New Dynamic Investment Criteria

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· Good review and discussion
· No programming, an example could be illustrated w/ Excel

NEW DYNAMIC INVESTMENT CRITERIA

PROJECT REPORT

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ABSTRACT

Four dynamic capital investment criteria were presented and discussed. Those criteria are: the discounted certainty equivalent (DCE) criterion, the time-weighted utility (TMU) criterion, and the horizon utility (HU) criterion, and the dynamic target-wealth criterion for the sequential budgeting process. After discussion of several points related to the criteria, several recommendations were made, including evaluating investment opportunities on the basis of wealth positions rather than cash flow.

CONTENT

INTRODUCTION

In the late 1970's and most of the 1980's, a flood of qualitatively superior and economically less expensive foreign products overwhelmed American manufacturers, seizing market share and profit from them. This situation brought increased attention to the difficulties in capturing all the aspects of manufacturing investment proposals in the discounted cash flow approach of the traditional economic justification methodologies may have inadequately evaluated investment proposals and consequently slowed the growth of possible solutions such as flexible manufacturing systems. Thus, new methodologies are being searched besides continuing study on old ones.

In capital investment evaluation, acceptance criteria, or measures of investment worth , plays an important role. The task of this project was to discuss several new investment decision criteria in order to find more significant evaluation methods , which might hopefully aid engineering managers to make good decisions and model their preference. Better criteria can make managers more efficient in allocating capital and accumulating more wealth for the firm.

In 1985, Robert A. Thompson and Gerald J. Thuesen put forward three new dynamic investment criteria: the discounted certainty equivalent (DCE) criterion, time-weighted utility (TWU) criterion, and horizon utility (HU) criterion[1]. Later, in a simulation experiment, these criteria together with two traditional criteria the expected present value (EPV) criterion and mean-variance (MV) criterion were tested. The time-weighted utility criterion produced only 14% as many bankruptcies as the other criteria [2]. This result is attracting. In 1991, one of the authors, Thuesen and another scholar Ramis extended the earlier work in dynamic decision criteria and presented the dynamic target-wealth criterion for sequential capital investment decisions [3], which gave many valuable insights into investment decision process.

This project analyzed several problems in the dynamic criteria and attempted to check the possibility of their application. This report will introduce those dynamic criteria at first. And then some analysis of several problems will follow. Several recommendations will appear at last.

DYNAMIC INVESTMENT CRITERIA FOR CAPITAL BUDGETING DECISIONS

The dynamic decision criteria to be considered are:

The discounted certainty equivalent (DCE) criterion,

The time-weighted utility (TWU) criterion, and

The horizon utility (HU) criterion.

Each of these criteria evaluates how the changes in the equity of the firm, the reflection of its ownership, vary over time.

Two traditional decision criteria were selected by the authors to provide basis for comparison. These criteria are

The expected present value (EPV) criterion, and

The mean-variance (MV) criterion.

Let:

S: equity change , a random variable,

t: time period index, t=1,... H,

H: time horizon (equal to the longest project life)

i: minimum acceptable rate of return (MARR),

E(St):expected equity change at time t,

V(St): variance of the equity change at time t, and

λ: variance penalty parameter.

Then, EPV is defined as:

