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ENGINEERING & TECHNOLOGY MANAGEMENT

# *Production Scheduling and Inventory Optimization*

## For a High-Tech Company to Manufacture Products Efficiently

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**Abstract**

Optimizing the production schedule to minimize operating costs and maximize profits is a challenge that a Portland, Oregon, based high-tech Company is facing. The objective of this project was to create a model that could be used as a tool by business analysts to enable informed and educated operational, production, and inventory decisions. An Integer Linear Programming model has been utilized to propose a build schedule and parts order process to meet the forecasted demand and production constraints for which the Company is contending. In addition to minimizing costs, the outputs of the model will allow the procurement team an opportunity to balance the parts necessary for the creation of the products by conforming to a just-in-time manufacturing methodology. Ultimately, this model can produce and will enable the analyst to explore the impacts of new and different constraints involved with creating an optimized production plan, which will improve the inventory, manufacturing, and production costs.

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# 1. Introduction

The overall strategy of this project is to deliver an optimization tool that will provide assistance to a Portland, Oregon, based High-Tech Company to better sequence their production schedule and parts ordering to maximize their profits with respect to the products being sold, by minimizing operating costs associated with inventory, procurement, and production.

## 1.1. Company Information

The Company that the optimization model has been generated for is a mid-sized high-tech company that specializes in developing and creating sophisticated equipment for high volume manufacturing in the semi-conductor and consumer electronics industries. At the core of this research, the Company has a rich history of technological and business/corporate innovation in Oregon since the 1940's. This Company has a proven track record of re-inventing itself, generating new business opportunities, and keeping pace with the changing industries and global markets. Adversity and several changes throughout the "dot.com" bubble have formed the lean corporation that exists today. Just-in-time (JIT) manufacturing processes are key in this industry and to this company with the competitive nature that surrounds the type of products that the business creates; delivering a product on-time and to the specifications of the product is necessary for continued success. Utilizing a just-in-time mentality, the Company can be more efficient with their storage limitations, minimize waste, and reduce the overall inventory costs freeing up capital to be used elsewhere within the corporation. The Company currently uses a SAP MRP system for scheduling and global production planning.

Although SAP is a widely used and an efficient program, the needs of the Company with specific regard to the problem faced requires a tool that is more flexible and dynamic, hence the reason and need for creating this integer linear program (ILP). [1] Through interviews and observations, we have learned that SAP is currently utilized as the enterprise production planning and inventory management tool. However SAP MRP engine is not utilized to the fullest extent and alludes to the fact that the business analysts do not possess the skills necessary to edit and modify the MRP algorithm [2] tied to the model on an enterprise scale. In reviewing the current method, it revealed that most analysis of the model and modification transpires outside of the MRP engine, where Excel spreadsheets are used to manually manipulate data to understand the impacts of scenario changes. This is a labor intensive task; correct application of an optimization model can largely automate this problem. Graphical data is presented and articulated in a manner in which visually based decisions will be made. The model that has been developed will assist in eliminating the need for the secondary work that typically takes place.

## 1.2. Problem Description

Like many companies, balancing the materials on-hand and carrying additional costs by extending corporate funds for additional costs for inventory is a constant financial challenge. The project team has been tasked with how to optimize [3] the inventory schedule for procurement purposes which drives the inventory optimization of the Company. This tool will allow for an opportunity to maximize output and profit of the products that are being created. We will determine what parts the Company needs to order and when to eliminate excessive materials on-hand. Building specific sequences for the inventory based on current lead times from sales is a crucial issue that has been posed. Expediting parts has become a concern and a costly part of the current process that needs to be eliminated, not only for the additional costs involved, but the slippage of the schedule and delivery constraints of the Company.

Optimally, the Company will be able to build and sequence a total count of 258 builds across 13 product types, taking into account the overlap of multiple orders and being able to ramp up and down to meet the demand forecasts across five quarters. There are several constraints and live/running inventory is a consistent problem. Resource planning is also another hurdle that must be overcome to become more efficient. Physical floor space in the manufacturing facilities is a constant concern which is compounded by a build time of three weeks. As previously stated, the Company uses the SAP MRP system for it is production planning, while SAP offers tremendous value to the Company, it also is rather complex and lacks the flexibility required by the individual business analyst to experiment and simulate multiple/different scenarios for optimality. Demand forecasts are updated weekly, SAP optimization is run once per week, and due to the intricacy and scale of the Company, it takes days to complete the computation. Ideally the our model and tool will help the analysts identify the critical part items for the proposed and optimized build schedule, which will be sensitive enough to analyze the appropriate ordering strategies to allow the user to optimize efficiency on a more frequent basis. Our tool will also minimize the time associated with running the model, due to the speed of Solver and reduction of the problem to a more manageable size.

# 2. Literature Review

To facilitate a better understanding of the overall issue and to resolve the inefficiency that attribute to the problem statement, we have reviewed several resources to better define the model that we have created. The first application that we reviewed, inspected the differences between the possibilistic linear programing (PLP) and the aggregate production planning (APP) methods. This enhanced understanding helps with the differences between APP and PLP and how it can assist corporations in better deciding which model to utilize in future pursuits. Analysis of another research paper, uncovered further details with regard to APP and the difference with regard to multiple criteria mixed integer linear programming (MCMILP). Taking our research further, we delved deeper into a genetic algorithm (GA) which was utilized to optimize base-stock levels of an organization that is focusing on optimizing the minimum needs related to the shortage and holding costs in the supply chain. [4] Lastly, the team reviewed another linear programming method which is material requirements planning (MRP). This is a method that prioritizes the necessary material requirements and the schedules for the various products of plants to meet the demand throughout the company. After reviewing all of these applicable models, an appreciation of why the MRP is the choice of how the Company currently monitors and controls the inventory for the business has been determined. However, to better understand similar concepts as they pair with the current process, we have analyzed the aforementioned linear programs appropriately.

## 2.1. APP versus PLP

*Applying possibilistic linear programming to aggregate production planning*

The objective of this type of process is to be more successful with understanding which direction is best for making the most lucrative decision for the Company. "Aggregate production planning (APP) determines the best way to meet forecast demand in the intermediate future, often 3 to 18 months ahead, by adjusting regular and overtime production rates, inventory levels, labor levels, subcontracting and back ordering rates, and other controllable variables." [5] This methodology has been around since the mid-1950's and has been refined over the years. The method describes the fuzzy inputs or parameters that a linear mathematical program cannot incorporate into solving. Due to these real-world constraints and issues, the fuzzy set theory is described and is taken into consideration when deriving an optimization process that will solve the situation.

Creating a more accurate optimization model does require flexibility with the LP. "Moreover, Zadeh presented the theory of possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable."[5] His theory takes into account the humanistic realities of a corporations function due to the decisions that are made based on the possibility of a choice versus the probability of it. APP models typically focus on the total costs and the PLP utilized in this article did as well with regard to the objective function. Minimizing the overall costs in the process is the goal of the Company with regard to fluctuating forecasts over a 3 to 18 month window. Even though this system appears to follow a similar desire and quest of efficiency, the MRP model is the accepted program that of which the Company uses.

## 2.2. APP and MCMILP

In reviewing APP further, an article specifically focuses on the APP model as it relates to a Portuguese firm in relation to the mixed integer linear programming. The model is predicated on minimizing late orders, maximizing profits, and using less employee resources. This type of evaluation technique is fairly involved and takes into consideration several variables. Ideally, this model unveils the quantity of workers for each specific type of work, the necessary hours (regular and additional), and the overall inventory for each of the products in the category, and any outsourcing necessary of a duration of a year.

The article that was examined researches further into the multiple criteria necessary for this specific case. An extreme amount of variables, the overall objective function, and constraints are complex and involved in order to achieve a minimizing objective function to derive and provide an efficient solution. Ultimately a decision support system (DSS) is proposed as part of the model. "We believe that the DSS presented in this study will really enhance the application of the MCMILP model in practice, making it a better practical alternative to more sophisticated and complex aggregate planning." [11] In order to be more dynamic like many of these linear programs try to be, having a more interactive type of DSS would help the executives contend with uncertainty that is not forecasted throughout the product lifecycle. This application is a broader net and model that surpasses the Company needs with regard to streamlining one process versus many. There are several great attributes to this style of linear programming but it is ultimately not what the management requires.

## 2.3. Genetic Algorithm

In evaluating research surrounding the Genetic Algorithm approach, utilizing a linear program that can mutate and change is an interesting perspective. After reviewing one article, better understanding supply chain and the impact it has on the value of a product is appealing. "The challenges facing business organizations grow rapidly because of the globalization of commerce, global distribution of manufacturing and warehousing facilities, rapid commoditization of products, demand for customized products, competitive pressures, rapid advance in information technology, etc." [6] Being able to optimize and control the supply chain presents an incredible opportunity for cost savings. [7] When you consider that approximately 30% of the costs associated with a product is due to the inventories that surround it. Ultimately, this paper has the objective to minimize the total supply chain cost (TSCC) by utilizing a genetic algorithm that essentially optimizes the base-stock levels while reducing the shortage and holding costs throughout the supply chain.

