WTA problem, all the inputs to the problem are fixed; all weapons and threats are known and all weapons engage to threats in a single stage whereas a dynamic problem is a generalization of the static problem[2]. A static model will be used for the sake of this project since dynamic models deal with detection, tracking and engaging dynamics with respect to time and range of threats.

Solving the equation will depend on the purpose of the analysis. Due to the nature of the problem neither the total quantity of weapons nor the targets can be less than one and must be integer based outputs, because one cannot have a fraction of a usable weapon or an incoming threat. Two assumptions are present in all WTA problems all weapons used must be assigned to all targets expected, and the individual probability of kill p_{ij} by assign *i*th target to *j*th weapon is for all of *i* and *j* [3].

4. Mathematical Model

The aim is to maximize the survival rate of the ship under given constraints. Survival rate is of the ship is related to combined effectiveness of defensive weapons against weighted threats. Expected "kill" of threat j if X_i weapons are used against it is given as [4]:

$$U_j\left(1-\prod_{i=1}^{W}\left(1-p_{ij}\right)^{X_{ij}}\right)$$

For combined effectiveness above value should be summed for all threats and maximized, giving the objective function for the problem as:

$$\max \sum_{j=1}^{T} U_j \left(1 - \prod_{i=1}^{W} (1 - p_{ij})^{X_{ij}}\right)$$

s.t.

Weight constraint:

$$\sum_{i=1}^{W} \left(M_i * \sum_{j=1}^{T} (X_{ij} * N_j)\right) \le M$$

Cost constraint:

$$\sum_{i=1}^{W} \left(C_i * \sum_{j=1}^{T} (X_{ij} * N_j)\right) \le C$$

Number of weapons allowed for a single target:

$X_{ij} \le 2 \;\; \forall i, j$ (i.e. it is assumed to be economically and strategically infeasible to engage more than 2 of the same weapon type to a single target)

(additional non-negativity and integer constraints described below)

Where;

i ≡ Air defense weapon type $i \in \{1,2,\dots,W\}$

j ≡ Threat/target type $j \in \{1,2,\dots,T\}$

C_i ≡ Resource usage cost for air defense weapon type i in [K$]

M_i ≡ Weight for air defense weapon type in [lbs]

U_j ≡ Mission specific threat coefficient for threat/target type j $U_j \in \{1,2,\dots,10\}$

N_j ≡ Number of threats of type j $N_j \in \mathbb{Z}^+$

p_{ij} ≡ Single-shot probability of kill of weapon i for target j $0 \le p_{ij} \le 1$

X_{ij} ≡ Number of weapon type i assigned to target type j $X_{ij} \in \mathbb{Z}^+$

$\forall i \in \{1,2,\dots,W\}$ *and* $\forall j \in \{1,2,\dots,T\}$

And;

T ≡ Total number of target/threat types available

W ≡ Total number of air defense weapon types available

M ≡ Total payload capacity of the naval vessel

C ≡ Total resource allocation limit for the naval vessel

Further define;

S_{ij} ≡ Probability of survival of target j from all weapons of type i deployed

K_j ≡ Multi-shot (sum of all weapons i assigned) kill probability of target type j

To clarify simplifications within the Excel model.

After all X_{ij} values are obtained using the optimal solution of this problem, number of each type of air defense weapons to be taken onboard, A_i is given by:

$$A_i = \sum_{j=1}^{T} (X_{ij} * N_j) \quad \forall i = 1,2, \ldots, \mathrm{W}$$

4.1. DATA

Single-shot probability of kill of weapon types with respect to target types (p_{ij})and their corresponding weights (W_i) and usage costs (C_i) are given in Table-1

								T = 7		
		j = 1	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7	C_i	M_i
		Single-shot probability of kill								
	p_{ij}	UAV	Fighter jet	Bomber	Light AC	BM	Subsonic Cruise Missile	Supersonic Cruise Missile	Resource Usage Cost [K$]	Weight [lbs]
i = 1	SHORAD	0,82	0,95	0,80	0,97	0,15	0,79	0,74	250,00	550,00
i = 2	MRAD	0,81	0,95	0,85	0,95	0,21	0,84	0,80	300,00	660,00
i = 3	LRAD	0,81	0,91	0,93	0,90	0,29	0,88	0,85	350,00	870,00
i = 4	A-BM	0,6	0,88	0,83	0,85	0,87	0,73	0,83	450,00	1100,00
i = 5	Barrel type (x1000)	0,82	0,76	0,48	0,92	0,10	0,87	0,60	10,00	110,00
W = 5										

Table 1 – Air defense weapons data table

This data is compiled from a large database of real air defense weapons specifications. Since the exact p_{ij} values would be different for each combination of exact models of weapon and target, a representative target is assumed to represent its own class and single-shot probability of kill of different AD weapons are given as an average for that target type.