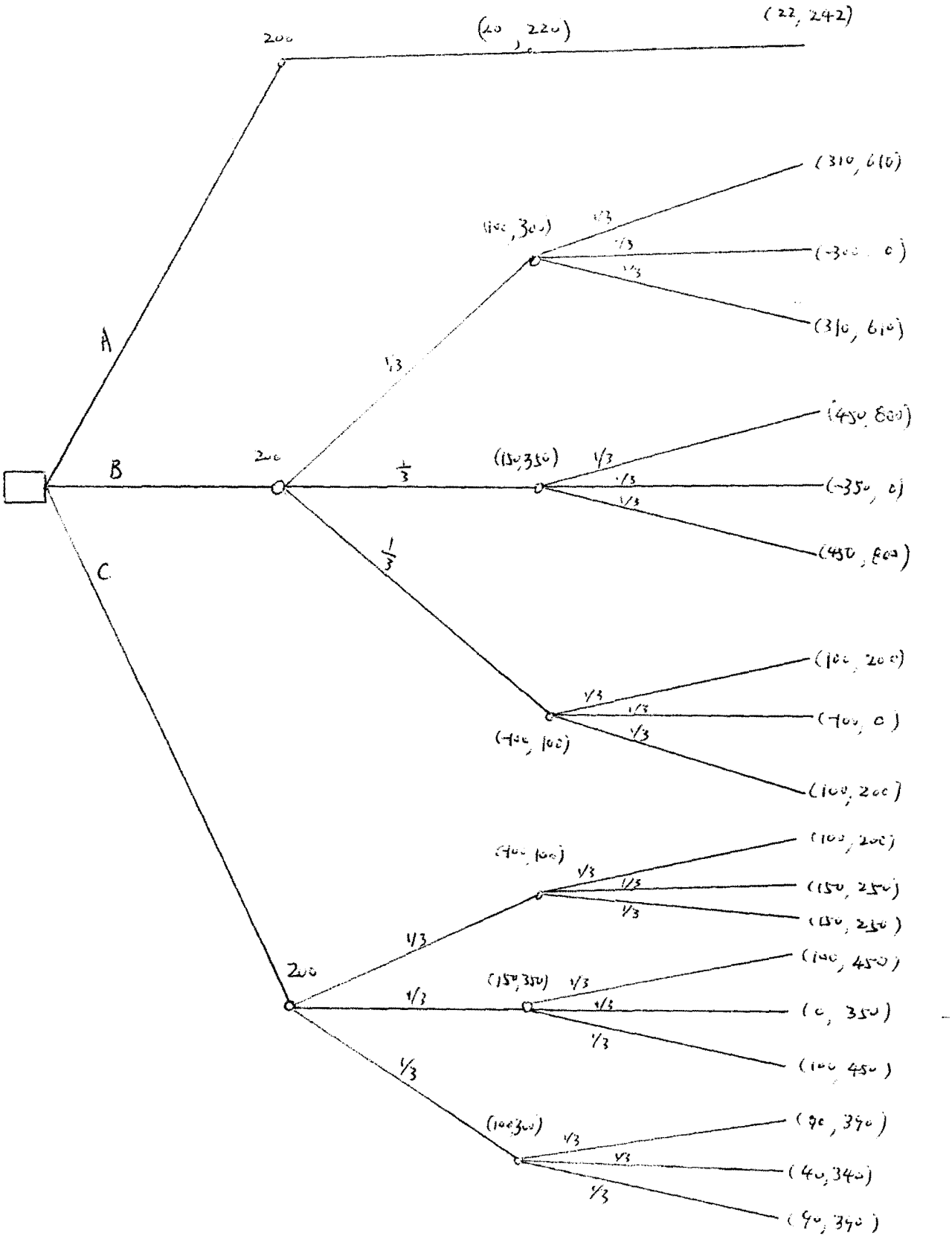
$$EPV = \sum_{t=1}^{n} E(St)/(1+i)^t$$

The mean-variance (MV) criterion adjusts for risk by deducting from the expected value of the measure of merit a penalty for variability, i. e. :

$$MV = EPV - \lambda \sum_{t=1}^{n} V(St)/(1+i)^{2t} \qquad \lambda > 0$$

MV criterion has received wide attention and comprises the most familiar risk-adjusted capital investment decision criterion (However, note that MV discounts variances by the factor of -2t).

The decision problem to be considered can be illustrated by the following example (showed by a decision diagram):


Period 0
Equity
Period 1
Equity change Equity
Period 2
Equity change Equity
200
(20, 220)
(22, 242)
A
B
C
200
1/3
(100, 300)
1/3
(310, 610)
(-300, 0)
(310, 610)
200
1/3
(150, 350)
1/3
(450, 800)
(-350, 0)
(450, 800)
1/3
(-100, 100)
1/3
(100, 200)
(-100, 0)
(100, 200)
200
1/3
(-100, 100)
1/3
(100, 200)
(150, 250)
(150, 250)
1/3
(150, 350)
1/3
(100, 450)
(0, 350)
(100, 450)
1/3
(100, 300)
1/3
(90, 390)
(40, 340)
(90, 390)

Alternative A: the firm's equity is invested in projects yielding a guaranteed 10% rate of return;

Alternative B,C: the firm invests in a set of new projects at period 0 and reinvest at period 1 in conjunction with the existing portfolio yielding equity changes in accordance with the probability showed.

