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Abstract: Two models were developed for the weekly optimal scheduling of airline ramp agents - the personnel who provide the ground services for aircraft, using the integer programming feature of the Super LINDO package.

Employee Scheduling Using Linear Programming Methods: A Case Study For Ground Services Personnel

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EMPLOYEE SCHEDULING USING LINEAR PROGRAMMING METHODS: A CASE STUDY FOR GROUND SERVICES PERSONNEL

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EXECUTIVE SUMMARY

In this study, our team undertook to develop a model for the weekly scheduling of airline ramp agents -- the personnel who provide the ground services for aircraft. In working on what became the first model, we encountered problems. These were severe enough for us to work on a second method of solving the problem indicated by literature. We still encountered difficulties in the capacity of the software we used which, in the end, allowed only a daily schedule solution. The final product was a pair of models which are both functional. They are limited in that they solve only a day to day scheduling problem and that changes in the constraints are time consuming for entry into the software. However, the model is good for use in small scheduling problems associated with the scenario given in the study if optimality is a concern. For larger problems a more fully developed model will be needed.

Both models were run for a first eight hours of the schedule as a test. This produced optimal solutions which had the same number of total hours but differing assignments for the agents. This is typical of this class of problems -- due to their structure they often have multiple optimal solutions We performed a sensitivity analysis for both runs of the models. This indicated that slight changes in the most of the constraints will produce a change in the solution, either decreasing or increasing the hours needed as well as the agent assignments. We were also able to use the second model for a full day test. It did, in fact, find an optimal solution. Preliminary sensitivity analysis indicates a similar lack of tolerance to changes as with the smaller runs.

At the end of the paper are suggestions on ways the study could be extended. The majority of these extensions use the model for more specific problems as opposed to this general solution.

INTRODUCTION

Scheduling forms the basis for almost every aspect of the airline industry. It determines when flights will arrive and depart, which crews will work those flights. It even affects how many passengers will be onboard. Key in making the flights operate properly and on-time is the ground services crew. These are the people who off-load and load the aircraft as well as providing other services. The scheduling of the ground services crews or ramp agents can be optimized as can any part of the industry. This paper will present two models which optimize the ramp agent staffing.

As a basis for starting the models, we obtained a copy of Alaska Airlines Portland, Oregon station flight schedule for the spring of 1993. The daily pattern was twenty-eight inbound and twenty-eight outbound flights, most of which were in one of four pushes or banks. The first of these banks occurred between 0615 and 0815 international time. The second was from 1000 to 1230. The heaviest of the pushes started at 1400 and ended at 1530 in which six aircraft arrived and three departed. The fourth bank is the longest in duration but has relatively fewer flights to work; it starts at 1650 and ends at 2110. A copy of this flight schedule is included in appendix A.

There are certain criteria which must be met to satisfy both federal and state regulations regarding airline safety and employment. The Oregon State Board of Labor has stipulated that *i*) once an employee has arrived at work for a regularly scheduled shift, the employee will be paid for half the scheduled time, not to be less than two hours; and *ii*) no employee may be scheduled for less than two consecutive hours of work. The Federal Aviation Administration (FAA) has determined that a minimum of three ramp agent will be used to marshal aircraft in to and out from the gates. There are other regulations which concern the operations on the ramp, but these three are the only ones which directly affect the scheduling of agents.

The goal of the project was to develop a weekly work schedule for the ramp agents working the Portland flights of Alaska Airlines using linear programming methods. In order to accomplish this, a determination of the hourly agent requirements was made for the flight schedule. For every flight arriving or departing in an hour, at least three ramp agents were needed. The following table summarizes this information:

Hr	1	2	3	4	5	6	7	8	9
# emp	12	15	3	3	15	12	6	3	15
Hr	10	11	12	13	14	15	16	17	18
# emp	15	9	12	3	12	9	6	6	6

Table 1

Note: Hr 1 is 0600 to 0700, Hr 2 is 0700 to 0800, etc.

CONSTRAINTS AND THE OBJECTIVE FUNCTION

The constraints we used were those set by the federal and state regulations. For a starting point we assumed for compliance with the first state employment regulation that once an employee showed up for work, he or she would finished the shift as scheduled. This is usually the case for the airlines. Even with flight cancellations, the agent's shift usually includes more than one flight. However, we were still left with a minimum number of hours for each employee; that is, each ramp agent must work at least two hours. But this is exactly what linear programming is best at. The FAA rule of three ramp agents per flight was left unchanged and stipulated that the right hand side (RHS) of the total per flight crew be a multiple of three. We also established that in fairness to the agents, each employee should start and end work only once each day. This formed the constraint set for our models.

In general, there are usually a variety of wages paid to the agents based on tenure and performance. However, the company which provides the ground services for Alaska Airlines' Portland station has a flat pay scale for employees who have past a six month trial period. This made the objective function very easy to formulate. Rather than minimize the total employee expenses (wages plus employer co-pay items such as taxes, SIAF, and unemployment insurance), we were able to minimize total hours. However, a varying pay scale model would be only negligibly harder to implement -- one would need use a coefficient which reflected employee expense for each agent's hours-worked decision variable.