The GA ultimately does optimize the inventory levels of the TSCC as predicted. An optimal solution was found by the GA and its overall effectiveness was benchmarked by the random search procedure (RSP). The RSP was used in an enumeration technique that delivered five times the number of solutions that were generated by the GA but they were comparable as the study suggests. A GA was more efficient than the RSP but still does not suit the method the corporation we have been working with requirements. Forecasting is the missing piece for our specific needs; however, this model would be great at streamlining the process as a whole throughout the various operations of the Company, if that was exactly our mission at this time.

## 2.4. MRP

Material Requirements Planning has become more commonplace since its inception in the 1970's by Olrick. The objective of this method is to deliver the correct materials at the time and place they are required to maximize efficiency. Utilizing MRP allows for corporations to calculate the material needs while understanding the market demand throughout the inventory on hand at various plants to create the products that are being sought after. The article touches on the catalyst for the development of this technique, which is to make a company more competitive which is more prevalent in the market due to ecommerce. Minimizing costs and maximizing quality is a constant pursuit of the Company for which this method focuses on that is often times using a just-in-time mentality.

This process requires four inputs to maximize the efficiency of a production procedure which include the schedule of production, the bill of materials, the production cycle times, and the supplier lead times. These components better define the scope of the work and assist in understanding the constraints of the project. The emphasis of MRP on a Company typically narrows the focus on inventory, priorities, and capacity. It was realized that overall motivators usually stem from management recognition of business opportunities, technical opportunities or the need of solving a manufacturing or inventory problem. Our overall mission and problem has been identified and requires a similar model. The Company is currently utilizing this method for which we are modeling as well. SAP provides visibility to this style of tracking and benchmarking and is incorporated throughout the fundamental process; however, an overall tool that can be used for checks and balances has been requested by the management of the Company. Our ultimate goal is to meld the existing MRP and SAP protocol with an Excel Solver instrument.

# 3. Platform and Optimization Solver Options

There are several types of platforms to consider when solving linear programs as they relate to the problem statement. Excel, R-Code, and Matlab were considered for the purpose of this project. When reviewing MatLab, the software can analyze the algorithm and has the features necessary to produce the final output and projection necessary for the scheduling portion or our solution. However, due to the cost involved with licensing and accessibility, this option was not considered.

R-code was discussed as well since it is open source and readily available for download. This program allows for branching, looping and modular programming. It can also review and analyze, and it is not limited to linear, non-linear, smoothing, and clustering models. Ultimately, it has the horsepower necessary to perform the functions necessary to resolve the challenges posed by the scheduling need of the Company. However, due to the ease of access to Excel and the functionality of the program, we decided to utilize it as our solving option to implement our algorithm. Excel also allows for ease of use and transferring between the team members. This flow and communication was straightforward when extending it to the Company for review of our progress as well.

## 3.1. OpenSolver

After making the determination to utilize Excel, organizing the model with the finalized objective function, the variables to be considered, and constraints, resulted in a rather large array and multiple tabs for part tracking and ordering. The scope of the problem required delivering to a forecast for a production portfolio containing 13 products over 65 weeks with ~1350 parts. Utilizing Excel, we had access to the default solver's three optimization engines, Simplex Linear Programming (LP), nonlinear generalized reduced gradient (GRG) and evolutionary. Excel solver has variable and constraint limitations (200 and 100 respectively for the LP) which make it suitable for small scale problems. As our model required 845 decision variables (build schedule for 13 products x 5 quarters) we chose to use

"OpenSolver" which does not have the same limitations as excel solver or expressive and provides more solver engine options. As our model is an ILP (Integer Linear Program) we chose to utilize the CBC solver in OpenSolver[9],[10].

Other attributes of utilizing OpenSolver include but are not limited to, no artificial limits on the size of problem you can solve, there is a built-in model visualizer that highlights the decision variables of the model, the objective and constraints can appear directly on your spreadsheet, it allows for quick iterations, and the algorithm can be created and updated with the details that are in the current spreadsheet[9]. One drawback of the OpenSolver is the sensitivity analysis reports for large scale problems did not seem to function correctly during our work.

# 4. Creating the Algorithm and Algebraic Model

In pursuit of the thesis, the first step in optimizing the production plan scheduling is to conceptually outline a framework to minimize the operating costs and define a set of decision variables to for each build cycles. To simplify the model, the timeframe was applied over a weekly period to ascertain a more manageable tool in terms of the decision variable count for a given solver solution. Building and implementing the model was an iterative process as it relates to the ILP. [12] Initially the framework was created with only total quantity constraints with the assumption that all of the parts were already on hand. This implied that the object function was the number of products produced. An introduction of implementing the basic model in the chosen solver ensued next, which allowed the team to understand how the solver can operate in simple progressive steps. Subsequently, more constraints were incorporated, such as production capacity, delivery demand constraints, and change of the objective function to a minimax for the production capacity. During the iterative process we consulted with the Company's production planning analyst for guidance and insight into costs, assumptions that were necessary, and overall limitations for which to be cognizant. This report compares and appraises three iterations of the model, which resulted in model 4, model 5 and model 6. Each model represents a different scenario. Model 4 is limited to only building products in the quarter they are forecast, no builds can start in weeks 12 or 13 of the quarter. Model 5 was constructed around the premise of allowing building products in prior quarters to reduce cost. The last model, Model 6, serves as a baseline for the minimum cost by ignoring the delivery demand constraints and allowing products to be built in the best possible sequence for the Company on a quarterly basis; there is a 2 week overlap where products that are required to meet the demand in Q2 can be built in weeks 12 and 13 in Q3. The following sections will discuss models 4, 5 and 6 as it relates to the formulation, implementation, and respective results. The inputs and outputs identified as part of the model are listed in Figure 1.0.

| Inputs | Outputs |
|---|---|
| Forecast: Product Quantity and delivery date - units per week per product | Build Sequence: Quantity and start date -units per week per product |
| Parts in inventory - units | Parts to Order and order date - units per week |
| Parts on order - units | Fixed Order/Procurement Costs $ |
| Production constraints min and max - unit build/week | Inventory Holding/Carrying costs $ |
| Part Lead Times in weeks | Fixed Production Costs $ |
| Part BOM Costs $ | Optimal order quantity - units |
| Inventory Holding/Carrying Rate % | Weekly production capacity - units |
| Production Cost Rate % | Total Operating Cost (TOC) $ |
| Order/Procurement Cost Rate % | |

Figure 1.0 Model Inputs and Outputs

## 4.1. Algebraic Model

The algebraic model is the mathematical formulation of the algorithm and serves as an encompassing depiction of the objective function, the appropriate constraints, and the relevant variables. The different variables are the foundation of what was used to build the objective function, constraints, and furthermore, the objective function is linked to the constraints by using the variables described in the report. Full mathematical notation for the algebraic model 4, 5, and 6 can be found in Appendix A.

In accordance with the aforementioned models 4, 5, and 6, an algebraic model and equation was consequently required and generated for each model. Since the algebraic model is a formalization of the algorithm, it is evident that there are differences between the models, which are concerned with how to produce on a quarterly or weekly accumulated basis, also in the respective algebraic forms. However, these changes solely occur in the constraints which allows for the objective function to be the same for all three models.

$$min \underbrace{\sum_i OC * O_i}_{\text{Order or Procurement Cost}} + \underbrace{\sum_i \sum_p IHR * C_p * PD_{i,p}}_{\text{Inventory Carrying Cost}} + \underbrace{\sum_i \sum_j P_{i,j} * MFC_1 * \frac{1}{3} H_j}_{\text{Production Cost @ 85\% Utilization}} + \underbrace{\sum_i \sum_j (P_{i,j} - P_{i\,min}) * (MFC_2 - MFC_1) * \frac{1}{3} H_j * X_i}_{\text{Production Cost @ > 85\% Utilization}}$$

Figure 2.0 Objective Function

Figure 2.0 above depicts the objective function, which has to be minimized, as well as the different types of costs and how they relate to each of the cost categories.

Model 5 has been chosen as the baseline and reference model which consists of twelve summarized and generalized constraints that divide into a large number of single constraints due to the usage of indices that run for products (j) from 1 to 13, for weeks (i) from 1 to 65 and for parts (p) from 1 to 1349. When appraising the model, constraint number eight for the stock balance, which says that the on-hand parts plus the to-order parts minus the for production used parts equal the on-hand parts in the next period, or constraint 1, which activates the additional manufacturing costs by using the 'Big M' if necessary, and all other constraints we want to focus in the following on constraint 5) of Model 5 (Figure 3.0).

$$5) \sum_{i=1}^{n} P_{i,j} \geq \sum_{i=1}^{n} D_{i,j} \ \forall j,\ n = 1, \dots, 63$$

Figure 3.0 Model Inputs and Outputs

This constraint states that the amount of products produced has to meet the demand and is critical in the analysis since this is the constraint that changes the overall model 4 or model 6. Model 5 uses the accumulative forecast from week to week which we algebraically solved by the summation of the produced products from i equals 1 to n while assigning the weeks 1 through 63 to n and, additionally, all products.