Dynamic Utility Criteria

The authors assumed decision maker (DM) has a risk averse utility function, and :

$U(0)=0$, the utility of zero change in the firms' equity is zero,

$U'(0^{+})=1$, the marginal utility where the equity change is barely positive is one. $0 = 0+\Delta$, where Δ is a very small amount,

$U(S1) > U(S2)$, $S1 > S2$

$U(S) <= -U(-S)$, $S >= 0$

$U(\alpha s) <= \alpha u(S)$, $S >= 0$ and $a >= 1$.

In their research, the authors found that it is usually possible to achieve a reasonable approximation of the decision-maker's risk preference relationship using a segmented utility function comprised of relatively simple mathematical forms. In most cases, the following forms appear adequate for the purpose:

In the positive equity change (gain) region, $S>=0$

$U(S) = a1S+b1$, $a1 > 0$, a1 and b1 are parameters,

and

$U(S) = \ln(b2S+ a2) + c2$, $b2 > 0$, $(b2S+a2)>0$
a2, b2 c2 ,parameters

In the negative (loss) region, $S < 0$

$U(S) = a3.\exp(-b3S) + c3$, $a3>0, b3>0$, parameters

Gains and losses tend to be viewed by how they impact the

firm, i.e. equity gains and losses tend to be viewed by firms relative to their equity position. More precisely, the marginal utility of an equity change id a function of the equity change and the equity position before the change:

$$U'(St|Q_{t-1}) = f(St, Q_{t-1})$$

Q_{t-1} , the equity position at period t-1
St , equity change at period t.

Further, suppose the firms has a risk preference relationship as follows:

$U(0) = 0$

$U'(0^{+}) = 1$

$U(S) = S, \quad S >= 0$

$U(S) = -aEXP(-Bs)+c, \quad S<0$

Set:

$S=-Q, \quad S<-Q$ (firms' losses cannot exceed its equity)

$U'(0^{-}) = 1$

$U'(-Q) = 5$ (the point of bankruptcy)

Parameters a, b, c are decided by above equations and prior period equity position. Thus, we are able to dynamically generate risk preference relationships based on changes to equity position over time and compute the expected utility of those changes at each time period.

The discounted certainty equivalent (DCE) criterion

Let:

E[U(S)]: expected utility of a set of randomly occurring equity change

$\hat{S}$: certainty equivalent

then

$U(\hat{S}) = E[U(S)]$

or

$\hat{S} = E[U(S)]$ $\quad E[U(S)] \geq 0$

$\hat{S} = -\ln((c-E[U(S)])/a)/b$, $\quad E[U(S)] < 0$

Given that the certainty equivalent for each future period has been determined, the future period certainty equivalents may be combined into a single index using standard discounting procedure. The total discounted certainty equivalent is defined as:

$$\hat{S}_T = \sum_t \hat{S}_t / (1+i)^t$$

DCE selects the alternative with the highest $\hat{S}_T$.

The time-weighted utility (TWU) criterion

Since the expected utility is based on equity changes that are the equivalent of changes in wealth (money) , it appears reasonable to infer that expected utility has some consistent time value similar to money, i. e. a time weighing of the form

$$E[U(S)]_T = \sum_t r^{(t-1)} \cdot E[U(S)]$$

r: a weighing parameter, $\quad 0<r<1$

The TWU criterion ranks the alternative with the highest

$E[U(S)]_T$ as the optimal.

The horizon utility (HU) criterion

The DCE and TWU criteria evaluate the effects of equity change at each future time to the horizon and combine the period evaluation into a single index. The HU criterion is to take a longer term view and examine the net change in equity from period 0 to the time horizon.

The expected utility is calculated by considering the total equity change from period 0 to horizon for each possible path. The HU criterion identifies the alternative with the highest expected utility as the best alternative.

Testing

In evaluating a decision criterion, it is important to know how the criterion performs over the range of circumstances under which it might be employed. Clearly, accumulating actual data for a meaningful evaluation of a wide range of circumstances would be an extremely difficult task. Consequently, a variety of investment situations were generated using simulation techniques.