While these constraints and objective function seem to be sparse, they do reflect what really occurs on the ramp, especially in union shops. In non-union companies, the same agents will be used on several flights in the same bank of flights, usually broken-down into a gate or gates to work. The agent will work one flight until an inbound flight on an adjacent gate arrives. At that point the agent will move to the inbound flight until it is off-loaded. Then he or she will return to the first flight he or she was working until departure of this or another flight or the arrival of yet another flight. In a union company, each agent is assigned a specific gate and a specific duty. This assignment does not change during the work period. Our constraints reflect this latter situation.

LITERATURE SURVEY

The problem we have undertaken in this study is a classic in the field of operations research. It is a specialized case of the transportation problem which was the original problem solved by George Dantzig in 1947. Subsequent to this he and his associates went on to solve the so-called traveling salesman problems (TSP) in more general terms[1]. While the statement of the TSP looks nothing like the problem of this study, in their formulations the problems are almost identical -- the exception being that the exact constraint sets may be different. The idea behind both is integer programming in which the decision variable are allowed only to be integers and more specifically zero-or-one variables[2].

Of particular concern to us during the development of a working model was the size of the problem. The scheduling problem generally grows very fast as regards the number of decision variables in the formulation. In fact, it has been shown in many papers that the growth of the problem is exponential based on a function of the number of constraints per decision variable[3]. This actually became critical in the both of the models as they grew to encompass the original scope of our goal. In the end, our models became too large for even Super Lindo to be viable for a whole week solution and we could solve only a daily schedule.

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[1] G. B. Dantzig, D. R. Fulkerson, and S. M. Johnson. "On a Linear-Programming Combinatorial Approach to the Traveling Salesman Problem," Operations Research 7, No. 1, January 1959.
[2] L. Lawler, ET AL. The Traveling Salesman Problem. New York: Wiley, 1985. pp 28 - 44.
[3] R. E. Bixby, ET AL. "Very Large-scale Linear Programming: a Case Study in Combining Interior Point and Simplex Methods. Operation Research, Sept-Oct 1992. v40 n5. pp 885.

The first of the models is similar to that presented by Browne, Propp, and Tribrewala[4]. This model uses a set of decision variables for the total individual employee hours, a set for the individual employee start times, and a set for the individual employee stop times. Due to the differences in the constraints from the Browne problem and our problem the actual constraint equations are not identical but the nature of the models is the same. In deed, the first constraint of our model is an inventory constraint and $E_{j,j+1}$ is the inventory variable[5].

The second model was first seen in a paper presented by Bakshi and Arora[6]. Their model uses a single set of decision variables. These variables are given for their job sequencing problem as waiting times while in our model they are given as hour that an agent worked. Again an exact match between the constraint equations for the two formulations fails to occur since the constraints themselves are different.

THE FIRST MODEL

The first model which we developed was, as stated earlier based on Browne, ET AL. In the model there are three main types of decision variables -- one for duration of work, one for start time, and one for stop or finish time. The model and the decision variable are defined for our case as follows:

The Decision Variables:

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[4]J. Browne, ET AL. "A paper presented at the TIMS/ORSA National Joint Meeting." TIMS/ORSA Symposium. 1978.

[5]W. L. Winston. Introduction to Mathematical Programming: Applications and Algorithms. Boston. PWS-Kent Publishing. 1991 pp 96 - 99

[6]M. S. Bakshi, S. R. Arora. "The Sequencing Problem." Management Science. v16, n4 1969. pp 247 - 263.

x_{ij} = 1 if ith agent starts work in jth hour

0 otherwise

y_i = Integer duration of work for the ith agent

z_{ij} = 1 if ith agent stops work at jth hour

0 otherwise

s_j = number of agents needed for jth hour

$E_{j,j+1}$= number of agents continuing to work from jth period to period j+1

i = 1..15; j = 1..7

Note that all variables are of integer type.

The Objective Function

$$\text{Min } g = \Sigma y_i + E_{7,T}$$

where T is the termination point for number of agents continuing past the last period.

The Constraints:

1. Number of agents needed in jth hour:

$$\Sigma x_{ij} \geq s_j \qquad j = \min j$$

$$\Sigma x_{ij} - \Sigma z_{ij} \geq s_j - E_{j-1,j} \qquad \text{for } \min j + 1 \text{ to } \max j$$

2. Each agent should start once per day:

$$\Sigma x_{ij} = 1 \qquad \forall i$$

3. Each agent should stop once each day:

$$\Sigma z_{ij} = 1 \qquad \forall i$$

Note that constraints 2 and 3 imply - *i)* the x_{ij} and the z_{ij} are zero-one variables

ii) each agents daily hours are continuous

4. The duration of work should be at least 2 hours:

$$2\,\Sigma x_{ij} - y_i \leq 0 \qquad \forall i$$

5. Computation of stop time for each agent:

$$\Sigma(j-1)x_{ij} + y_i = \Sigma(j-1)z_{ij} \qquad \forall i$$

or in canonical form:

$$\Sigma(j-1)x_{ij} + y_i - \Sigma(j-1)z_{ij} = 0 \quad \forall i$$

6. Stop time of each agent should not go beyond the last period

$$\Sigma(j-1)x_{ij} + y_i \leq \max j$$

7. Non-negativity constraints:

$$x_{ij}, y_i, z_{ij}\ E_{j,j+1} \geq 0 \qquad \forall i,j$$

The full model equations as used by the computer are in appendix B.

