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Measuring Warehouse Efficiency With Weight Restricted DEA

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MEASURING WAREHOUSE EFFICIENCY

With Weight Restricted DEA

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to

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Abstract

It is increasingly recognized that logistics can play a vital role in affecting organization performance and competitiveness. The purpose of this study is to examine one particular aspect of logistics: warehouse performance.

Data Envelopment Analysis (DEA) is used to measure the relative operating efficiencies of 58 warehouses. In particular, the dual version of the CCR input-oriented multiplier model, involving two inputs and five outputs, is utilized for this purpose. Use of this model allows us to consider the impact on relative operating efficiencies of imposing additional constraints on three of the output weights appearing in the model.

The impact on the relative efficiencies of some warehouses was dramatic. Almost half the warehouses originally identified as being efficient were no longer so. The analysis illustrates the value of using constraints on factor weights to better characterize the particular application area under consideration.

Section 1: Introduction

Historically, the area of logistics has not received the level of academic attention reserved for marketing, finance, and operations management. In fact, it would be accurate to say that organizations themselves did not place the emphasis on logistics that was deserved.

There are many possible explanations for this historical state of affairs. Along with logistics, even operations management did not receive the proper level of attention after World War II. Perhaps it was the lack of glamour associated with the areas of operations and logistics. For many people a warehouse represented a facility where material entered, was stored, and finally shipped. Nothing too exciting about that.

Well, the times have changed. Organizations now recognize the critical role logistics can play in affecting overall organization performance in an increasingly competitive global environment. Technological developments in logistics operations have dramatically accelerated. For example, many of today's modern warehouses are filled with new and expensive technology designed to achieve lower costs, increased accuracy, and higher levels of customer service.

This study concentrates on one area of logistics that of warehouse performance. Modern warehouses can be very complex and expensive facilities. Their performance can no longer be ignored or taken for granted.

More specifically, the purpose of this study is to re-evaluate the performance of a collection of warehouses using the tool of weight restricted Data Envelopment Analysis (DEA).

Data Envelopment Analysis was introduced in 1978 by Charles, Cooper, and Rhodes (CCR) as a method for evaluating the relative efficiencies of decision-making units (DMUs) having a similar mission (Charnes et al., 1979). One of the strengths of DEA is that it readily allows for the analysis of organizations using multiple inputs to produce multiple outputs. Another strength is that DEA does not require an explicit representation of the production relationship linking inputs to outputs. Finally, DEA uses a mathematical technique called Linear Programming that is well-established and relatively easy to implement.

There are, in fact, many versions of DEA models which have been developed since the seminal work of CCR (Charnes et al., 1994). The particular model selected will often depend on the purpose of the study. This study will utilize a model referred to as the dual version of the CCR input-oriented multiplier model. In order to better understand the purpose of this study, it would be useful to examine this model in somewhat greater detail.

Toward this end, let us consider a collection of n DMUs, each using m different inputs to produce s different outputs. The quantity of output j $(j=1,\ldots,s)$ produced by DMU k $(k=1,\ldots n)$ will be denoted by $y_{j,k}$. Similarly, $x_{i,k}$ will denote the quantity of input i $(i=1,\ldots,m)$ used by DMU k.

Let the letter o represent the particular DMU under consideration. A weighted sum of outputs for DMU o is defined to be

$$\sum_{j=1}^{s} u_j y_{jo}$$

where u_j is referred to as the multiplier or factor weight corresponding to output j for DMU o. We require that $u_j \geq 0$ for $j=1,...,s$.

Similarly, a weighted sum of inputs for DMU o is defined to be

$$\sum_{i=1}^{m} v_i x_{io}$$

where v_i is referred to as the multiplier or factor weight corresponding to input i for DMU o. We also require that $v_i \geq 0$ for $i=1,...,m$.

The efficiency of DMU o is then defined to be the ratio of the weighted sum of outputs to the weighted sum of inputs. Letting h_o denote the efficiency of DMU o, we have that

$$h_o = \frac{\sum_{j=1}^{s} u_j y_{jo}}{\sum_{i=1}^{m} v_i x_{io}}$$

The problem faced by DMU o can then be stated as follows:

Model A: To find that set of output weights u_j $(j=1,...,s)$ and input weights v_i $(i=1,...,m)$

which

$$\text{maximize } h_o = \frac{\sum_{j=1}^{s} u_j y_{jo}}{\sum_{i=1}^{m} v_i x_{io}} \tag{1}$$

subject to the constraints

$$\frac{\sum_{j=1}^{s} u_j y_{jk}}{\sum_{i=1}^{m} v_i x_{ik}} \leq 1 \ (k=1,\ldots,n) \tag{2}$$

$$v_i \geq 0 \ \ (i=1,\ldots m) \tag{3}$$

$$u_j \geq 0 \ \ (j=1,\ldots s) \tag{4}$$

The left side of the constraints given by (2) can be interpreted as the efficiency of DMU k, where the factor weights u_j and v_i used to assess that efficiency are the factor weights selected by DMU o to maximize its own efficiency score. In other words, the left-hand side of (2) can be interpreted as the efficiency of DMU k from the perspective of DMU o.

Thus, the constraints given by (2) require that the efficiency of DMU k using DMU o's factor weights cannot exceed 1. And this must be true for each $k=1,\ldots,n$. In particular, DMU o's efficiency also cannot exceed 1. It should also be clear from the required non-negativity of the factor weights that the efficiency of DMU o must be at least 0.

The problem as stated above is non-linear in nature, the solution to which can be difficult to obtain. A linearized version of the problem will lend itself more readily to solution. Such a linear version can be obtained by making two changes to the above problem formulation.

The first change involves simply rewriting the constraints given by (2) as follows:

$$\sum_{j=1}^{s} u_j y_{jk} \leq \sum_{i=1}^{m} v_i x_{ik} \quad (k=1,\ldots,n)$$

or

$$\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0 \quad (k=1,\ldots,n)$$

The second change requires making the additional assumption that

$$\sum_{i=1}^{m} v_i\, x_{io} = 1$$

The linearized version of the problem faced by DMU o can now be stated as follows:

Model B: To find that set of output weights u_j $(j=1,\ldots,s)$ and input weights v_i $(i=1,\ldots,m)$ which

$$maximize\ w_o = \sum_{j=1}^{s} u_j y_{jo}$$

subject to the constraints

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0 \qquad (k=1,\ldots,n)$$

$$v_i \geq 0 \ (i=1,\ldots m)$$

$$u_j \geq 0 \ (j=1,\ldots s)$$

Model B actually represents a set of *n* separate linear programs, one for each DMU. Thus, the final result of the analysis is an efficiency score and a set of non-negative factor weights for each of the *n* DMUs under consideration. And, to repeat an observation made earlier, the efficiency score for each DMU must be a number between 0 and 1, with higher values representing higher levels of efficiency.

The above examination of the dual version of the CCR input-oriented multiplier model sheds light on a potential weakness of DEA. A DMU, in its effort to maximize its efficiency score, would emphasize those outputs where it was relatively successful and de-emphasize those outputs where it was relatively unsuccessful. In the extreme case, a DMU might very well place all of its weight on just a single output.