A fair representation of the capital investment environments in which firm typically operate can be simulated if a variety of risk levels and abandonment options (WHIch reflects the firm' s ,management style) in combination is considered. Three risk levels (high,mid, and low variabilities) and three abandonment options (no abandonment, partial abandonment, and full abandonment) interact to form nine risk-abandonment combinations. For each combination, ten cases were generated. A case is a sequence of investment situations spanning 13 times periods. At each time period from two to six projects were presented for possible investment. The number of projects and the characteristics of each project were stochastically generated. Actual realizations generated from these characterizations are presented. These realizations are used to determine actual outcomes resulting from investment selections.

Performance Measures

1 Profitability measures:

How an decision criterion has performed over the course of the study period was first judged by the equity at the end of the period , which is measured by:

Expected equity at the end of the study period.

Expected utility of equity change over the study period.

This measure assumes that the decision maker id risk averse and has a risk attitude that is consistent with the dynamic utility decision criteria developed by the authors' research.

2 Safety measure

The frequency of bankruptcies was chosen to gauge the relative safety.

3. Variability measures

The sample standard deviations of equity and utility in 1 were used as risk surrogates.

4. Safety measures

Orderly sustained growth is generally viewed as being indicative of good management. Following four measures were used to reflect stability of growth:

. Variation in equity growth rate-- the average mean square deviation from the expected period equity.

.Average amount of equity loss--the average period fractional loss,

.Average amount of equity gain above the target--the average period fractional gain above the target

.Average stability ranking. The criteria were evaluated using a Wileven rank-sum test[4] to provide an overall stability assessment.

Result

Table 4. Summary of Comparative Test Results

Performance Measure	Decision Criterion				
	EPV	MV	DCE	TWU	HU
Expected Equity (in 1,000 equity units)	1.4259	1.2327	1.0928	1.2668	1.0289
Comment: EPV is preferred to HU and DCE with 90% statistical confidence.					
Expected Utility	0.3479	0.2486	0.0422	0.7232	0.3657
Comment: TWU is preferred to DCE and MV with 95% statistical confidence and to EPV and HU with 90% statistical confidence.					
Frequency of Bankruptcy	0.2333	0.2111	0.2667	0.0111	0.0778
Comment: At the 95% level of statistical confidence, TWU is preferred to all other decision criteria; and HU is preferred to DCE, EPV, and MV.					
Sample Standard Deviation of Equity	1.7176	1.4033	1.2882	1.0857	1.0212
Comment: HU, TWU, and DCE are preferred to EPV with 95% statistical confidence.					
Sample Standard Deviation of Utility	2.4720	2.1612	2.0276	1.4791	1.6262
Comment: At the 95% level of statistical confidence, TWU and HU are preferred to EPV, MV, and DCE; and DCE is preferred to EPV.					
Expected Standard Deviation of Period Equity Growth	0.5990	0.7255	0.5470	0.4329	0.5683
Comment: No statistically significant difference exist.					
Expected Period Fractional Equity Loss	-0.094	-0.088	-0.085	-0.019	-0.057
Comment: At the 95% level of statistical confidence, TWU is preferred to all other criteria; and HU is preferred to EPV, MV, and DCE.					
Expected Period Fractional Equity Gain above the Target	0.192	0.170	0.151	0.063	0.126
Comment: At the 95% level of statistical confidence, TWU is preferred to all other criteria; HU is preferred to EPV and MV; and DCE is preferred to EPV.					
Average Stability Ranking	4.67	3.61	3.28	1.00	2.44
Comment: At the 95% level of statistical confidence, TWU is preferred to all other decision criteria; HU is preferred to EPV, MV, and DCE; and DCE and MV are preferred to EPV.					

A summary of the important comparison s between these five criteria follows:

1 TWU is ranked above DCE for all nine measures;

2 TWU is ranked above HU for eight of the nine measures;

3 HU is ranked above DCE for seven of the nine measures;

4 TWU is ranked above EPV for all the measures except for expected equity measure;

5 TWU is ranked above MV for all nine measures.

Apparently, TWU criterion outperforms all the other criteria. TWU selects investments that yield an accumulated wealth amount at the horizon comparable to the other criteria, but TWU produced only 4% to 14% as many bankruptcies as other criteria. These dramatic improvements in the reduction of downside losses indicates the improvements in decision making that are possible by utilizing a dynamic decision criterion that can appropriately adjust to changes in the investment environment.