Figure 4.0 illustrates five constraints of Model 6 that altogether are the equivalent of the above described constraint 5) of Model 5. Since Model 6 is interested in a quarterly perspective we calculate the sum of the produced products over each quarterly time span which has to be greater than or equal to the forecast for every product. In order to fulfill the characteristics of Model 6, the two week overlap for building in the previous quarters, we shortened the 1st quarter by two weeks, so that the time span is just eleven weeks.  The model than optimizes the start of second quarter in week twelve which is usually the first quarters second last week. From this point forward every quarter embraces the typical 13 weeks. This has the effect that the start of the production time of quarter 5 ends in week 63 so that all of the products can be finished within their common three week production timeline.

$$6)\ \sum_{i=1}^{11} P_{i,j} \ \geq\ FC1_j \ \forall\, j$$

$$7)\ \sum_{i=12}^{24} P_{i,j} \ \geq\ FC2_j \ \forall\, j$$

$$8)\ \sum_{i=25}^{37} P_{i,j} \ \geq\ FC3_j \ \forall\, j$$

$$9)\ \sum_{i=38}^{50} P_{i,j} \ \geq\ FC4_j \ \forall\, j$$

$$10)\ \sum_{i=51}^{63} P_{i,j} \ \geq\ FC5_j \ \forall\, j$$

Figure 4.0 Model Inputs and Outputs

Figure 5.0 below shows the five constraints of Model 4 that reflect the produced products and how they have to meet the demand of each quarter. Due to the fact that Model 4 does not allow the manufacturing or delivery of products in an adjacent quarter, we have taken the formulation of constraint 5) from model 5 and apportioned it into the five single quarters. By taking the sum from 'i equal to the first week of quarter' to n and assigning the respective week numbers to n, the algorithm goes from one week to the other and constantly sums up when the next week has started. With this notion, we guarantee that, although the demanded products have to be produced in the same quarter, we still can chose in which week the products shall be built.

$$6)\ \sum_{i=1}^{n} P_{i,j} \ \geq \sum_{i=1}^{n} D_{i,j} \ \ \forall\, j,\ n = 1, \dots, 13$$

$$7)\ \sum_{i=14}^{n} P_{i,j} \geq \sum_{i=14}^{n} D_{i,j}\ \forall j,\ n = 14, \dots, 26$$

$$8)\ \sum_{i=27}^{n} P_{i,j} \geq \sum_{i=27}^{n} D_{i,j}\ \forall j,\ n = 27, \dots, 39$$

$$9)\ \sum_{i=40}^{n} P_{i,j} \geq \sum_{i=40}^{n} D_{i,j}\ \forall j,\ n = 40, \dots, 52$$

$$10)\ \sum_{i=53}^{n} P_{i,j} \geq \sum_{i=53}^{n} D_{i,j}\ \forall j,\ n = 53, \dots, 65$$

Figure 5.0 Model Inputs and Outputs

The full algebraic notation representing the models of the above-mentioned, including all constraints as well as the model differences can be found in appendix section A.

## 4.2. Assumptions

Several assumptions were made to account for the three influential burdened costs which are procurement, residual inventory, and production costs. Assumptions were also made based on input from the Company analyst for the production minimum and maximum capacities.

First, we calculated the fixed order costs of \$130 by taking 5% of the sum of the BOM costs of all the parts that are ordered over the five quarters. They were then divided by the total number of purchase orders over the same period. The 5% fixed order cost rate was provided by the Company's production planning analyst as an average figure for the Company. Furthermore based on input from the subject matter expert, the model was based on the assumption that there would be unlimited storage space in the plant and regional facilities, resulting in a fixed inventory carrying rate of 5%. As the inventory storage capacity cannot be exceeded, no additional costs are incurred to expand or ramp up the warehousing capacities. The inventory carrying cost is calculated as 5% of the sum of the BOM costs of all parts. In models 4,5 and 6 the inventory cost will be fixed at ~\$4m due to the fixed product demand, in reality the demand will be constantly changing as the model progresses through time. Overall production was provided at a rate of 10%. We calculated the production fixed cost of \$260 by taking 10% of the sum of the BOM value for all parts ordered, divided by the total products forecast for the 5 quarters and then divided again by time to build each product in hours (120) to arrive at a fixed cost per production hour. A production scaling factor of 120% of the fixed production cost is $260, and was assumed for production ramping, to account for overtime, training and additional resources. [14]This scaling factor is applied to all production builds that exceed the minimum capacity constraint previously mentioned and result in fixed cost of $312 per production hour.

We arrived at a minimum production capacity constraint of 9 by calculating 85% which is the average weekly build count, and then the equation multiplies this number by 3, because it takes three weeks to build one product. The average weekly build count is calculated by dividing the total products built per quarter by the number of weeks in a quarter, then rounding the number to the nearest integer value. This number then is used to set the minimum production constraint based on production slots and by the number of products that can be built at any given week. The maximum production constraint on the other hand is artificial and provides a simple upper limit/bound of 40 product builds per 3 weeks; this prevents the solver making all products in the same week to minimize the order costs.

These three fixed costs equate to the total operations burden of 20%. By using a percentage of total resource values it is assumed that the Company's operations will naturally scale to the forecasted demand. A burdened rate of 30% is normal for similar companies; however the Company's property portfolio, large cash reserves and zero debt enable lower burdened rates than industry average.

| **Assumption** | **Value** |
|---|---|
| Fixed order/procurement cost | $130 |
| Fixed inventory holding rate | 5% of BOM |
| Fixed production cost | $260 or $312 |
| Minimum production capacity | 9 |
| Maximum production capacity | 40 |

Figure 6.0 Model Assumptions

## 4.3. Creating Optimization Model in MS Excel with OpenSolver

Throughout this process the various Excel-functions that were introduced in the class were utilized to the fullest extent, as well as other formulas to depict our model in a spreadsheet and make it possible to solve. Formulas in the spreadsheet included the SUMPRODUCT function, the TRANSPOSE function to transform vertical rows into horizontal columns and vice versa, the OFFSET and INDEX functions to choose specific values out of matrices like our product forecast, IF and OR functions to describe the cases of deciding respectively exceeding production capacities, as well as the name manager to assign names to certain areas in order to ease the use of handling with the data in the formula bar.

Furthermore we used colored cell backgrounds and explanation texts above the respective sections in the Excel file to make constraints and specific changes clear and comprehensible for every team member. The file itself contains 11 tabs, by which every single tab illustrates a necessary intermediate step to set up the model with sufficient variables, constraints, inputs and outputs. The initial data comprises information about the parts used per product (bill of materials - BOM), the lead times for each product and the amount of parts that are available in stock, as provided by the Company. We were furthermore given the demand forecast for the 13 products for the following five quarters on a weekly basis. Using this data, we were able to set up further tabs with information about the point when parts need to be ordered to arrive at the plant in the same week as it is demanded. This was crucial in order to implement the Just-in-Time approach. Another tab, the 'Cost Tracker', is summarizing each week the purchase order costs per part, the procurement costs as a fixed price that always occurs if parts ordered, inventory costs as a percentage of the purchasing costs per part and the manufacturing costs for each product. This sum represents the total operating costs and their sum over all weeks which serve as the objective function cell of our model-tab that is going to be minimized by the solver.

The model tab itself was designed in order to provide the highest possible overview and to imply the underlying logic as intuitively as possible. The model's name and characteristics are stated in the first row over multiple columns so that it is the first element that attracts the attention. The space at the left edge of the spreadsheet contains all the parameters and the objective function cell. To the right of this section is the decision variable matrix and below this matrix is followed by the build forecast constraints on a quarterly basis. The build constraints that determine the production costs are located right below and the weekly based product constraints are attached to their right. A graph representing the total costs and the purchase order costs located at the lower left edge completes the spreadsheet.

We created the main model in a way that allows being slightly modified in order to produce 3 different scenarios for benchmarking purposes and to determine which scenarios are optimal and most applicable to the practicality of the problem. The scenarios only differ in the way the demand forecast is being satisfied. All the other constraints and the objective function maintain the same and are valid for all the scenarios. The first scenario depicts the initial and main model 5 that allows the preproduction of products already in weeks that do not consist to the quarter in which the product demand actually occurs. Nevertheless, the product demands need to be met by the week they become valid. The results of Scenario 1/Model 5 serve as our benchmark to compare the other solutions against.