The possible result of all of this is that a DEA analysis could result in multiple DMUs achieving an efficiency score of 1. The phenomenon of multiple DMUs being viewed as efficient has the potential to undermine the usefulness of the results since it may not allow us to sufficiently discriminate between DMUs in terms of their performance.

Accordingly, we seek a means to further discriminate between the multiple DMUs achieving an efficiency score of 1. The method chosen involves imposing additional constraints on factor weights that more closely reflect the underlying characteristics of the application area under consideration. By doing this, we will be able to achieve a more meaningful discrimination between the performance of DMUs.

The first objective is to reformulate a warehouse performance model originally developed by Hackman and Frazelle (1994) using the dual version of the CCR input-oriented multiplier model.

This lays the foundation for the second objective, which is to impose additional constraints on the output weights that reflect the nature of the warehouse environment, and then examine the impact of these additional constraints on the resulting efficiency scores (Roll & Golany, 1993; Roll et al., 1991).

Section 2: Literature Review

The literature of warehouse models can be categorized in a number of ways. One approach is to use the following three categories: Strategic Planning Models, Warehouse Design Models, and Operations Planning Models (Cormier & Gunn, 1992; Hollingsworth, 1995; Warehouse Education and Research Council, 1998).

1. <u>Strategic Planning Models</u>:

One type of strategic planning model treats a warehouse or warehouses as part of an overall logistics network or supply chain. Such a model is concerned with the design of an entire distribution network. Key questions addressed by such a model would include the following:

- □ How many warehouses should there be in the distribution network?
- □ What should be the general geographic location of these warehouses?
- □ What should be the storage and throughput capacity of these warehouses?
- □ How should customers be allocated to the warehouses?
- □ How should products be allocated to the warehouses?

The benefits of improved distribution network design can be the following:

- □ Reduced transportation costs.
- □ Improved customer service.
- □ Reduced inventory.
- □ Reduced warehouse costs.

- Reduced warehouse costs.

Network design models use modeling tools such as mathematical programming and computer simulation.

Another type of warehouse model that can be placed in the strategic planning category involves the selection of a specific site for the warehouse. This goes beyond the identification of the best geographic area to locate a warehouse. Much more specific decision factors must now be considered. These include the following:

- Land and construction costs.
- Specific tax and utility rates.
- Labor availability and wage structure.
- Zoning requirements.
- Transportation factors.

Site selection models use a variety of multi-criteria techniques, an example being the Analytic Hierarchy Process (AHP).

Warehouse cost models represent a third type of model that can be placed in the strategic planning category. Simply put, this type of model answers the question: how much does it cost to operate your warehouse?

Such cost models enable management to identify high-cost operations and then carry out efforts to reduce these costs. They can also be used to investigate the possibility of using third-party providers.

Four main categories of costs are usually considered. These are: direct handling, direct storage, operating administration, and general administration. The modeling tool involves financial analysis using spreadsheets.

One frustration with the traditional approach to financial modeling of warehouse costs is that it is often unable to isolate the costs associated with specific activities. For example, while the overall cost of handling may turn out to be unexpectedly high, it may not be possible to identify the specific activities that led to this result.

An alternative to the traditional approach that addresses this issue is activity-based costing (ABC). As its name suggests, ABC identifies costs at the individual activity level. While holding great potential, the complexity of implementing it has limited its acceptance.

2. Warehouse Design Models:

Warehouse design models address the internal arrangement of the facility as well as its overall size and shape.

Many functions must be performed in fulfilling the mission of a warehouse. These functions include receiving, inspection, storage, order picking, sorting, shipping, and many others. At the highest level, warehouse design involves identifying where these various functions must be located in order to optimize warehouse operations. The modeling approaches to accomplish this include relationship diagraming, mathematical programming, and computer simulation.

At a more detailed level, the internal arrangement of the facility deals with issues such as:

- The number and orientation of storage racks;
- The number and size of aisles;
- The allocation of scarce storage space among competing uses.

Again, mathematical programming and computer simulation are two of the main modeling tools used for carrying out this more detailed design analysis.

A Model of Warehouse Efficiency Using Data Envelopment Analysis

Hackman and Frazelle utilized Data Envelopment Analysis (DEA) to evaluate the relative efficiencies of 58 warehouses (1994). The Hackman-Frazelle model of warehouse operations involved two inputs and five outputs. They are defined as follows:

Inputs:

1. Equipment:

Equipment was measured in terms of the replacement cost of storage and material-handling equipment. This included the replacement cost of three types of equipment: vehicles, small parts storage systems, and conveyer systems. Replacement costs were used in lieu of actual costs to allow consistent comparisons among firms in their database.

2. Labor

Labor was measured in terms of annual labor hours. Annual labor hours was the sum of two components: direct and indirect labor hours. Direct labor hours involved labor hours spent performing operations related to storing and shipping. Indirect hours included those hours spent on maintenance, supervision, and management. Hours spent on other indirect operations such as security, customer satisfaction, traffic, and personnel were not included. Many of the warehouses in their database did not have accurate estimates of hours used by function. Therefore, to achieve somewhat consistent comparisons among firms in their database, the total number of hours was calculated from the headcounts used in each function, assuming 2000 hours per person per year.

Outputs:

1. Annual broken-case lines shipped, denoted by BC.
2. Annual full-case lines shipped, denoted by FC.
3. Annual pallet lines shipped, denoted by P.
4. Accumulation, denoted by A.
5. Storage, denoted by S.

Some additional explanation of the nature of warehouse outputs used in the Hackman-Frazelle model is needed. An order may be comprised of a single line or of multiple lines, where a line denotes one or more of a specific item being ordered. Total lines shipped were disaggregated into the three standard categories of broken-case, full-case, and pallet lines because the resources required to complete the associated picking activities are typically different in type and amount.

Accumulation is a measure of the resources required to bring together the individual lines picked to form the orders that are to be shipped. Accumulation is measured as Annual lines picked - Annual orders shipped. Thus, a single-line order would have an accumulation index equal to zero. On the other hand, the accumulation index for an order would increase as the number of lines comprising the order increased. Finally, storage is a measure of the resources required to store inventory in the warehouse. Hackman and Frazelle used the CCR input-oriented envelopment model to generate their efficiency scores. That model is given as follows:

$$\min_{\Theta, \lambda_k} \Theta \text{ for DMU o}$$

subject to:

$$\sum_{k=1}^{n} \lambda_k x_{ik} \leq \Theta \; x_{io} \; \textit{for each } i=1,...,m$$

$$\sum_{k=1}^{n} \lambda_k y_{jk} \leq y_{jo} \; \textit{for each } j=1,..., s$$

$$\lambda_k \geq 0 \; \textit{for each } k=1,...,n$$

where

$x_{i,k}$ = *the quantity of the ith input for DMU k.*

$y_{j,k}$ = *the quantity of the jth output for DMU k.*

DMU o represents the particular DMU for which the above linear program is solved. Thus, the above linear program is solved *n* times, once for each DMU.

The result of the above analysis will be, for each DMU, an efficiency score θ between 0 and 1 and non-negative values of $\lambda_1,\ldots,\lambda_n$.

The above DEA model does not lend itself to the imposition of additional constraints that reflect the special characteristics of the inputs and outputs for the particular application under consideration.

Consequently, a different formulation, the dual version of the CCR input-oriented multiplier model, was selected to investigate the impact of additional constraints. It should be emphasized that, without any additional constraints, both models will generate the same efficiency scores for the DMUs under consideration.