DYNAMIC TARGET-WEALTH CRITERION

A dynamic decision criterion for the sequential budgeting process is developed. Using utility functions, this decision criterion models a decision maker's preferences for sequences of wealth positions generated by a set of investment opportunities relative to targeted wealth positions developed by the firm. The derivation of this criterion is related to cash flow analysis and the use of sequences of total wealth accumulations. This decision criterion allows the decision maker to express preferences for how future wealth is accumulated and not just the obtainment of the final level of wealth. It also allows these preferences to change over time, depending on the wealth level at the time of decision. This dynamic capability provides new insights in a sequential decision making process in which goals can change due to fluctuations in the financial position of the firm.

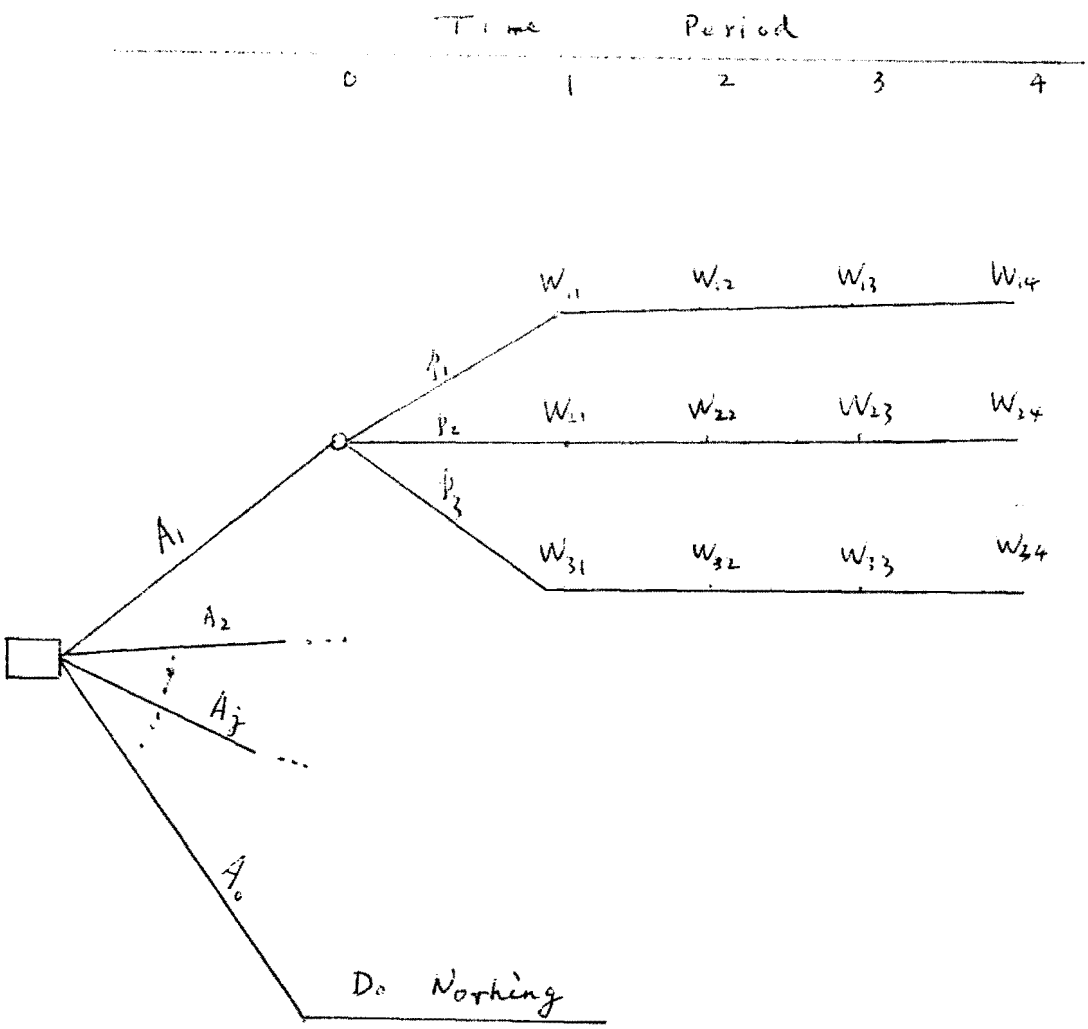
A sequential capital budgeting process is a multi-stage decision process wherein investments are made on a regular , periodic basis. In this characterization, a decision is one in a long sequence of periodic decisions.