Scenario 2/Model 4 is expanding the forecast building constraints and hence the optimal solution space is further restricted. In this model products can only be built and delivered in the same quarter as their demands becomes valid. Furthermore, the orders still have to be met in every week within each quarter. However, this still allows products to be fabricated before the demand becomes valid, as long as it is in the same quarter. This scenario is expected to be more expensive since there will be a need to order parts more often and the savings in possible inventory costs are outweighed by the higher order costs. [13]

Evaluating Model 6, it is realized that it is the most unrestricted scenario. In Model 6 only the total product count demands on a quarterly basis need to be met, regardless of the delivery demand in actual weeks within the quarter. So a product that is actually demanded in week 3 of a quarter can also be built in week 6 of the same quarter if that leads to lower costs. This scenario only describes the ideal case with the lowest theoretical costs. In all actuality, it is not feasible in reality, because demands need to be met; Model 6 is our lowest cost benchmark that is assimilated to be close to other models as possible.

In addition to the optimization models being resolved by the solver, the Economic Order Quantity (EOQ) formula was incorporated in order to determine the optimal order quantity for each part that has reduced the accumulated costs. Figure 7.0 describes the basic form of the equation with D representing the annual demand per part; C reflecting the purchasing costs for each part, S symbolizes the fixed costs of procurement (order costs), i signifies the inventory burdened costs at 5% of the purchase costs per part and Q refers to the optimal order quantity. Hence, to determine the optimal order quantity the equation was manipulated with respect to Q and ultimately solving for Q results in an optimal order quantity Q* as seen in Figure 7.0.

The results of calculating Q* for each part is available in the spreadsheet in the tab Order Optimizer. However, unfortunately the team was unable to integrate the specific order quantities into the spreadsheet model; since there would be a need to create macros with the VBA function of excel due requirement of IF statements in the inventory management section [8]. This impact would make the constraints non-linear. The optimal order quantities can nevertheless be used as a foundation for further development of the model in the future and can provide insight into the potential savings for additional optimization beyond JIT inventory management.

$$Total\ Annual\ Costs = DC + \frac{D}{Q}S + \frac{Q}{2}C_i$$

$$Q = \sqrt{\frac{2DS}{C_i}}$$

Figure 7.0: Base form of EOQ model

# 5. Analysis

## 5.1. Model Findings/Results

We experienced several iterations to the model to arrive at the optimal build schedule as it relates to the forecast through a five quarter projection. Overall cost savings realized by optimizing the ordering process is approximately $2-3M. This is a fairly substantial reduction that is predicated on efficiently ordering materials in a JIT fashion, clearly defining the hours of production, the time that the manufacturing process is in production, labor hours and overtime requirements, minimizing cash expenditures, and the burden costs. Practically speaking Model 5 was found to be optimal vs Model 4, as expected Model 6 provided the lowest cost as a benchmark. The inputs and outputs of the spreadsheet to Model 5 can be found in appendix section B. The decision variables that are emphasized pertain to the optimized production schedule; in our model the decision variables are also an output. The weekly demand and total forecast are model inputs. Outputs for the ordering requirements and schedule are for the model scenarios are noted in Figure 8.0.

| Model | Scenario |
|---|---|
| 4 | Cannot build in prior quarters |
| 5 | Can build in prior quarters |
| 6 | No delivery constraints - low cost benchmark |

Figure 8.0 Model Decoder for Charts

## 5.2. Total Operating Cost

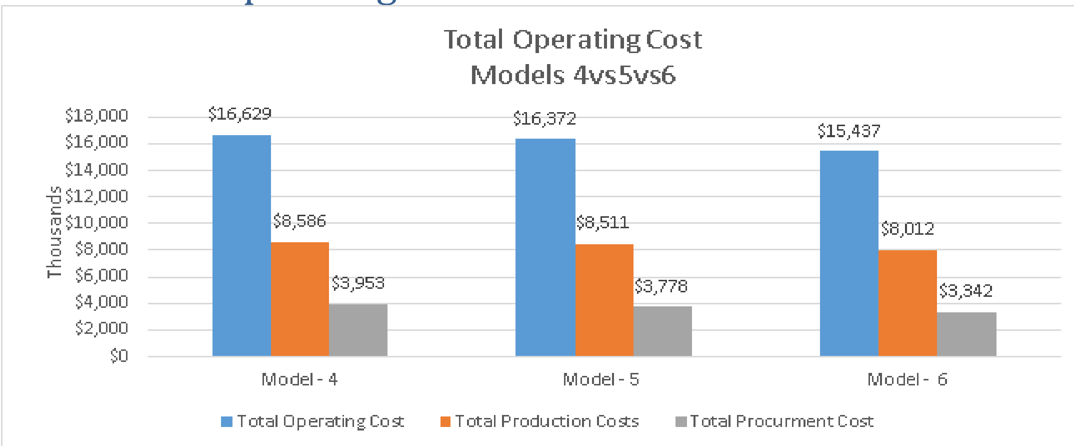


Figure 9.0 Total Operating Costs

The chart represented in Figure 9.0 displays the total costs for each model over the 5 forecast quarters. The total operating cost is a sum of the total production cost, total procurement cost and total inventory carrying cost. Given that the product demand forecast is the same for all models, the total part requirement will also be equal, which in turn will make the inventory holding cost fixed across all three models at ~$4M for a JIT system. As such, inventory holding cost is not displayed in the chart above, but it is included in the TOC value.

Model 6 provides the lowest total costs at ~$15.4M, which is the benchmark for minimum cost which was computed by removing the delivery constraints from the ILP optimization. Model 5 is the proposed build schedule from the model and results in a total cost of $16.372M over 5 quarters. This scenario establishes the delivery demand constraint being accumulated over the 5 quarters provides ~$260k in savings when compared with Model 4's total cost of $16.63M. Model 5 essentially allows the production plan to pre-build products in previous quarters to meet future demand, this is advantageous if the firm wants to maximize the production utilization but adds a certain amount of risk to holding capital in finished goods, if a forecast is inaccurate or and order is cancelled. By adding a reasonable maximum production capacity, the firm can minimize this risk. Model 4's limitation results in products that can only be built in the quarter they are ordered, which practically will cost the Company more but could be an attractive plan if the firm focuses on quarterly expenses.  This mentality is advantageous if the expenses in the quarter are reflected and revenue is recognized.  Model 4 also reduces the risk that forecasting errors will result in the firm holding capital in a finished product if the forecast is inaccurate or the order is cancelled.

## 5.3. Weekly Operating Costs

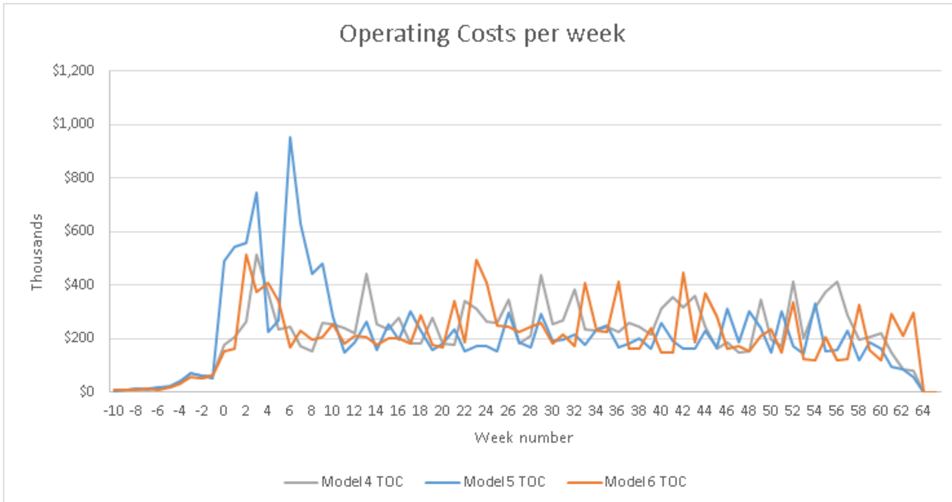


Figure 10.0 Operating cost per week

The chart in Figure 10.0 presents the cash flow out for the operating costs on a weekly basis. Model 5 shows a heavily front loaded cash flow in Q1, and a lower cash flow in Q5. If the forecast was not limited to 5 quarters (closed loop or continuous/wrapping) and had demand both before and after the forecast period, the shape of the graph would be smoother. Cash flow is an important metric for the firm; minimizing the maximum of funds extended could be an alternative objective or used in a multi-objective LP as a weighted decision variable. This output allows the firm to forecast the expenses and have an indicator that would allow for making more informed business decisions.