Section 3: Methodology

It would be useful at this point to restate the purpose of this study, which is to examine warehouse performance utilizing the tool of Data Envelopment Analysis. To accomplish this purpose the study was divided into two parts.

Part I of the study involved reformulating the original Hackman-Frazelle model in the form commonly referred to as the dual version of the CCR input-oriented mutliplier model, hereinafter referred to as the CCR_D-I model. The CCR_D-I model is a linear programming model that explicitly uses input and output weights in its formulation.

Part II of the study involved modifying the CCR_D-I model by introducing additional constraints on three of the output weights. These additional constraints reflect specific characteristics of the warehouse environment not captured in the original model. The results of the two models were then compared in order to better understand the impact of introducing additional constraints into the model.

Part I: The CCR_D-I Model Applied to Warehouse Performance

The following notation will be used in the CCR_D-I model applied to warehouse performance:

Let $x_{i,k}$ denote the quantity of the *ith* input for DMU k, where $i=1,2$ *and* $k=1,...,58$.

In particular,

$x_{1,k}$ = the replacement cost of equipment for DMU *k*,

$x_{2,k}$ = total labor hours for DMU *k*.

Let $y_{j,k}$ denote the quantity of the jth output for DMU k, where $j=1,...,5$.

In particular,

$y_{1,k}$ = Annual broken case lines shipped by DMU *k*,

$y_{2,k}$ = Annual full-case lines shipped by DMU *k*,

$y_{3,k}$ = Annual pallet lines shipped by DMU *k*,

$y_{4,k}$ = Storage for DMU *k*,

$y_{5,k}$ = Accumulation for DMU *k*.

The CCR_D-I model is then given as follows:

$$\underset{v_i,\, u_j}{Maximize} \qquad w_o = \sum_{j=1}^{5} u_j y_{jo}$$

subject to:

$$\sum_{i=1}^{2} v_i x_{io} = 1$$

$$\sum_{j=1}^{5} u_j y_{jk} - \sum_{i=1}^{2} v_i x_{ik} \leq 0 \qquad (k=1,...,58)$$

$$v_i \geq 0 \;\; (i=1,\, 2)$$

$$u_j \geq 0 \;\; (j=1,...5)$$

Additional Terminology:

w_o denotes the efficiency of DMU o,

v_i is referred to as the weight (or multiplier) for input i,

u_j is referred to as the weight (or multiplier) for output j.

It should be emphasized that the above problem represents 58 separate linear programs, one for each DMU. The result of each individual program will be values for w_o, v_i $(i=1, 2)$ and u_j $(j=1, \ldots, 5)$.

The AMPL version of the above CCR_D-I model, the data, and the results of the calculations can be found in Appendix A of this report. Note that the warehouse inputs and outputs are abbreviated in the AMPL model as follows:

RC = Replacement cost for equipment

LH = Total labor hours

BC = Broken-case lines shipped

FC = Full-case lines shipped

P = Pallet lines shipped

S = Storage

A = Accumulation

Part II: The CCR_D-I Model With Constraints on Output Weights

As stated earlier, Part II of the study involved modifying the CCR_D-I model by introducing additional constraints on three of the output weights. The constraints were of the following form:

$$u_1 \geq u_2 \geq u_3 \qquad (1)$$

Recall that:

Output 1 = Broken-case lines

Output 2 = Full-case lines

Output 3 = Pallet lines

The constraints imposed on the output weights represented by inequality (1) reflect the fact that the resources required to carry out broken-case, full-case, and pallet picking activities are usually different in type and amount. More specifically, the inequality (1) expresses the idea that the resources required to carry out broken-case picking activities are at least as great as those required to carry out full-case picking activities which, in turn, are at least as great as those required to carry out pallet picking activities.

The CCR_D-I model, now including the additional constraints given by (1), is as follows:

$$\underset{v_i,\, u_j}{Maximize} \quad w_o = \sum_{j=1}^{5} u_j y_{jo}$$

subject to:

$$\sum_{i=1}^{2} v_i x_{io} = 1$$

$$\sum_{j=1}^{5} u_j y_{jk} - \sum_{i=1}^{2} v_i x_{ik} \leq 0 \quad (k=1,\ldots,58)$$

$$u_1 \geq u_2 \geq u_3$$

$$v_i \geq 0 \quad (i=1, 2) \ .$$

$$u_j \geq 0 \quad (j=1,...,5)$$

The AMPL version of the above CCR_D-I model, the data, and the results of the calculations can be found in Appendix B of this report.

A comparison of the results from the two models is given in Chapter 4 of this report.

It would be useful at this point to discuss how the additional constraints given by inequality (1) were implemented using AMPL (Fourer et al., 1993).

Inequality (1) can be rewritten as the following two inequalities:

$$u_1 \geq u_2 \qquad (2)$$

$$u_2 \geq u_3 \qquad (3)$$

Inequality (2) can then be written as follows:

$$u_1 - u_2 \geq 0$$

or

$$u_1 - u_2 + 0u_3 + 0u_4 + 0u_5 \geq 0 \qquad (2a)$$

Inequality (2a) is in a form that lends itself to easy implementation in AMPL. This involves introducing a new parameter, call it a, which is indexed over the set Y of outputs and whose values are 1, -1, 0, 0, and 0.

Inequality (2a), expressed in AMPL, will look as follows:

sum {j in y} u[j] ∗ a[j] > = 0;

where a[1] = 1, a[2] = -1, a[3] = a[4] = a[5] = 0.

Using the same logic, inequality (3) can be written in AMPL as

sum {j inY} u[j] ∗ b[j] > = 0,

where now we've introduced a new parameter b indexed over Y and having values

b[1] = 0, b[2] = 1, b[3] = -1, b[4] = b[5] = 0.

The values of the parameters a and b will appear in the AMPL Data File as follows:

Param:	a	b	: =
BC	1	0	
FC	-1	1	
P	0	-1	
S	0	0	
A	0	0 ;	

Section 4: Analysis of Results

The table below compares the results of Model 1 with Model 2

DMU	Efficiency (Model 1)	Efficiency (Model 2)	Change in Efficiency (Model 2 - Model 1)	Rank (Model 1)	Rank (Model 2)	Change in Rank (Model 2- Model 1)
1	1	0.24	-0.76	1	46	45
2	0.36	0.36	0	53	36	-17
3	1	1	0	1	1	0
4	0.48	0.36	-0.12	47	36	-11
5	1	1	0	1	1	0
6	1	0.31	-0.69	1	39	38
7	0.51	0.18	-0.33	43	48	5
8	0.78	0.78	0	18	14	-4
9	0.78	0.43	-0.35	18	28	10
10	0.78	0.78	0	18	14	-4
11	1	1	0	1	1	0
12	0.64	0.56	-0.08	27	18	-9
13	0.46	0.15	-0.31	49	51	2
14	0.7	0.46	-0.24	24	25	1
15	0.82	0.8	-0.02	17	13	-4
16	0.69	0.55	-0.14	26	20	-6
17	0.97	0.94	-0.03	13	8	-5
18	0.41	0.14	-0.27	52	52	0
19	0.64	0.4	-0.24	27	30	3
20	0.49	0.18	-0.31	45	48	3
21	0.98	0.98	0	12	7	-5
22	0.13	0.13	0	57	53	-4
23	0.09	0.09	0	58	57	-1
24	0.18	0.09	-0.09	56	57	1
25	0.49	0.18	-0.31	45	48	3
26	0.29	0.11	-0.18	54	55	1
27	0.87	0.63	-0.24	16	17	1
28	1	0.87	-0.13	1	12	11
29	1	0.92	-0.08	1	9	8