In addition to air defense weapons data, data regarding expected threats and constraints regarding weight and cost should be input by the commander in order to initialize the problem. Table 2 represents the input console of the ship commander specific to each mission/ship combination with dummy values for each cell.

	UAV	Fighter jet	Bomber	Light AC	BM	Subsonic Cruise Missile	Supersonic Cruise Missile	
Number of threats	15	10	3	4	2	8	5	N_j
Mission spesific threat coefficient (1-10)	8	6	3	1	1	8	10	U_j

Mission spesific payload capacity [lbs]	85.000	M
Mission spesific resource allocation [K$]	30.000	C

Table 2 – Input console of the ship commander (with dummy values)

4.2. Decision Variables

The decision variables will be the number of weapon type i assigned to threat type j:

X_{ij} ≡ Number of weapon type i assigned to target type j $\qquad X_{ij} \in \mathbb{Z}^{+}$

For 5 type of weapons and 7 type of threats there is a total of 35 variables for this scenario but can be more depending on types of threats and weapons.

4.3. Constraints

The total payload would be constrained by the capacity of the vessel, which provides the upper limit for total payload.

$$\sum_{i=1}^{W}\left(M_i * \sum_{j=1}^{T}(X_{ij} * N_j)\right) \leq M$$

The total cost would be constrained by the higher echelon, which provides the upper limit for total cost.

$$\sum_{i=1}^{W}\left(C_i * \sum_{j=1}^{T}(X_{ij} * N_j)\right) \leq C$$

Number of weapons allowed for a single target will be constrained assuming it is economically and strategically infeasible to engage more than 2 of the same weapon type to a single target.

$X_{ij} \leq 2 \;\; \forall i, j$ (i.e.)

Number of weapon types assigned to each target type are positive integers.

Additionally;

The number of each type of threat will be predetermined by intelligence reports for the mission and the value of each type of threat will be evaluated by the Commander. (Scale of 1-10).

5. Results & Analysis

Everything considered problem presented here is a "non-linear integer optimization problem with 35 decision variables". With this definition, it is within capabilities of Excel solver. However as expected for a problem of this complication, solution takes quite some computation time to find an optimal solution (approx 1250 seconds on a 6–core 4GHz Intel CPU). Sample solutions for 2 cases are provided within the report with the working model attached for further examples.

First example is of a destroyer vessel with maximum payload of 40,000lbs and a resource allocation of $20 million. There are 7 of each type of targets expected with different coefficients of importance.

INPUTS

	UAV	Fighter jet	Bomber	Light AC	BM	Subsonic Cruise Missile	Supersonic Cruise Missile	
Number of threats	7	7	7	7	7	7	7	N_j
Mission spesific threat coefficient (1-10)	8	6	3	1	1	8	10	U_j

Mission spesific payload capacity [lbs]	40.000	M
Mission spesific resource allocation [K$]	20.000	C

DECISION VAIRABLES

	Xij	j=1 UAV	j=2 Fighter jet	j=3 Bomber	j=4 Light AC	j=5 BM	j=6 Sub CM	j=7 Sup. CM	
i=1	SHORAD	1	1	0	0	0	0	1	3
i=2	MRAD	0	0	0	0	0	0	1	1
i=3	LRAD	0	0	1	0	0	0	0	1
i=4	A-BM	0	0	0	0	1	0	0	1
i=5	Barrel (x1000)	2	2	2	2	0	2	2	12

Sij = (1-pij)^Xij

0,180	0,050	1,000	1,000	1,000	1,000	0,260
1,000	1,000	1,000	1,000	1,000	1,000	0,200
1,000	1,000	0,070	1,000	1,000	1,000	1,000
1,000	1,000	1,000	1,000	0,130	1,000	1,000
0,032	0,058	0,270	0,006	1,000	0,017	0,160

Kj = 1-Π(Sij)

0,99417	0,99712	0,981072	0,9936	0,87	0,9831	0,99168

OBJECTIVE FUNCTION

Σ(Uj*Kj)

7,95334	5,98272	2,943216	0,9936	0,87	7,8648	9,9168	36,5245	MAX

SHORAD	7	7	0	0	0	0	7	21 SHORAD
MRAD	0	0	0	0	0	0	7	7 MRAD
LRAD	0	0	7	0	0	0	0	7 LRAD
A-BM	0	0	0	0	7	0	0	7 A-BM
Barrel (x1000)	14	14	14	14	0	14	14	84 Barrel (x1000)

Total number of weapons needed onboard

CONSTRAINTS

SHORAD	MRAD	LRAD	A-BM	Barrel (x1000)	LHS		RHS	
21	7	7	7	84				NOT a constraint (copied from above)
550,00	660,00	870,00	1100,00	110,00	39.200	≤	40.000	Weight constraint
250,00	300,00	350,00	450,00	10,00	13.790	≤	20.000	Resource cost constraint

Figure 1 – Destroyer example

Analysis showed that the most important weapons to take on board were the bullets and the short range air defense missiles. The binding constraint was the ship's weight capacity which was 98% used up. Only about 69% of the cost constraint was used. It is realized that weight is the binding constraint because increasing any of the decision variables by 1 causes weight constraint to go ver the allowed limit.