## 5.4. Optimal Order Quantity

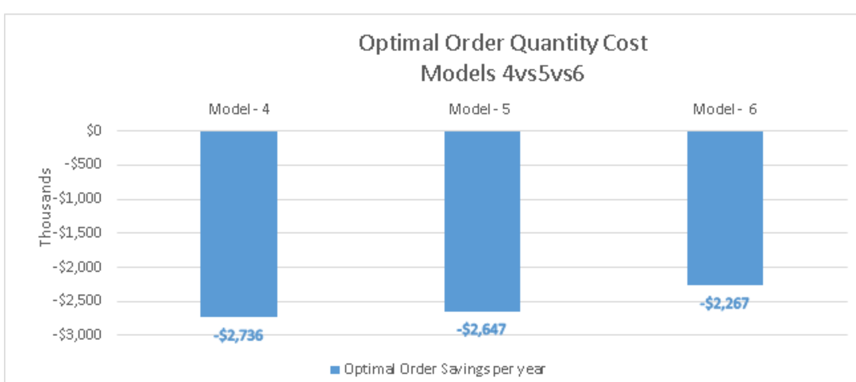


Figure 11.0 Optimal Order Quantity Cost

**Product Scheduling and Industry Optimization**

An examination represented in Figure 11.0 provides insight into the potential savings per year to the procurement costs if the optimal order quantity number is implemented as a minimum order quantity. The EOQ model is non-linear in nature and requires further expansion of the model to understand the impact of this method to cash flow and inventory holding costs. To estimate the potential savings from deviating from a pure JIT model, factoring in the cost of procuring a part ($130), and inventory holding cost of 5%, we derived the total annual cost equation and applied the resultant equation on a part by part basis. By subtracting the sum of the total annual costs for the JIT method and the EOQ method, the conclusion was drawn that there is ~$2.2-2.7m in savings to procurement expenses by developing the model further. Model 4 was found to be the least optimal in JIT part ordering, while Model 6 is the most profitable of the three models due to the lack of delivery constraints.

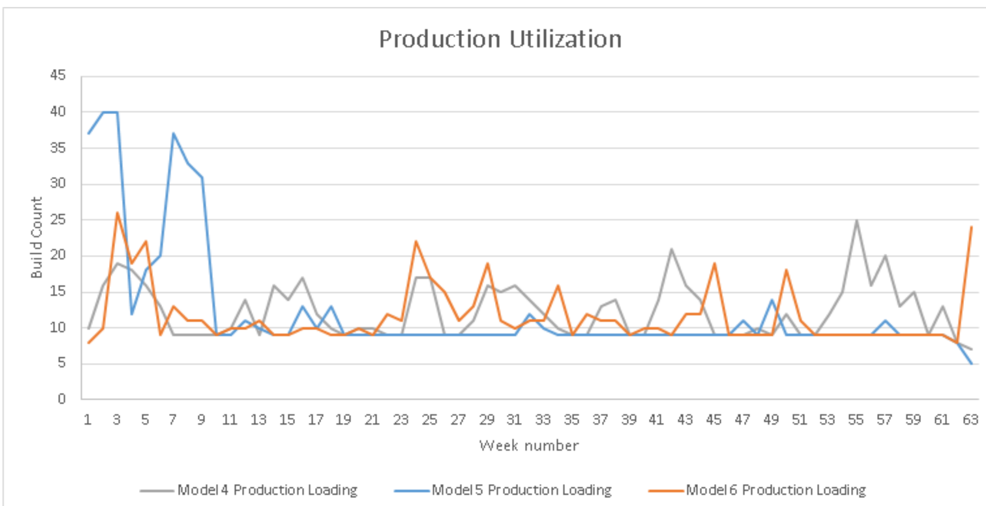


Figure 12.0 Production Utilization

The production utilization embodied in Figure 12.0 is the weekly number of products being built for each scenario modeled. A minimum production utilization constraint insures that at least 85% of the average weekly forecast is produced each week. The constraint in this case has been rounded down to 3 build starts per week, and it takes 3 weeks to build a product, therefore, the constraint is 9 units. An assumption of max capacity of 40 units per week has been set as the upper limit. When utilized correctly, the user can empirically determine optimal min and max constraints in order to meet business objectives for production planning purpose. This process could assist in answering the question as to whether the current production facility is adequate to meet future forecasts. Correctly setting the constraint allows for more stable production rates, but at a higher cost. If a model is found infeasible for a given minimum utilization constraint it may indicate that there is a need to change the production facilities capability.

# 6. Further Steps/Work

Considerations have been made with regard to the next steps which take aim at further defining the inputs and the parameters of the orders. Streamlining the ordering per the projections is an integral piece of the next steps. Adding more detailed actual costs for labor rates, the fluctuation of materials, and the impacts that lead times have would be a secondary step to gain further insights into how the model can optimize the process. Having a more detailed understanding of how ordering certain bulk materials versus JIT can assist with cost savings, especially if the materials that are necessary, take up little space and do not require substantial commitments of cost which could make the model more realistic. Having a more dynamic representation of the costs associated with building the equipment will reflect true costs which prove what budgets are necessary and what additional funds may be needed or can be reduced. The following items are actions for further development of the model beyond the scope of is paper:

- Implement EOQ as minimum order quantity to optimize order frequency to leverage better quote rates based on volume and procurement costs.
- Lead time sensitivity analysis: Focus procurement on which lead times to renegotiate lead times for/find a second source…etc. based on impact to cash flows and production planning.
- Implement a Multi-objective ILP where the user can define the weights for TOC, cash flow, production capacity...etc., would provide further value and add complexity to the model.
- Adding a “cash flow turns” variable or constraint based on the optimal order quantity.
- Implement a real time date/period tracking system, allow for updated forecast and ability to modify the model as time advances.

The development of the model has proven to have validity with the analyst that has been involved with the project. Further steps have already been discussed and are planned to be evaluated and curtailed to meet the Company’s needs to ultimately assist in making operational decisions which can have a significant impact on costs for the Company’s future.

# 7. Conclusion

In conclusion, our research and design of the ILP has delivered a useful tool that the Company can utilize to provide flexibility for the analysts to experiment with different operating constraints and scenarios which will enable better business decisions. A model utilizing a scenario where the products can be built in prior quarters will save the Company from extending additional costs and liabilities, with some additional risk of holding capital in finished goods if forecasts are erroneous. Implementing a model where the customer demand is aligned to the optimal build schedule, would further reduce costs; however this is not practical in today's market and customer driven ecosystem. Predicated on this notion, utilizing Model 5 would be the most appropriate option for evaluating a real-time and more dynamic model that can be manipulated to meet the customer demand.

Creating the models and evaluating all three respectively, the team was able to arrive at the desired conclusion which was to create an optimal build schedule using ILP methodologies to minimize the costs that are inefficiently spent in the current process. This goal was achieved by utilizing Excel Solver and MRP planning which meets the needs of the Company and has provided the team with a valuable tool for future pursuits. Model 5 has delivered the lowest Total Operating Costs (TOC) which result in an approximate savings of over $300K for a 5 quarter period. It was determined that JIT planning does not provide the lowest order costs because of, but not limited to, higher expedience shipping expenses and missing the opportunity of volume discount ordering. Introducing EOQ to the equation results in an approximate $2-3M savings as it relates to procurement costs for the projected timeframe with the assumption of fixed order costs.

# 8. Acknowledgements

Our team would like to sincerely thank Dr. Anderson and Dr. Lim for their continued provisions and guidance throughout the creation and iterations that were necessary to develop the models that have been generated throughout the term. The feedback was greatly appreciated and has not only led to an improvement in the current process for the Company but it has provided the team with instruments to be utilized in future pursuits academically and in our respective careers. We would like to express our gratitude for you all of your time, consideration, and patience.

Also, we would like to thank the analyst of the Company for which we have worked closely with the project on for their time, involvement, and the access to the system; so that we could generate the tool

that we have created for the various models. Without this opportunity, we would not have had the chance to take we have learned into practice, for which we are grateful. It is our hope and desire that the model is implemented and used to make the process more streamlined and efficient.