DMU	Efficiency (Model 1)	Efficiency (Model 2)	Change in Efficiency (Model 2 - Model 1)	Rank (Model 1)	Rank (Model 2)	Change in Rank (Model 2- Model 1)
30	0.7	0.42	-0.28	24	29	5
31	0.99	0.89	-0.1	11	10	-1
32	0.77	0.39	-0.38	21	32	11
33	1	1	0	1	1	0
34	0.56	0.56	0	36	18	-18
35	0.55	0.55	0	38	20	-18
36	0.57	0.26	-0.31	33	44	11
37	0.48	0.12	-0.36	47	54	7
38	0.59	0.28	-0.31	32	42	10
39	0.57	0.28	-0.29	33	42	9
40	0.56	0.26	-0.3	36	44	8
41	0.63	0.24	-0.39	29	46	17
42	0.55	0.55	0	38	20	-18
43	0.46	0.4	-0.06	49	30	-19
44	0.55	0.29	-0.26	38	41	3
45	0.57	0.31	-0.26	33	39	6
46	0.55	0.45	-0.1	38	26	-12
47	0.53	0.32	-0.21	42	38	-4
48	0.62	0.44	-0.18	30	27	-3
49	0.74	0.54	-0.2	22	23	1
50	1	1	0	1	1	0
51	0.5	0.39	-0.11	44	32	-12
52	0.43	0.38	-0.05	51	34	-17
53	0.71	0.71	0	23	16	-7
54	1	1	0	1	1	0
55	0.89	0.89	0	15	10	-5
56	0.9	0.53	-0.37	14	24	10
57	0.6	0.37	-0.23	31	35	4
58	0.25	0.11	-0.14	55	55	0

TABLE 1

Analysis of Results: Comparing Model 1 to Model 2

1. Comparison of Efficiency Scores:

The imposition of constraints on three of the output weights had the effect of reducing, or leaving unchanged, the efficiency score of each DMU (warehouse).

The above result is to be expected. For the CCR_D-I model under consideration, each DMU faces the problem of selecting those values of factor weights that maximize its own efficiency, subject to those factor weights obeying certain constraints. Imposing additional constraints on three of the output weights will generally reduce (but certainly not expand) the possible choices of factor weights. Thus, efficiency scores calculated with the additional constraints in place will certainly be no higher than the original efficiency scores and may very well be lower.

Under Model 1, ten warehouses achieved efficiency scores of 1.0. Under Model 2, six of these warehouses retained their efficiency scores of 1.0 while four saw their scores decline, sometimes dramatically. This will be examined in greater detail in a later section.

Since each efficiency score in Model 2 is reduced (or at least unchanged) from its corresponding value in Model 1, the average efficiency score for Model 2 will be smaller than for Model 1. The results are as follows:

Model 1: Average efficiency score = 0.674

Model 2: Average efficiency score = 0.504

2. Calculation of Measures of Correlation:

As another method for comparing the efficiency scores of Model 1 with Model 2, it would be useful to calculate measures of correlation. Two measures were considered: the Pearson correlation coefficient r, and the Spearman correlation coefficient r_S.

A. Pearson Correlation Coefficient r:

The Pearson correlation coefficient r is a measure of the strength of the linear relationship between two numerical variables. Two variables are linearly related, if in a scatterplot, the points cluster around a straight line.

The Pearson correlation coefficient r ranges in value from -1 to 1. A value of $r=1$ indicates that all of the points fall exactly on a line with positive slope. If the points fall exactly on a line with negative slope then $r=-1$. The magnitude of r tells us how tightly the points cluster around the line.

For the two sets of efficiency scores under consideration, the Pearson correlation coefficient $r=0.808$ indicates a relatively strong linear relationship between the two sets of efficiency scores.

B. Spearman Correlation Coefficient r_S:

The Spearman correlation coefficient r_S is a measure of the degree to which the relationship between two numerical variables is monotonic. An increasing monotonic relationship is one where the value of y increases as the value of x increases. A decreasing monotonic relationship is one where the value of y decreases as the value of x increases.

The Spearman correlation coefficient is calculated by first replacing the actual data values with their corresponding ranks and then calculating a Pearson r for the ranked data. For this reason, r_s also ranges between -1 and 1.

The formula for r_s is given by

$$r_s = 1 - \left[6 \cdot \sum \frac{(d^2)}{n(n^2-1)} \right]$$

where n is the sample size and d is the difference for each pair of scores between the ranks of the two scores. Considering for a moment just the scores for Model 1, rankings for tied scores are calculated by averaging the ranks that would have been assigned to these tied values. The same approach would be used for dealing with tied scores for Model 2.

Why use a Spearman correlation coefficient? It has the advantage of being sensitive to a broader range of relationships than the Pearson coefficient r. Any linear relationship is monotonic, but not all monotonic relationships are linear. Thus, a low value for r would indicate a relatively weak linear relationship between two variables, but the corresponding value of r_s could be relatively high, indicating a strong monotonic relationship between the two variables.

Another reason for using the Spearman correlation coefficient r_s is that the magnitude of the Pearson correlation coefficient r can be greatly affected by outlying values. Outliers can cause r to either greatly increase or greatly decrease, depending upon where they appear in the scatterplot. One way to bring outliers under control is to use ranks rather than the raw data and then use r_s to calculate correlation.

In our particular situation, the value of the Pearson correlation coefficient r=0.808 indicates a relatively strong linear relationship between the two sets of efficiency scores. But it is also possible that this value of r overstates the strength of the linear relationship between the two sets of scores because of the number of DMUs achieving efficiency scores of 1.0 in both Model 1 and Model 2. If the strength of the positive linear relationship is not as great as seemingly indicated, is there at least evidence for a strong increasing monotonic relationship between the two sets of efficiency scores? A value of the Spearman correlation coefficient r_s=0.791 indicates this is the case.

3. <u>The DMUs Experiencing the Largest Changes in Efficiency</u>

Two DMUs stand out in terms of experiencing the largest changes in efficiency:

DMU	Efficiency	Efficiency	Δ Efficiency
#1	=1.0	=0.24	-0.76
#6	=1.0	=0.31	-0.69

It would be useful to understand how the imposition of the additional constraints on three of the output weights resulted in such large reductions in efficiency. For DMU #1, the analysis is as follows:

Outputs	Values of Outputs	Factor Weights Model 1	Factor Weights Model 2
BC	396	0.00000	0.00006
FC	2430	0.00041	0.00006
P	0	0.00000	0.00000
S	633	0.00000	0.00011
A	595	0.00000	0.00000

Under Model 1, output BC (broken-case lines shipped) where DMU #1 is a relatively low performer compared to other DMUs received a zero weight, meaning output BC made no contribution to the efficiency score of DMU #1. Meanwhile, output FC (full-case lines shipped), where DMU #1 is the highest performer among all DMUs, received a relatively large factor weight. It can also be seen from the above chart that the remaining three outputs, P, S, and A, make no contribution to DMU #1's efficiency score since the products of their respective output values and factor weights are all zero. Thus, DMU #1's efficiency score of 1.0 is due *solely* to output FC.