Second example is of a battleship vessel with maximum payload of 85,000lbs and a resource allocation of $30 million. This time there are different number of each type of targets expected again with different coefficients of importance.

INPUTS

	UAV	Fighter jet	Bomber	Light AC	BM	Subsonic Cruise Missile	Supersonic Cruise Missile	
Number of threats	15	10	3	4	2	8	5	N_j
Mission spesific threat coefficient (1-10)	8	6	3	1	1	8	10	U_j

Mission spesific payload capacity [lbs]	85.000	M
Mission spesific resource allocation [K$]	30.000	C

DECISION VAIRABLES

	Xij	j = 1 UAV	j = 2 Fighter jet	j = 3 Bomber	j = 4 Light AC	j = 5 BM	j = 6 Sub CM	j = 7 Sup. CM
i = 1	SHORAD	1	1	0	0	0	0	1
i = 2	MRAD	0	0	0	0	0	0	1
i = 3	LRAD	0	0	1	0	0	0	0
i = 4	A-BM	0	0	0	0	1	0	0
i = 5	Barrel (x1000)	2	2	2	2	0	2	2

Sij = (1-pij)^Xij

0,180	0,050	1,000	1,000	1,000	1,000	0,260
1,000	1,000	1,000	1,000	1,000	1,000	0,200
1,000	1,000	0,070	1,000	1,000	1,000	1,000
1,000	1,000	1,000	1,000	0,130	1,000	1,000
0,032	0,058	0,270	0,006	1,000	0,017	0,160

Kj = 1-Π(Sij)

0,99417	0,99712	0,981072	0,9936	0,87	0,9831	0,99168

OBJECTIVE FUNCTION Σ(Uj*Kj)

7,95334	5,98272	2,943216	0,9936	0,87	7,8648	9,9168	36,5245	MAX

SHORAD	15	15	0	0	0	0	15	45 SHORAD
MRAD	0	0	0	0	0	0	15	15 MRAD
LRAD	0	0	15	0	0	0	0	15 LRAD
A-BM	0	0	0	0	15	0	0	15 A-BM
Barrel (x1000)	30	30	30	30	0	30	30	180 Barrel (x1000)

Total number of weapons needed onboard

CONSTRAINTS

SHORAD	MRAD	LRAD	A-BM	Barrel (x1000)	LHS		RHS	
45	15	15	15	180				NOT a constraint (copied from above)
550,00	660,00	870,00	1100,00	110,00	84.000	≤	85.000	Weight constraint
250,00	300,00	350,00	450,00	10,00	29.550	≤	30.000	Resource cost constraint

Figure 2 – Battleship example

The number of weapons to take on board changes with this scenario, as expected. However this time, the binding constraints were both the ship's weight capacity and the cost constraint which were both 99% used up when the optimal solution was found.

As the results of different scenarios indicate, the model is working and the results given are as expected.

6. CONCLUSION

This model can be used by ship commanders in the Navy in assistance to assist the decision of how many air defense weapons of each type to take onboard for specific missions. By making use of a static WTA, the model provides ship commander with the optimal allocation of weapons that will fit within the payload and cost constraints to give the highest probability of kill for the types and number of threats he can expect to encounter. However since this is a non-linear model, it requires a lot of computation time and even still, the optima found at the end of the Excel run can't be guaranteed to be a global optima. Despite this fact, it is expected to provide a better estimate than pre-determined catalog values as it considers mission specific data.

The model can be further improved with:

- More type of targets or weapons
- Incorporating the launch platforms as one-time weight addition
- Allowing sets of munitions instead of standalone ones
- An implementation in a better/faster computational tool (such as R)

as matters of future research.

This approach to allocation can also be adopted outside of the world of defense into other industries where problems have the same basic framework. Such an example is already found in the use of media sourcing and budgeting for advertising campaigns[5] during the literature survey. In this approach the advertising campaign must reach the maximum amount of people through the a minimum number of air time per set media outlets under a set budget. The cost is dictated by the rate at which the media outlet charges for the time and the amount of time allocated. The audience are not unlike the targets in the standard WTA problem and the media outlets are not unlike the weapons. The allocation of the amount of time per media outlet aimed at the target outlets audience is the WTA model.

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