The DM's objective is to maximize the firm's future wealth

which is defined as the net equity value associated with the capital assets. The DM uses as decision criterion which compares opportunities on the basis of streams of total wealth positions rather than on streams of cash flows(wealth positions for an investment opportunity result from a combination of the cash flow streams of the opportunity and the firm's current wealth position).

A target-wealth utility based criterion compares each investment opportunity s' sequence of wealth positions to the sequence of targeted wealth positions which are determined from a firm"s initial wealth and a desired or targeted growth rate. This criterion selects projects not only for the resulting final wealth position , but also for how the firm reaches that final wealth position.

The decision problem can be represented by following decision diagram.


Time Period
0
1
2
3
4
W_{11} W_{12} W_{13} W_{14}
P_1
W_{21} W_{22} W_{23} W_{24}
P_2
P_3
W_{31} W_{32} W_{33} W_{34}
A_1
A_2
A_j
A_0
Do Nothing

A1 represents many possible alternatives, and in general, if A1 is selected, this could result in three different scenarios with probabilities p1, p2, p3 respectively. Within each alternative, wealth positions $W_{k,n}$ will be realized , where k denotes a possible sequence os outcomes (paths) fora particular alternative and n is the time period. Aj represents a single alternative which describes one of many possible combinations of projects including the existing portfolio of investments resulting from previous investments.

Because there can be hundreds of thousands of paths for each alternative in above decision tree, a sample scheme is used. Let A_{jk} represents the kth path for a single alternative Aj. For a single path, current wealth position W_{k0} is known, future wealth positions and target wealth position are W_{k1} , W_{k2} ,.... and t_{k1} , t_{k2} ,.... respectively. The utility function for this path can be represented by following form:

$$u(A_{jk}) = u\,[(W_{k1};t_{k1}), (W_{k2};t_{k2}),\ldots,(W_{kN};t_{kN})]_{jk}$$

For example, for a two period situation, utility function can be assumed to be of the form:

$$u[(w1;t1),(w2;t2)] = u(w1;t1)+ku(w2;t2),\ k>0 \text{ (parameter)}$$

The single period utility function for final wealth u(wn;tn) depends on the target t_n , and for different targets there may be different preferences, i.e.different utility functions.

An overall estimates of the desirability of the alternative is then determined by finding the expected utility based on the sample of paths selected for that alternative, if given M paths, then

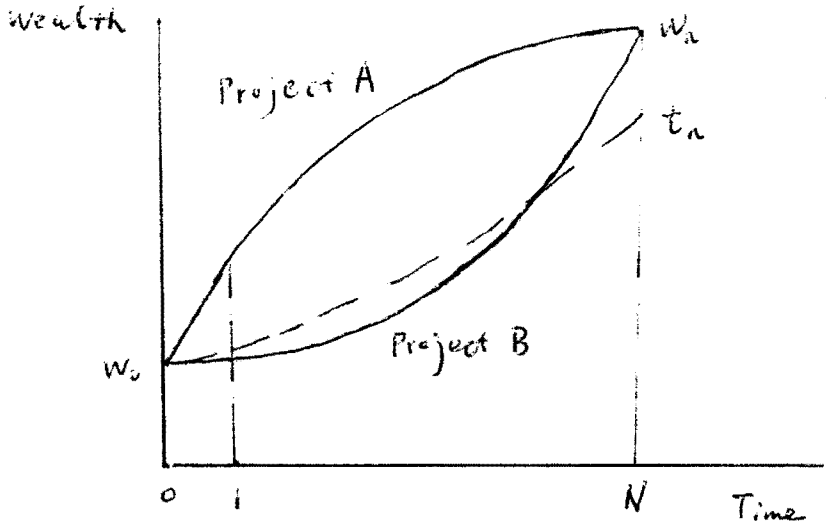
$$E[u(A_{jK})] = \sum_{k=1}^{M} u(A_{jK})p_k$$

where p is the probability of the path occurring in the sample.