The impact of Model 2 is to increase the importance of output BC (where DMU #1 is relatively weak), while decreasing the importance of output FC (where DMU #1 is extremely strong). The end result is to only marginally increase the factor weight associated with BC, while significantly reducing the factor weight associated with FC. Of the remaining three outputs, only S (storage) made a small contribution to DMU #1's efficiency score. Consequently, the net result was to significantly reduce DMU #1's efficiency score.

For DMU #6, the analysis is as follows:

Outputs	Values of Outputs	Factor Weights Model 1	Factor Weights Model 2
BC	2695	0.00002	0.00006
FC	2205	0.00038	0.00006
P	0	0.00000	0.00000
S	317	0.00000	0.00009
A	4651	0.00002	0.00000

Compared to other DMUs, DMU #6 had the second-highest value of FC (full-case lines shipped), just below DMU#1 and significantly above all other DMUs. In terms of output BC (broken-case lines shipped), DMU #6 was relatively strong, with only 13 DMUs having larger values of output BC.

The dynamics at work which explain DMU #6's significant reduction in efficiency are similar to that of DMU #1. Under Model 1, output FC (where DMU #6 was a very high performer) received a relatively large factor weight. The remaining outputs all received very low or zero weights. Therefore, output FC contributed 0.84 to DMU #6's efficiency score of 1.0, with outputs BC and A contributing the rest.

As was true for DMU #1, Model 2 had the effect of significantly reducing the factor weight associated with output FC while only marginally increasing the factor weight associated with output BC. The factor weight associated with output A was reduced to zero, thus eliminating output A's contribution to DMU #6's efficiency score. Finally, the factor weight associated with output S marginally increased from zero to 0.00009, but the very small value of S (=317) resulted in S making only a small contribution DMU #6's efficiency score. Thus, the net result was to significantly reduce DMU #6's efficiency score.

4. <u>DMU Experiencing No Change In Efficiency</u>:

To provide a different perspective, it would be useful to examine a DMU that experienced no change in efficiency as a result of the imposition of additional constraints on output weights.

Such would be the case for DMU #3 which had an efficiency score of 1.0 in both models.

<u>DMU #3</u>

Outputs	Values of Outputs	Factor Weights (Model 1)	Factor Weights (Model 2)
BC	9700	0.00009	0.00010
FC	300	0.00041	0.00010
P	0	0.00169	0.00010
S	168	0.00000	0.00000
A	9048	0.00000	0.00000

Compared to other DMUs, DMU #3 had the third highest value for output BC while having a relatively low value for output FC. Model 1 assigned a relatively low weight to output BC while assigning a relatively high weight to output FC. Output BC contributed 0.87 to DMU #3's efficiency score of 1.0, with output FC contributing the remainder.

The impact of Model 2 is to increase the importance of output BC (where DMU #3 is relatively strong) while decreasing the importance of output FC (where DMU #3 is relatively weak). As was true for both DMU #1 and DMU #6, Model 2 results in just a marginal increase in the factor weight associated with output BC, while there is a large decrease in the factor weight associated with output FC. But this time, because of the relative sizes of outputs BC and FC, the net result is to leave DMU #3's efficiency score unchanged at 1.0.

In summary, for all three DMUs examined, the impact of Model 2 was to significantly decrease the factor weight associated with output FC while only marginally increasing the factor weight associated with output BC. For DMUs #1 and 6 which were both very strong in output FC, this had the effect of significantly reducing their efficiency scores. For DMU #1, the additional constraints imposed by Model 2 were in line with the relative importance of outputs BC and FC, so that there was no reduction in efficiency score.

5. <u>Examining DMUs Experiencing the Largest Change in Rank</u>:

Ranking was done as follows: first, the rank assigned to a DMU was inversely related to its efficiency (i.e., lower efficiency scores correspond to higher values of rank). Second, DMUs having the same efficiency score received the same rank. Finally, if 10 DMUs each had an efficiency score of 1 (and thus rank = 1), the DMU having the next highest efficiency score received a rank = 11.

As is clear from the results summarized on Page 23, the two DMUs experiencing the largest change in rank (DMUs #1 and #6) were also the two DMUs experiencing the largest decreases in efficiency when going from Model 1 to Model 2. But this phenomenon is not universally true. For example, DMU #25 experienced a relatively large decrease in efficiency equal to 0.31, but its change in rank was only 3 units. This is explained by the fact that DMU #25's efficiency score of 0.49 under Model 1 gave it a rank of 45, ahead of only 13 DMUs. While its efficiency score dropped to 0.31 under Model 2, the 13 DMUs below it were also experiencing reductions in efficiency. Thus, it is possible that a relatively large reduction in efficiency does not automatically translate into a relatively large reduction in rank.

6. Analysis of Output Weights:

A careful examination of the results for Model 2 (see Appendix B) reveals that, for 28 DMUs

$$\text{Model 2:} \qquad U_{BC} = U_{FC} = U_P \qquad (1)$$

This equal weighting did not occur for any DMU in Model (1). What might explain this phenomenon?

The table shown below provides a summary of the relationships between

U_{BC}, U_{FC}, and U_P in Model 1 for the 28 DMUs satisfying the equality given by (1) in Model 2:

Relationship Between U_{BC}, U_{FC}, and U_P	Number of DMUs
$U_{BC} < V_{FC} < V_P$	18
$U_{BC} < U_P < U_{FC}$	6
$U_{BC} = U_P < U_{FC}$	1
$U_{FC} < U_{BC} < U_P$	2
$U_{FC} = U_{BC} < U_P$	1

Table 2. Summary of the relationships between U_{BC}, U_{FC}, and U_P in Model 1 for the 28 DMUs satisfying (1) in Model 2.

Recall that for Model 2, the additional constraints imposed on three of the output weights were given by

$$U_{BC} \geq U_{FC} \geq U_P \qquad (2)$$

Thus, we see that for 18 of 28 DMUs, the weights placed on the three outputs in Model 1 are in complete conflict with the additional constraints represented by (2). For the remaining 10 DMUs, the weights placed on the three outputs in Model 1 are in partial conflict with (2).

It follows that for the 28 DMUs in question conformance to the additional constraints represented by (2) requires changes to the factor weights U_{BC}, U_{FC}. U_P such that $U_{BC} = U_{FC} = U_P$.

Section 5: Conclusions and Future Research

The results of this study illustrate the impact of imposing additional constraints on factor weights in the context of a data envelopment analysis approach to measuring warehouse efficiency. The additional constraints on factor weights, reflecting the unique characteristics of the application area under consideration, had a dramatic impact on at least some of the warehouses. In four cases, warehouses that were originally viewed as efficient lost that status as a result of explicitly taking into account the special characteristics of three of the warehouse outputs used in the model.

Future research could involve a more sophisticated set of constraints on factor weights, considering both input and output weights. This would require a better understanding of the technology and economics underlying warehouse operations, as well as the elicitation of factor weight constraints using experts in the field.