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# 10. Appendices

## 10.1. Appendices - A

**ALGEBRAIC MODEL FOR #4**

Changes versus Model 5: Add constraints 6),7),8),9),10)

$$min \sum_i OC * O_i + \sum_i \sum_p IHR * C_p * PD_{i,p} + \sum_i \sum_j P_{i,j} * MFC_1 * \frac{1}{3} H_j + \sum_i \sum_j (P_{i,j} - P_{i\,min}) * (MFC_2 - MFC_1) * \frac{1}{3} H_j * X_i$$

Subject to:

$$1) \sum_j P_{i,j} \leq P_{i\,min} + X_i * M \;\forall\, i \;[Additional\ Manufacturing\ Fixed\ Costs\ on\ or\ off]$$

$$2) \sum_j P_{i,j} \geq P_{i\,min} \;\forall\, i \;[Amount\ of\ produced\ products\ must\ exceed\ the\ minimum\ production\ capacity]$$

$$3) \sum_j P_{i,j} \leq P_{i\,max} \;\forall\, i \;[Amount\ of\ produced\ products\ can\ not\ exceed\ the\ maximum\ production\ capicity]$$

$$4) \sum_p \sum_j OH_{p,i,j} + \sum_p \sum_j TO_{p,i,j} - \sum_p \sum_j P_{i,j} * PR_{p,i,j} = \sum_p \sum_j OH_{p,i+1,j} \;\forall\, i \;[Stockblanace]$$

$$5) \sum_j TO_{p,i-Lp,j} - \sum_j OH_{p,i,j} = \sum_p \sum_j PR_{p,i,j} * P_{i,j} \;\forall\, i, L_p [Reordering\ of\ To-Order\ parts]$$

$$6) \sum_{i=1}^{n} P_{i,j} \geq \sum_{i=1}^{n} D_{i,j} \;\forall\, j,\ n = 1, \ldots, 13\ [Amount\ of\ produced\ products\ has\ to\ meet\ the\ Demand\ of\ quarter\ 1]$$

$$7) \sum_{i=14}^{n} P_{i,j} \geq \sum_{i=14}^{n} D_{i,j} \;\forall\, j,\ n = 14, \ldots, 26\ [Amount\ of\ produced\ products\ has\ to\ meet\ the\ Demand\ of\ quarter\ 2]$$

$$8) \sum_{i=27}^{n} P_{i,j} \geq \sum_{i=27}^{n} D_{i,j} \;\forall\, j,\ n = 27, \ldots, 39\ [Amount\ of\ produced\ products\ has\ to\ meet\ the\ Demand\ of\ quarter\ 3]$$

$$9) \sum_{i=40}^{n} P_{i,j} \geq \sum_{i=40}^{n} D_{i,j} \;\forall\, j,\ n = 40, \ldots, 52\ [Amount\ of\ produced\ products\ has\ to\ meet\ the\ Demand\ of\ quarter\ 4]$$

$$10) \sum_{i=53}^{n} P_{i,j} \geq \sum_{i=53}^{n} D_{i,j} \;\forall\, j,\ n = 53, \ldots, 65\ [Amount\ of\ produced\ products\ has\ to\ meet\ the\ Demand\ of\ quarter\ 5]$$

$$11)\ P_{i,j} \geq 0 \;\forall\, i, j\ [Non-Negativity]$$

$$12)\ P_{i,j} \in N_0\ [Integer]$$

$$13)\ X_i \in \{0,1\} \forall\, i\ [\ Binary\ Variable]$$

Where

$OC = Order\ Costs$

$IHR = Inventory\ Holding\ Rate$

$C_p = Costs\ per\ part\ p$

$PD_{i,p} = Parts\ delivered\ per\ part\ p\ in\ week\ i$

$P_{i,j} = Amount\ of\ produced\ products\ j\ in\ week\ i$

$MFC_1 = Manufacturing\ Fixed\ Costs\ 1$

$MFC_2 = Manufacturing\ Fixed\ Costs\ 2$

$P_{i\,min} = Minimum\ Production\ Capacity\ in\ week\ i$

$P_{i\,max} = Maximum\ Production\ Capacity\ in\ week\ i$

$H_j = Hours\ production\ per\ product\ j$

$X_i = Binary\ Variable\ responsible\ for\ additional\ Manufacturing\ Fixed\ Costs$

$M = Big\ number$

$O_i = Orders\ in\ week\ i$

$PR_{p,i,j} = Part\ p\ required\ to\ meet\ demand\ for\ product\ j\ in\ week\ i$

$OH_{i,j} = On\ hand\ parts\ for\ product\ j\ in\ week\ i$

$TO_{i,j} = To\ order\ parts\ for\ product\ j\ in\ week\ i$

$L_p = Lead\ Time\ for\ part\ p$

$D_{i,j} = Demand\ per\ product\ j\ in\ week\ i$

$p \in \{1, \ldots, 1349\} = Parts$

$i \in \{1, \ldots, 65\} = Weeks$

$j \in \{1, \ldots, 13\} = Products$

**ALGEBRAIC MODEL FOR #5**

Changes for Model 4: Erase Constraints 2),5)6),7); Erase $Y_i, BOM_{pj}, Z_{ip}$
Changes for Model 6: Erase constraints 2),5),6),7); Erase $D_{ij}, Y_i, BOM_{pj}, Z_{ip}$

$$min \sum_i OC * O_i + \sum_i \sum_p IHR * C_p * PD_{i,p} + \sum_i \sum_j P_{i,j} * MFC_1 * \frac{1}{3} H_j + \sum_i \sum_j (P_{i,j} - P_{i\,min}) * (MFC_2 - MFC_1) * \frac{1}{3} H_j * X_i$$

Subject to:

$$1) \sum_j P_{i,j} \le P_{i\,min} + X_i * M \; \forall \, i \; [Additional\ Manufacturing\ Fixed\ Costs\ on\ or\ off]$$

$$2) \sum_j P_{i,j} \le O_i + Y_i * M \; \forall \, i \; [Amount\ of\ produced\ products\ can\ not\ exceed\ the\ amount\ of\ orders\ ]$$

$$3) \sum_j P_{i,j} \ge P_{i\,min} \; \forall \, i \; [Amount\ of\ produced\ products\ must\ exceed\ the\ minimum\ production\ capacity]$$

$$4) \sum_j P_{i,j} \le P_{i\,max} \; \forall \, i \; [Amount\ of\ produced\ products\ can\ not\ exceed\ the\ maximum\ production\ capicity]$$

$$5) \sum_{i=1}^{n} P_{i,j} \ge \sum_{i=1}^{n} D_{i,j} \; \forall \, j, \; n = 1, \ldots, 63 \; [Amount\ of\ produced\ products\ must\ meet\ the\ demand]$$

$$6) \sum_j BOM_{p,j} * D_{i,j} \le Z_{i-Lp,\,p} * M \; \forall \, i, p \; [Amount\ of\ required\ parts\ can\ not\ exceed\ the\ amount\ of\ ordered\ parts]$$

$$7) \; PD_{i,p} = \sum_j D_{i,j} * BOM_{p,j} \; \forall \, i, p \; [Amount\ of\ delivered\ parts\ must\ equal\ the\ required\ parts]$$

$$8) \sum_p \sum_j OH_{p,i,j} + \sum_p \sum_j TO_{p,i,j} - \sum_p \sum_j P_{i,j} * PR_{p,i,j} = \sum_p \sum_j OH_{p,i+1,j} \; \forall \, i \; [Stockblanace]$$

$$9) \sum_j TO_{p,i-Lp,j} - \sum_j OH_{p,i,j} = \sum_p \sum_j PR_{p,i,j} * P_{i,j} \; \forall \, i, L_p [Reordering\ of\ To-Order\ parts]$$

$$10) \; P_{i,j} \ge 0 \; \forall \, i, j \; [Non-Negativity]$$

$$11) \; P_{i,j} \in N_0 \; [Integer]$$

$$12) \; X_i, \; Y_i, \; Z_{i,p} \in \{0,1\} \forall \, i, p \; [Binary\ Variable]$$

Where

$OC = Order\ Costs$
$IHR = Inventory\ Holding\ Rate$
$C_p = Costs\ per\ part\ p$
$PD_{i,p} = Parts\ delivered\ per\ part\ p\ in\ week\ i$
$P_{i,j} = Amount\ of\ produced\ products\ j\ in\ week\ i$
$MFC_1 = Manufacturing\ Fixed\ Costs\ 1$
$MFC_2 = Manufacturing\ Fixed\ Costs\ 2$
$P_{i\,min} = Minimum\ Production\ Capacity\ in\ week\ i$
$P_{i\,max} = Maximum\ Production\ Capacity\ in\ week\ i$
$D_{i,j} = Demand\ per\ product\ j\ in\ week\ i$
$H_j = Hours\ production\ per\ product\ j$
$X_i = Binary\ Variable\ responsible\ for\ additional\ Manufacturing\ Fixed\ Costs$
$Y_i = Binary\ Variable\ resposnible\ for\ ordering\ in\ week\ i$
$M = Big\ number$
$O_i = Orders\ in\ week\ i$
$BOM_{p,j} = Amount\ of\ parts\ p\ in\ product\ j$
$Z_{i,p} = Binary\ Variable\ responsible\ for\ ordering\ part\ p\ in\ week\ i$
$PR_{p,i,j} = Part\ p\ required\ to\ meet\ demand\ for\ product\ j\ in\ week\ i$
$OH_{i,j} = On\ hand\ parts\ for\ product\ j\ in\ week\ i$
$TO_{i,j} = To\ order\ parts\ for\ product\ j\ in\ week\ i$
$L_p = Lead\ Time\ for\ part\ p$
$p \in \{1, \ldots, 1349\} = Parts$
$i \in \{1, \ldots, 65\} = Weeks$
$j \in \{1, \ldots, 13\} = Products$