The selection of the desired alternative is determined by the alterative having the maximum expected utility. Those projects defining the most preferred alternative are selected and added to the portfolio . When the next time for a decision occurs, the portfolio outcomes are realized for the past period. A new current wealth position results, and new targets are calculated.

This decision criterion allows the DM to express preferences for how future wealth is accumulated and not just the obtainment of the final level of wealth. It allows these preference to change over time depending on the wealth level at the time of decision.

For an example, two projects have equal final wealth but goes through different paths:


Wealth
Project A
W_n
t_n
Project B
W_0
0
1
N
Time

If decisions are to be made sequentially over time and/or the future wealth positions are probabilistic, there are two advantages in accumulating wealth according to project A rather than B. First, the greater the wealth at each point in time the less likely the firm is to end up bankrupt when unpredictable future losses occur. There fore, there is greater margin of safety with project A compared to project B because possible undesirable consequences can be absorbed without threatening the firm"s financial ability to survive. Second, the rate of cash inflow early in the time frame from period 0 to N is greater for A than B. This early return of cash provides more flexibility regarding possible future investment options since a shortage of cash flow could preclude the acceptance of other outstanding investment opportunities occurring early during the time horizon.

A fundamental issue for this decision criterion is the determination of wealth positions at each time period. The authors recommended to use the following formula:

$$Wn = Wn-1 + Xn - Dn + (g)\sum_{m=1}^{n-1} Xm(1+g)^{(n-m-1)} \quad n=1,\ldots N$$

Wn: the wealth position at time period n,

Xm: cash flow at time period m

Dn the depreciation at time period n,

g: average reinvestment rate

DISCUSSION

In analyzing the dynamic criteria, several points seem to deserve further discussion.

1 In derivation of DCE, TWU and HU criteria, the authors claimed that " it is usually possible to achieve a reasonable approximation of the decision-maker's risk preference relationship using a segmented utility comprised of relatively simple mathematical forms."They suggested to use a linear function and a logarithmic function for positive equity change region. Yet later, they used only linear one. Since the following calculation covered large range of equity change, this linear approximation could produce great error while true utility functions are not risk neutral. This problem might become more serious in HU criterion calculation which covers a longer period and so a larger range of equity change. Besides, the combination of linear and logarithmic function cannot represent the preference relationship of risk-seeking.

2 For "dynamic", utility function at each period was determined by the equity position at prior period.It seems to be better to use total wealth to substitute equity change.In this way, "dynamic" requirement is also met .(Utility as a function of wealth will be discussed later).

3 In application of TWU criterion:

$$E[U(S)]_T = \sum_t E[U(St)] \cdot \gamma^{(t-i)}$$

Considering deterministic situation, the above formulation is equivalent to

$$U(S) = \sum_t \gamma^{t} U(St)$$

If U(st) at different time period are independent the above relationship might hold. Yet, in derivation of the criterion, it is clear that U(St) depends on prior period equity position or Q Q depends on S , S affects the value of U(S). So U(S) and U(S) might not be independent.Then, additive utility model cannot hold[5]. This problem needs more examination.

4 One of the fundamentals in application of TWU is the decision of the weighing parameter r. Unfortunately, the authors did not tell how to decide it at all, neither in original paper nor in later one(in [2],the authors mentioned that "Determination of appropriate values for the time-weighing parameter is discussed later). If this parameter cannot be decided, the attracting testing result would meaningless.

RECOMMENDATION

1 In capital investment evaluation, it seems to be better to compare opportunities on the basis of streams of total wealth positions rather than streams of cash flow (Wealth positions for an investment opportunity result from a combination of cash flow streams of the opportunity and the firm"s current wealth position).

It is the dynamics of the change in total wealth that determines bankruptcy or economic prosperity. It is critical that the firms understand this dynamics.

People might have different preference relationship or utility function when they have different amount of wealth.

2 While evaluating investment opportunities, it is necessary to consider not only the final wealth position but also how the future wealth is accumulated. Many investment opportunities involve risk in it.Sometimes, trading-off the advantage of having increased investment flexibility or safety with the final wealth position expected from projects needs be made.

3 TWU was shown much better performance than other criteria in the simulation experiment conducted by the authors. Although there seem to be some problems with TWU, it deserves further study.

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