With regard to factor weights, another avenue of research could involve the impact of introducing "standards" into the model. This could be accomplished by selectively adding DMUs (warehouses) to the basic set of DMUs being studied, where these additional DMUs represent standards of excellence in some dimension of performance. Golany and Roll provide a model that illustrates how this additional information could be used to set bounds on factor weights.

Within the framework of the existing Hackman-Frazelle model the following additional questions could be examined:

- What is the impact of expanding the existing inputs to include warehouse size?
- What additional types of equipment should be considered?
- What would be the impact of desegregating labor hours into indirect and direct labor hours?
- What alternatives are there for measuring the amount of equipment utilized by a warehouse?

Another direction for future research would be to significantly alter the basic framework of the model. In particular, the outputs utilized by Hackman-Frazelle could be replaced by more conventional measures of warehouse performance. Such measures might include:

- Inventory turns per year
- Inventory accuracy
- Return rates due to inaccurate shipment
- Total cases picked
- Service level, etc.

The inputs would continue to include measures of labor, equipment, and warehouse size. The efficiency scores generated by a DEA analysis could then be regressed against a variety of factors reflecting warehouse characteristics and utilization of "best practices," which are supposedly correlated with warehouse performance. This would provide a potentially more meaningful analysis of factors influencing efficiency than was carried out in the original Hackman-Frazelle paper.

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Appendix A

Model 1: No Constraints on Weights

Model 1: No Constraints on Weights

```
set DMU_set;                                  # Set of DMUs
set X;                                        # Set of Inputs
set Y;                                        # Set of Outputs
set Factors;                                  # Set of Inputs & Outputs
param Data {DMU_set, Factors};                # Data indexed by DMU & Factor
param o symbolic;                             # DMU whose efficiency is being maximized

var v {X} >= 0;                               # Input weights indexed over the set X of inputs
var u {Y} >= 0;                               # Output weights indexed over the set Y of outputs

maximize Efficiency : sum {j in Y} u[j]*Data[o, j];
subject to Input_constraints: sum {i in X} v[i]*Data[o, i] = 1;
subject to Output_constraints {k in DMU_set} : sum {j in Y} u[j]*Data[k,j] - sum {i in X} v[i]*Data[k, i]
<= 0;
```

Data File for Model 1: No Constraints on Weights

```
set DMU_set :=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58;
set Factors := RC LH BC FC P S A;
param Data : RC LH BC FC P S A
:=

1 4.17 434 396 2430 0 633 595
2 4.00 400 5000 0 0 101 2817
3 3.70 200 9700 300 0 168 9048
4 1.60 150 19 300 56 1054 315
5 0.31 48 0 5 95 874 90
6 4.08 513 2695 2205 0 317 4651
7 12.65 1584 6598 1628 887 465 7847
8 0.40 574 2249 395 330 219 2669
9 1.86 508 2756 650 394 230 3145
10 0.90 968 4641 955 548 304 5085
11 0.23 306 1660 544 255 211 2075
12 0.45 196 968 164 110 220 1059
13 15.99 1112 4781 1261 605 457 5749
14 0.95 244 1526 258 163 190 1694
15 0.50 484 2534 563 319 228 2795
16 0.86 528 2365 424 322 248 2780
17 0.33 274 1745 381 218 225 2023
18 13.95 1212 4679 1058 611 366 5386
19 0.97 230 1248 211 151 213 1389
20 9.98 734 3891 783 435 357 4409
21 0.69 1174 4856 1019 634 337 5617
22 7.00 158 188 120 0 468 120
23 8.87 822 2489 0 0 263 44
24 10.13 428 887 213 83 541 566
25 2.57 128 422 211 70 315 590
26 4.16 218 422 211 70 315 580
27 0.54 27 8 198 0 421 152
28 0.59 23 10 238 0 489 187
29 0.54 18 6.5 179 0 415 143
30 1.06 65 811 255 0 158 897
31 0.62 22 10.6 220 0 482 172
32 1.33 100 856 426 0 258 1047
33 0.27 14 4.6 104 0 382 82
34 7.61 1286 17131 34 69 485 16078
35 0.75 70 1212 82 0 140 1044
36 1.50 92 336 357 7 273 550
37 16.50 612 179 581 469 1658 600
38 2.46 112 589 512 11 298 874
39 1.40 80 319 326 7 288 512
40 3.30 140 763 611 14 294 1091
41 3.00 232 985 836 37 348 1275
42 1.50 240 3000 0 0 332 2125
43 1.07 114 0 156 7 885 147
44 2.29 336 0 703 37 1424 691
45 1.53 214 0 476 25 1012 468
46 1.11 114 0 237 12 972 228
47 1.51 314 0 490 26 1133 482
48 1.08 118 0 319 17 923 313
49 0.50 64 667 152 0 210 570
50 4.20 1164 20000 0 0 255 12746
51 0.72 48 6 120 24 431 106
52 2.00 160 3 316 48 1312 295
53 0.11 50 270 0 0 103 176
54 0.70 80 2641 139 0 92 2748
55 1.50 250 5225 220 55 110 5448
56 1.80 182 2500 630 0 304 3112
57 2.60 158 1954 353 47 270 2090
58 3.20 342 164 432 0 444 336
;

set X := RC LH ;
set Y := BC FC P S A ;
```