**ALGEBRAIC MODEL FOR #6**

Changes versus Model 5: Add constraints 6),7),8),9),10); Add $FC1_j$, $FC2_j$, $FC3_j$, $FC4_j$, $FC5_j$ to variable description

$$min \sum_i OC * O_i + \sum_i \sum_p IHR * C_p * PD_{i,p} + \sum_i \sum_j P_{i,j} * MFC_1 * \frac{1}{3} H_j + \sum_i \sum_j (P_{i,j} - P_{i\,min}) * (MFC_2 - MFC_1) * \frac{1}{3} H_j * X_i$$

Subject to:

$$1) \sum_j P_{i,j} \le P_{i\,min} + X_i * M \;\forall\, i \;[Additional\ Manufacturing\ Fixed\ Costs\ on\ or\ off]$$

$$2) \sum_j P_{i,j} \ge P_{i\,min} \;\forall\, i \;[Amount\ of\ produced\ products\ must\ exceed\ the\ minimum\ production\ capacity]$$

$$3) \sum_j P_{i,j} \le P_{i\,max} \;\forall\, i \;[Amount\ of\ produced\ products\ can\ not\ exceed\ the\ maximum\ production\ capicity]$$

$$4) \sum_p \sum_j OH_{p,i,j} + \sum_p \sum_j TO_{p,i,j} - \sum_p \sum_j P_{i,j} * PR_{p,i,j} = \sum_p \sum_j OH_{p,i+1,j} \;\forall\, i \;[Stockblanace]$$

$$5) \sum_j TO_{p,i-Lp,j} - \sum_j OH_{p,i,j} = \sum_p \sum_j PR_{p,i,j} * P_{i,j} \;\forall\, i, L_p [Reordering\ of\ To-Order\ parts]$$

$$6) \sum_{i=1}^{11} P_{i,j} \ge FC1_j \;\forall\, j \;[Amount\ of\ produced\ products\ has\ to\ meet\ the\ Forecast\ of\ quarter\ 1]$$

$$7) \sum_{i=12}^{24} P_{i,j} \ge FC2_j \;\forall\, j \;[Amount\ of\ produced\ products\ has\ to\ meet\ the\ Forecast\ of\ quarter\ 2]$$

$$8) \sum_{i=25}^{37} P_{i,j} \ge FC3_j \;\forall\, j \;[Amount\ of\ produced\ products\ has\ to\ meet\ the\ Forecast\ of\ quarter\ 3]$$

$$9) \sum_{i=38}^{50} P_{i,j} \ge FC4_j \;\forall\, j \;[Amount\ of\ produced\ products\ has\ to\ meet\ the\ Forecast\ of\ quarter\ 4]$$

$$10) \sum_{i=51}^{63} P_{i,j} \ge FC5_j \;\forall\, j \;[Amount\ of\ produced\ products\ has\ to\ meet\ the\ Forecast\ of\ quarter\ 5]$$

$$11) P_{i,j} \ge 0 \;\forall\, i,j \;[Non-Negativity]$$

$$12) P_{i,j} \in N_0 \;[Integer]$$

$$13) X_i \in \{0,1\} \forall\, i \;[\,Binary\ Variable]$$

Where

$FC1_j = Forecasts\ for\ quarter\ 1\ per\ product\ j$

$FC2_j = Forecasts\ for\ quarter\ 2\ per\ product\ j$

$FC3_j = Forecasts\ for\ quarter\ 3\ per\ product\ j$

$FC4_j = Forecasts\ for\ quarter\ 4\ per\ product\ j$

$FC5_j = Forecasts\ for\ quarter\ 5\ per\ product\ j$

$OC = Order\ Costs$

$IHR = Inventory\ Holding\ Rate$

$C_p = Costs\ per\ part\ p$

$PD_{i,p} = Parts\ delivered\ per\ part\ p\ in\ week\ i$

$P_{i,j} = Amount\ of\ produced\ products\ j\ in\ week\ i$

$MFC_1 = Manufacturing\ Fixed\ Costs\ 1$

$MFC_2 = Manufacturing\ Fixed\ Costs\ 2$

$P_{i\,min} = Minimum\ Production\ Capacity\ in\ week\ i$

$P_{i\,max} = Maximum\ Production\ Capacity\ in\ week\ i$

$H_j = Hours\ production\ per\ product\ j$

$X_i = Binary\ Variable\ responsible\ for\ additional\ Manufacturing\ Fixed\ Costs$

$M = Big\ number$

$O_i = Orders\ in\ week\ i$

$PR_{p,i,j} = Part\ p\ required\ to\ meet\ demand\ for\ product\ j\ in\ week\ i$

$OH_{i,j} = On\ hand\ parts\ for\ product\ j\ in\ week\ i$

$TO_{i,j} = To\ order\ parts\ for\ product\ j\ in\ week\ i$

$L_p = Lead\ Time\ for\ part\ p$

$p \in \{1, \dots, 1349\} = Parts$

$i \in \{1, \dots, 65\} = Weeks$

$j \in \{1, \dots, 13\} = Products$

## 10.2. Appendices - B

**Model 5 build schedule - decision variables output.**

| | Q1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Week | | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Product1 | | 2 | | | 1 | | | | | | | | |
| Product2 | 1 | 1 | | | | 1 | | | | | | | |
| Product3 | 2 | | | | 1 | | | | | | | | |
| Product4 | | | | | | | 1 | 1 | | | | | |
| Product5 | | | | 1 | 1 | 1 | | | | | | | |
| Product6 | | | | | 3 | | | | | | | | 3 |
| Product7 | | | | 1 | | | | | | 2 | | | |
| Product8 | | | | | 3 | | | | | | | 2 | |
| Product9 | 3 | | | | | | | | | | | | |
| Product10 | 11 | | | | | | | | | | | | |
| Product11 | 20 | | | | | | | | | | | | |
| Product12 | | | | 7 | | | | 4 | | 2 | 5 | | |
| Product13 | | | | | | | 25 | | | | | | |

| | Q2 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| Product1 | | | | 1 | | | | | | | 4 | | |
| Product2 | | | | | | | | | | | 1 | | |
| Product3 | | | | | | | 1 | | | | | | 1 |
| Product4 | | | 1 | | | | | | | 1 | | | |
| Product5 | | | | | | 3 | | | | | | | |
| Product6 | | | | | | | | 2 | | | | | |
| Product7 | | | | | | | | 3 | | | | 1 | |
| Product8 | | 2 | | | | | | | | | | | |
| Product9 | | | 3 | | | | | | | | | 2 | |
| Product10 | | | | | | | | | | | | | |
| Product11 | | | | | | | | | | | | | |
| Product12 | 4 | | 3 | | 5 | | | | 3 | | | | |
| Product13 | | | | | | | | | | | | | |

| | Q3 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| Product1 | | | | | | | | | | | | | |
| Product2 | | | | | | 2 | | | | | | | 2 |
| Product3 | | | | | 1 | | | | | | | 1 | |
| Product4 | | | | | | | | 2 | | | | 2 | |
| Product5 | | | | | | 2 | 3 | | | | | 1 | |
| Product6 | | | 1 | | | | | | | | 2 | | |
| Product7 | | 3 | | | | | | | | | | | |
| Product8 | | | | | 1 | | | | | | | | |
| Product9 | | | | | | | | | 2 | | | | |
| Product10 | | | | | | | | | | | | | |
| Product11 | | | | | | | | | | | | | |
| Product12 | 3 | | | 5 | 1 | | | | | 3 | | | 1 |
| Product13 | 2 | | | | | | | | 2 | | | | |

| | Q4 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 |
| Product1 | | | | | 1 | | | | | | | | |
| Product2 | | | | | | | | | 1 | | | | |
| Product3 | | | | | | | 2 | | | | | | |
| Product4 | | | | | | | | | | 2 | | | |
| Product5 | | | | | | | | | | | | | |
| Product6 | | | | | | | | | | | | | |
| Product7 | 2 | | | | | | | | | | | | |
| Product8 | | | | | 3 | | | | | | | 1 | |
| Product9 | | | | 2 | | | | | | | | | 2 |
| Product10 | | | 3 | | | | | | | | | | |
| Product11 | | | | | | | | | | | | | |
| Product12 | | | | | | 3 | | 6 | | | 1 | | 5 |
| Product13 | | 4 | | | | | | | | 5 | | | |

| | Q5 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 |
| Product1 | | | | | | | | | | | | 0 | 0 |
| Product2 | | | | | | | | | | | | 0 | 0 |
| Product3 | | | | | | | | | | | | 0 | 0 |
| Product4 | | | | | | | | | | | | 0 | 0 |
| Product5 | | | | | | 3 | | | | | | 0 | 0 |
| Product6 | | | | | 3 | | | | | | | 0 | 0 |
| Product7 | | | | | | 1 | | | 2 | | | 0 | 0 |
| Product8 | | 1 | | | | | 1 | | | | | 0 | 0 |
| Product9 | | | | 1 | | | | | | | | 0 | 0 |
| Product10 | | | | | | | | | | | | 0 | 0 |
| Product11 | | | | | | | | | 2 | | | 0 | 0 |
| Product12 | 1 | | | | | 1 | | 3 | 1 | | | 0 | 0 |
| Product13 | | | 7 | | | | | | | | | 0 | 0 |