Results for Model 1: No Constraints on Weights

DMU	EFF	V_{RC}	V_{LH}	U_{BC}	U_{FC}	U_P	U_S	U_A
1	1.00	0.17756	0.00060	0.00000	0.00041	0.00000	0.00000	0.00000
2	0.36	0.11347	0.00137	0.00007	0.00000	0.00106	0.00000	0.00000
3	1.00	0.13372	0.00253	0.00009	0.00041	0.00169	0.00000	0.00000
4	0.48	0.35049	0.00293	0.00000	0.00096	0.00171	0.00009	0.00000
5	1.00	0.00000	0.02083	0.00000	0.00000	0.00350	0.00076	0.00000
6	1.00	0.14079	0.00083	0.00002	0.00038	0.00000	0.00000	0.00002
7	0.51	0.01918	0.00048	0.00001	0.00009	0.00030	0.00000	0.00000
8	0.78	2.50000	0.00000	0.00035	0.00000	0.00000	0.00000	0.00000
9	0.78	0.06886	0.00172	0.00005	0.00033	0.00107	0.00000	0.00000
10	0.78	0.69047	0.00039	0.00017	0.00000	0.00000	0.00000	0.00000
11	1.00	0.12727	0.00317	0.00009	0.00062	0.00199	0.00000	0.00000
12	0.64	0.48547	0.00399	0.00025	0.00000	0.00360	0.00000	0.00000
13	0.46	0.00000	0.00090	0.00002	0.00009	0.00045	0.00000	0.00000
14	0.70	0.14221	0.00354	0.00011	0.00069	0.00222	0.00000	0.00000
15	0.82	0.22344	0.00184	0.00011	0.00000	0.00166	0.00000	0.00000
16	0.69	0.19243	0.00158	0.00010	0.00000	0.00143	0.00000	0.00000
17	0.97	0.38752	0.00318	0.00020	0.00000	0.00287	0.00000	0.00000
18	0.41	0.02264	0.00056	0.00002	0.00011	0.00035	0.00000	0.00000
19	0.64	0.14919	0.00372	0.00011	0.00072	0.00233	0.00000	0.00000
20	0.49	0.00000	0.00136	0.00002	0.00013	0.00068	0.00000	0.00000
21	0.98	1.44928	0.00000	0.00020	0.00000	0.00000	0.00000	0.00000
22	0.13	0.00000	0.00633	0.00013	0.00000	0.00108	0.00023	0.00000
23	0.09	0.04893	0.00069	0.00003	0.00000	0.00000	0.00006	0.00000
24	0.18	0.00000	0.00234	0.00004	0.00022	0.00117	0.00000	0.00000
25	0.49	0.00000	0.00781	0.00014	0.00075	0.00391	0.00000	0.00000
26	0.29	0.00000	0.00459	0.00008	0.00044	0.00229	0.00000	0.00000
27	0.87	1.50739	0.00689	0.00000	0.00370	0.00000	0.00027	0.00019
28	1.00	1.27852	0.01068	0.00000	0.00350	0.00623	0.00034	0.00000
29	1.00	0.23832	0.04841	0.00000	0.00206	0.01351	0.00115	0.00108
30	0.70	0.58757	0.00580	0.00023	0.00201	0.00174	0.00000	0.00000
31	0.99	0.00000	0.04545	0.00083	0.00293	0.01643	0.00069	0.00000
32	0.77	0.43149	0.00426	0.00017	0.00147	0.00128	0.00000	0.00000
33	1.00	3.40457	0.00577	0.00000	0.00410	0.00000	0.00150	0.00000
34	0.56	0.09855	0.00019	0.00003	0.00000	0.00011	0.00000	0.00000
35	0.55	0.56064	0.00828	0.00037	0.00016	0.00000	0.00065	0.00000
36	0.57	0.41519	0.00410	0.00016	0.00142	0.00123	0.00000	0.00000
37	0.48	0.00000	0.00163	0.00003	0.00016	0.00082	0.00000	0.00000
38	0.59	0.19041	0.00475	0.00014	0.00092	0.00297	0.00000	0.00000
39	0.57	0.45661	0.00451	0.00018	0.00156	0.00135	0.00000	0.00000
40	0.56	0.00000	0.00714	0.00013	0.00068	0.00357	0.00000	0.00000
41	0.63	0.18900	0.00187	0.00007	0.00065	0.00056	0.00000	0.00000
42	0.55	0.52934	0.00086	0.00016	0.00000	0.00000	0.00023	0.00000
43	0.46	0.79166	0.00134	0.00000	0.00095	0.00000	0.00035	0.00000
44	0.55	0.29022	0.00100	0.00000	0.00066	0.00000	0.00006	0.00000
45	0.57	0.44129	0.00152	0.00000	0.00100	0.00000	0.00009	0.00000
46	0.55	0.76736	0.00130	0.00000	0.00092	0.00000	0.00034	0.00000
47	0.53	0.48970	0.00083	0.00000	0.00059	0.00000	0.00022	0.00000
48	0.62	0.67301	0.00231	0.00000	0.00153	0.00000	0.00014	0.00000
49	0.74	1.16805	0.00650	0.00034	0.00302	0.00000	0.00024	0.00000
50	1.00	0.15394	0.00030	0.00005	0.00000	0.00000	0.00000	0.00000
51	0.50	0.68912	0.01050	0.00000	0.00272	0.00741	0.00000	0.00000
52	0.43	0.29970	0.00250	0.00000	0.00082	0.00146	0.00008	0.00000
53	0.71	6.38058	0.00596	0.00165	0.00000	0.00000	0.00259	0.00000
54	1.00	1.29263	0.00119	0.00028	0.00000	0.00000	0.00000	0.00009
55	0.89	0.57802	0.00053	0.00013	0.00000	0.00000	0.00000	0.00004
56	0.90	0.30950	0.00243	0.00000	0.00092	0.00039	0.00000	0.00010
57	0.60	0.15350	0.00380	0.00009	0.00072	0.00236	0.00000	0.00003
58	0.25	0.20995	0.00096	0.00000	0.00052	0.00000	0.00004	0.00003

Appendix B

Model 2: Constraints on Output Weights

Model 2: Constraints on Output Weights

```
set DMU_set;                              # Set of DMUs

set X;                                    # Set of Inputs

set Y;                                    # Set of Outputs

set Factors;                              # Set of Inputs & Outputs

param Data {DMU_set, Factors};            # Data indexed by DMU & Factor

param o symbolic;                         # DMU whose efficiency is being maximized

param a {Y};                              # Coefficients of output weights for first constraint
                                          # on output weights

param b {Y};                              # Coefficients of output weights for second constraint
                                          # on output weights


var v {X} >= 0;                           # Input weights indexed over the set X of inputs

var u {Y} >= 0;                           # Output weights indexed over the set Y of outputs


maximize Efficiency : sum {j in Y} u[j]*Data[o, j];

subject to Input_constraints: sum {i in X} v[i]*Data[o, i] = 1;

subject to Output_constraints {k in DMU_set} : sum {j in Y} u[j]*Data[k,j] - sum {i in X} v[i]*Data[k, i]
<= 0;

subject to First_constraint: sum {j in Y} u[j] * a[j] >= 0;       # First constraint on output weights

subject to Second_constraint: sum {j in Y} u[j] * b[j] >= 0;      # Second constraint on output weights
```

Data File for Model 2: Constraints on Output Weights

```
set DMU_set :=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58;
set Factors := RC LH BC FC P S A;
param Data : RC LH BC FC P S A
:=

1 4.17 434 396 2430 0 633 595
2 4.00 400 5000 0 0 101 2817
3 3.70 200 9700 300 0 168 9048
4 1.60 150 19 300 56 1054 315
5 0.31 48 0 5 95 874 90
6 4.08 513 2695 2205 0 317 4651
7 12.65 1584 6598 1628 887 465 7847
8 0.40 574 2249 395 330 219 2669
9 1.86 508 2756 650 394 230 3145
10 0.90 968 4641 955 548 304 5085
11 0.23 306 1660 544 255 211 2075
12 0.45 196 968 164 110 220 1059
13 15.99 1112 4781 1261 605 457 5749
14 0.95 244 1526 258 163 190 1694
15 0.50 484 2534 563 319 228 2795
16 0.86 528 2365 424 322 248 2780
17 0.33 274 1745 381 218 225 2023
18 13.95 1212 4679 1058 611 366 5386
19 0.97 230 1248 211 151 213 1389
20 9.98 734 3891 783 435 357 4409
21 0.69 1174 4856 1019 634 337 5617
22 7.00 158 188 120 0 468 120
23 8.87 822 2489 0 0 263 44
24 10.13 428 887 213 83 541 566
25 2.57 128 422 211 70 315 590
26 4.16 218 422 211 70 315 580
27 0.54 27 8 198 0 421 152
28 0.59 23 10 238 0 489 187
29 0.54 18 6.5 179 0 415 143
30 1.06 65 811 255 0 158 897
31 0.62 22 10.6 220 0 482 172
32 1.33 100 856 426 0 258 1047
33 0.27 14 4.6 104 0 382 82
34 7.61 1286 17131 34 69 485 16078
35 0.75 70 1212 82 0 140 1044
36 1.50 92 336 357 7 273 550
37 16.50 612 179 581 469 1658 600
38 2.46 112 589 512 11 298 874
39 1.40 80 319 326 7 288 512
40 3.30 140 763 611 14 294 1091
41 3.00 232 985 836 37 348 1275
42 1.50 240 3000 0 0 332 2125
43 1.07 114 0 156 7 885 147
44 2.29 336 0 703 37 1424 691
45 1.53 214 0 476 25 1012 468
46 1.11 114 0 237 12 972 228
47 1.51 314 0 490 26 1133 482
48 1.08 118 0 319 17 923 313
49 0.50 64 667 152 0 210 570
50 4.20 1164 20000 0 0 255 12746
51 0.72 48 6 120 24 431 106
52 2.00 160 3 316 48 1312 295
53 0.11 50 270 0 0 103 176
54 0.70 80 2641 139 0 92 2748
55 1.50 250 5225 220 55 110 5448
56 1.80 182 2500 630 0 304 3112
57 2.60 158 1954 353 47 270 2090
58 3.20 342 164 432 0 444 336
;

set X := RC LH ;
set Y := BC FC P S A ;

param:          a       b :=
        BC      1       0
        FC      -1      1
        P       0       -1
        S       0       0
        A       0       0 ;
```