**Product demand - used to build constraints for when to build a tool....ie delivery**

| Product Demand | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Product1 | | | | | 1 | | 1 | | | | | | |
| Product2 | | | 1 | 1 | | | | | | | | | |
| Product3 | | | 2 | | | | | | | | | | |
| Product4 | | | | | | | | | 1 | 1 | | | |
| Product5 | | | | | | 1 | 1 | 1 | | | | | |
| Product6 | | | | | | | 1 | 2 | | | | | |
| Product7 | | | | | | | | | | | 1 | 1 | 1 |
| Product8 | | | | | | | | 2 | 1 | | | | |
| Product9 | | | 3 | | | | | | | | | | |
| Product10 | | | | | 2 | 1 | | | | | | | |
| Product11 | | | | | | | | 5 | | | | | |
| Product12 | | | | | | 1 | 2 | 2 | 2 | 2 | 2 | 2 | |
| Product13 | | | | | | | | | 5 | 4 | | | |

| | Q2 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| Product1 | | | | 1 | | 1 | | | | | | | |
| Product2 | | | | | | | | 1 | | | | | |
| Product3 | | | 1 | | | | | | 1 | | | | |
| Product4 | | | | | | | | | | | 1 | 1 | |
| Product5 | | | | | | | | | 1 | 2 | | | |
| Product6 | | | | | | | | 2 | 1 | | | | |
| Product7 | | | | | | | | | | | 3 | | |
| Product8 | | | | | | | 3 | | | | | | |
| Product9 | | | | | 1 | 1 | 1 | | | | | | |
| Product10 | | | | | | | | 1 | | 1 | | | |
| Product11 | | 1 | | 1 | | 1 | | | | | | | |
| Product12 | | | | | 2 | 3 | 3 | 3 | 3 | 3 | | | |
| Product13 | | | | | | | | | | 4 | | | 4 |

| | Q3 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| Product1 | | | | | 1 | | 1 | | | | | | |
| Product2 | | | | | | | | 2 | | | | | |
| Product3 | | | | | | | | | 2 | | | | |
| Product4 | | | | | | | | | | 2 | | | |
| Product5 | | | | | | | | 1 | 1 | | 1 | | |
| Product6 | | | | | 1 | 1 | 1 | | | | | | |
| Product7 | | | | 1 | 1 | 1 | | | | | | | |
| Product8 | | | | | | | | 2 | | | | | |
| Product9 | | | | | | | | | | | | 3 | |
| Product10 | | | | | | | | | | | | 3 | |
| Product11 | | | | | | | | 1 | 1 | 1 | 1 | 1 | |
| Product12 | | | | 6 | | | | 6 | | | | | |
| Product13 | | | | | 2 | 2 | 2 | | 2 | 2 | 1 | | |

| | Q4 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 |
| Product1 | | 1 | 1 | | | | | | | | | | |
| Product2 | | | | | 2 | | | | | | | | |
| Product3 | | | | | | 1 | | | | | | | |
| Product4 | | | | | 1 | 1 | | | | | | | |
| Product5 | | | | | | | | | 3 | | | | |
| Product6 | | | | | | | | | | | 2 | | |
| Product7 | | | | | | | | | | 3 | | | |
| Product8 | | | | | | | 1 | 1 | 1 | | | | |
| Product9 | | | | | | | | 2 | 1 | | | | |
| Product10 | | | | | | | 1 | 2 | | | | | |
| Product11 | | 2 | 2 | | | | | | | | | | |
| Product12 | | | | | | | | 3 | 3 | 2 | 3 | | |
| Product13 | | | | 5 | | | | | | | | 5 | |

| | Q5 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 |
| Product1 | | 1 | | | | | | | | | | | |
| Product2 | | | 2 | | | | | | | | | | |
| Product3 | | | | 2 | | | | | | | | | |
| Product4 | | | | | 1 | 1 | | | | | | | |
| Product5 | | | | | | | | 3 | | | | | |
| Product6 | | | | | | | 1 | 1 | 1 | | | | |
| Product7 | | | | | | | | | | 1 | 1 | 1 | |
| Product8 | | | | | | | 2 | | 1 | | | | |
| Product9 | | | | | 1 | 2 | | | | | | | |
| Product10 | | | | | | 1 | 1 | 1 | | | | | |
| Product11 | | | | | | | | 1 | 1 | 1 | 1 | 1 | |
| Product12 | | | | | | | | 3 | 3 | 3 | 3 | 2 | |
| Product13 | | | | | 4 | 3 | | | | | | | |

**Product forecast totals**

| Forecast | Q1 | Q2 | Q3 | Q4 | Q5 |
|---|---|---|---|---|---|
| Product1 | 2 | 2 | 2 | 2 | 1 |
| Product2 | 2 | 1 | 2 | 2 | 2 |
| Product3 | 2 | 2 | 2 | 1 | 2 |
| Product4 | 2 | 2 | 2 | 2 | 2 |
| Product5 | 3 | 3 | 3 | 3 | 3 |
| Product6 | 3 | 3 | 3 | 2 | 3 |
| Product7 | 3 | 3 | 3 | 3 | 3 |
| Product8 | 3 | 3 | 2 | 3 | 3 |
| Product9 | 3 | 3 | 3 | 3 | 3 |
| Product10 | 3 | 2 | 3 | 3 | 3 |
| Product11 | 5 | 3 | 5 | 4 | 5 |
| Product12 | 13 | 17 | 12 | 11 | 14 |
| Product13 | 9 | 8 | 11 | 10 | 7 |

Part Ordering output - tells when and how many parts to order based on the optimal build schedule - parts are named A, AA, AAA....etc. Row 1 contains the week count, with 1 being the start of the 1st quarter. Due to lead times for the parts we projected orders back into the past to ensure delivery prior to the build starts.

## Product Scheduling and Industry Optimization

| | A | AD | AE | AF | AG | AH | AI | AJ | AK | AL | AM | AN | AO | AP | AQ | AR | AS | AT | AU | AV | AW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | -12 | -11 | -10 | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | A | | | | | | | 1869 | 172 | | 1074 | 400 | 176 | 3496 | 598 | | 368 | 690 | 92 | 171 | 552 |
| 3 | AA | | | | | | | | 22 | 12 | | 99 | 25 | 12 | 329 | 56 | | 34 | 65 | 4 | 9 |
| 4 | AAA | | | | | | 4 | 2 | | | 4 | 2 | | | | | | | 3 | | |
| 5 | AAB | | | | | | | | | | 4 | 2 | | | 4 | 2 | | | | | |
| 6 | AAC | | | | | 4 | 2 | | | 4 | 2 | | | | | | | 3 | | | |
| 7 | AAD | | | | | | | | | | 4 | 2 | | | 4 | 2 | | | | | |
| 8 | AAE | | | | | 2 | 2 | | | 3 | 2 | | | | | | | 3 | | | |
| 9 | AAF | | | | | | | | | 8 | 8 | | | 10 | 4 | | | | | | |
| 10 | AAG | | | | | 10 | 8 | | | 11 | 4 | | | | | | 4 | | | 4 | |
| 11 | AAH | | | | | | | | | | | | 8 | 8 | | | 10 | 4 | | | |
| 12 | AAI | | | | | | | | | | 4 | 2 | | | 4 | 2 | | | | | |
| 13 | AAJ | | | | | | | | | | 4 | 4 | | | 8 | 2 | | | | | |
| 14 | AAK | | | | | | 4 | 2 | | | 7 | 2 | | | | | | 2 | 3 | | 2 |
| 15 | AAL | | | | | | 4 | 2 | | | 7 | 2 | | | | | | 2 | 3 | | 2 |
| 16 | AAM | | | | | | | | | | 48 | 48 | | | 60 | 24 | | | | | |
| 17 | AAN | | | | | | | | | | 4 | 2 | | | 4 | 2 | | | | | |
| 18 | AAO | | | | | | | | | | 68 | 4 | | | 4 | 4 | | | | | |
| 19 | AAP | | | | | 2 | 2 | | | | 2 | | | | | | | | | | |
| 20 | AAQ | 2 | 2 | | | 3 | 2 | | | | | | | 3 | | | | | | | |
| 21 | AAR | | | | | | | | | | 32 | 18 | | | 9 | 6 | | | | | |
| 22 | AAS | | | | | | | | | 12 | 6 | | | 3 | 6 | | | | | | |

... Customer Demand | Forecast | Product_Requirements | Parts_Consumed | Parts_to_Order | Order_tracker | Parts_delivered | Inventory_on_Hand | C ...