Results for Model 2: Constraints on Output Weights

DMU	EFF	V_{RC}	V_{LH}	U_{BC}	U_{FC}	U_P	U_S	U_A
1	0.24	0.11147	0.00123	0.00006	0.00006	0.00000	0.00011	0.00000
2	0.36	0.11347	0.00137	0.00007	0.00000	0.00000	0.00000	0.00000
3	1.00	0.15641	0.00211	0.00010	0.00010	0.00010	0.00000	0.00000
4	0.36	0.30680	0.00339	0.00017	0.00017	0.00000	0.00029	0.00000
5	1.00	0.80968	0.01560	0.00000	0.00000	0.00000	0.00114	0.00000
6	0.31	0.20426	0.00032	0.00006	0.00006	0.00000	0.00009	0.00000
7	0.18	0.06393	0.00012	0.00002	0.00002	0.00002	0.00003	0.00000
8	0.78	2.50000	0.00000	0.00035	0.00000	0.00000	0.00000	0.00000
9	0.43	0.35469	0.00067	0.00010	0.00010	0.00010	0.00015	0.00000
10	0.78	0.69047	0.00039	0.00017	0.00000	0.00000	0.00000	0.00000
11	1.00	1.23778	0.00234	0.00036	0.00036	0.00036	0.00053	0.00000
12	0.56	1.21928	0.00230	0.00036	0.00036	0.00036	0.00052	0.00000
13	0.15	0.03011	0.00047	0.00002	0.00002	0.00002	0.00003	0.00000
14	0.46	0.70881	0.00134	0.00021	0.00021	0.00021	0.00030	0.00000
15	0.80	1.05777	0.00097	0.00023	0.00000	0.00000	0.00000	0.00008
16	0.55	0.61533	0.00089	0.00000	0.00000	0.00000	0.00025	0.00017
17	0.94	1.57159	0.00176	0.00039	0.00000	0.00000	0.00065	0.00006
18	0.14	0.03057	0.00047	0.00002	0.00002	0.00002	0.00003	0.00000
19	0.40	0.71206	0.00134	0.00021	0.00021	0.00021	0.00030	0.00000
20	0.18	0.04685	0.00073	0.00003	0.00003	0.00003	0.00005	0.00000
21	0.98	1.44928	0.00000	0.00020	0.00000	0.00000	0.00000	0.00000
22	0.13	0.00000	0.00633	0.00013	0.00000	0.00000	0.00023	0.00000
23	0.09	0.04893	0.00069	0.00003	0.00000	0.00000	0.00006	0.00000
24	0.09	0.00000	0.00234	0.00005	0.00005	0.00005	0.00007	0.00000
25	0.18	0.00000	0.00781	0.00015	0.00015	0.00015	0.00024	0.00000
26	0.11	0.00000	0.00459	0.00009	0.00009	0.00009	0.00014	0.00000
27	0.63	0.00000	0.03704	0.00072	0.00072	0.00072	0.00115	0.00000
28	0.87	0.00000	0.04348	0.00085	0.00085	0.00085	0.00135	0.00000
29	0.92	0.00000	0.05556	0.00108	0.00108	0.00000	0.00173	0.00000
30	0.42	0.48392	0.00749	0.00032	0.00032	0.00032	0.00053	0.00000
31	0.89	0.00000	0.04545	0.00089	0.00089	0.00000	0.00141	0.00000
32	0.39	0.34742	0.00538	0.00023	0.00023	0.00023	0.00038	0.00000
33	1.00	2.35360	0.02604	0.00127	0.00127	0.00000	0.00226	0.00000
34	0.56	0.11372	0.00010	0.00002	0.00000	0.00000	0.00000	0.00001
35	0.55	0.56064	0.00828	0.00037	0.00016	0.00000	0.00065	0.00000
36	0.26	0.34194	0.00529	0.00023	0.00023	0.00023	0.00037	0.00000
37	0.12	0.00000	0.00163	0.00003	0.00003	0.00003	0.00005	0.00000
38	0.28	0.00000	0.00893	0.00017	0.00017	0.00017	0.00028	0.00000
39	0.28	0.37898	0.00587	0.00025	0.00025	0.00025	0.00041	0.00000
40	0.26	0.00000	0.00714	0.00014	0.00014	0.00014	0.00022	0.00000
41	0.24	0.15169	0.00235	0.00010	0.00010	0.00010	0.00016	0.00000
42	0.55	0.52934	0.00086	0.00016	0.00000	0.00000	0.00023	0.00000
43	0.40	0.42897	0.00475	0.00023	0.00023	0.00000	0.00041	0.00000
44	0.29	0.35406	0.00056	0.00010	0.00010	0.00000	0.00016	0.00000
45	0.31	0.25658	0.00284	0.00014	0.00014	0.00000	0.00025	0.00000
46	0.45	0.42173	0.00467	0.00023	0.00023	0.00000	0.00040	0.00000
47	0.32	0.49767	0.00079	0.00014	0.00014	0.00000	0.00022	0.00000
48	0.44	0.41921	0.00464	0.00023	0.00023	0.00000	0.00040	0.00000
49	0.54	1.66174	0.00264	0.00047	0.00047	0.00000	0.00073	0.00000
50	1.00	0.15394	0.00030	0.00005	0.00000	0.00000	0.00000	0.00000
51	0.39	0.72824	0.00991	0.00044	0.00044	0.00044	0.00075	0.00000
52	0.38	0.26525	0.00293	0.00014	0.00014	0.00000	0.00025	0.00000
53	0.71	6.38058	0.00596	0.00165	0.00000	0.00000	0.00259	0.00000
54	1.00	1.29263	0.00119	0.00028	0.00000	0.00000	0.00000	0.00009
55	0.89	0.57802	0.00053	0.00013	0.00000	0.00000	0.00000	0.00004
56	0.53	0.15733	0.00394	0.00000	0.00000	0.00000	0.00022	0.00015
57	0.37	0.19816	0.00307	0.00013	0.00013	0.00013	0.00022	0.00000
58	0.11	0.14319	0.00158	0.00008	0.00008	0.00000	0.00014	0.00000