THE SECOND MODEL

Initially, only one model was under consideration. However, difficulties were encountered in the development of the first model. As a consequence, efforts were put toward trying to find a viable formulation using the job scheduling model of Bakshi and Arora. The formulation was actually quickly derived. However, the sheer volume of equation entry makes the model difficult for quick usage. Later, it was indicated that a generator program using FORTRAN, Pascal, C, or a spreadsheet could have been used to simplify that equation process. The definitions of the decision variables and the equations are as follows for this second model:

The Decision Variables:

Let

$$x_{ij} = \begin{cases} 1 & \text{if ith agent works at time j} \\ 0 & \text{otherwise} \end{cases}$$

s_j ~~x_{ij}~~ = The required number of agents per hour.

Where, i = 1..16 $(1, \cdots 37)$

j = 1..8 $(1, \cdots 18)$

The Objective Function:

$$\text{Min } z = \Sigma\, \Sigma\, x_{ij}$$

The Constraints:

1. Each agents work less than or equal to 8 hours.

$$\Sigma\, x_{ij} \leq \cancel{\max j}\ 8$$

2. There should be more than or an equal number of agents available than required for each hour.

$$\Sigma x_{ij} \geq s_j \qquad \forall j$$

3. Each agent should work at least 2 hours per day.

$$x_{i(j-1)} - x_{ij} + x_{i(j+1)} \geq 0 \ \forall i,j$$

$$x_{i2} - x_{i1} \geq 0 \qquad \forall\, i$$

$$x_{i7} - x_{i8} \geq 0 \qquad \forall\, i$$

$(X_{i17} - X_{i18} \geq 0)$

4. Continuity requirement or one start and stop time per day.

$$x_{i(j-1)} - x_{ij} + x_{i(j+1)} \leq 1 \qquad \forall i,j$$

$$x_{i(j-1)} - x_{ij} + x_{i(j+2)} \leq 1 \qquad \forall i,j$$

...

5. Non-negativity Constraints

$$x_{ij} \geq 0 \qquad \forall\, i,j$$

The full model equations as used by the computer are in appendix C and D.

SOLUTIONS

The first model was used for the first seven hours of the daily schedule. The second model was used for both the first and then the whole day (eighteen hours total). Because of Super Lindo's capacity limitations, availability of the agents had to be constrained for the whole day run. The availability was constrained by defining each employees range of working hour -- the x_{ij} -- to an eight hour shift. This alteration of the problem, while not necessarily reflecting reality, is close enough for the size of the model.

The solutions as found by Super Lindo are presented in tabular form on the following pages. They represent a bar chart type schedule in which each employee's duration of work is indicated. The full Super Lindo print our of the solutions are in: appendix B for the first model; appendix C for the second model first eight hour run; appendix D for the second model full day run.

The first model when run for the first seven hours resulted with 90 total hours divided between fifteen agents for differing durations. The second model over the same run resulted in the same total hours, the assignments were different. This is because there are usually multiple optimal solutions to the TSP. The run of the whole day resulted with 191 total hours divided amongst thirty-seven employees.

The First 7 hrs Schedule of The First Model

Hours	1	2	3	4	5	6	7
Req. # of agents	12	15	3	3	15	12	6
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							

FINSCHE.XLS

18 Hours Schedule of The Second Model

Hours	1	2	3	4	5	6
Req. # of Agents	12	15	3	3	15	12
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
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32						
33						
34						
35						
36						
37						

FINSCHE.XLS

Hours	7	8	9	10	11	12
Req. # of Agents	6	3	15	15	9	12
1						
2						
3						
4						
5						
6						
7						
8						
9						
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34						
35						
36						
37						

FINSCHE.XLS

Hours	13	14	15	16	17	18
Req. # of Agents	3	12	9	6	6	6
1						
2						
3						
4						
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6						
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SENSITIVITY ANALYSIS

Since integer programming in Super Lindo does not allow for sensitivity analysis, several different runs were made with changes in the constraints. With first model, changes in the minimum working hours requirement was analyzed.

Min Hr	2	3	4	5	6	7
Z-value	90	90	90	90	93	105

Table 2

It can be seen from Table 2 that with two hours minimum working hour time limitation, the total hours needed comes out to be 90 hours. This minimum z-value continues until the minimum work time is greater than or equal to six hours. At six minimum hours the z-value starts to increase. At seven minimum working hours, all agents work continuously for a full shift. This gives a result of the maximum number of total hours of 105.

With in the second model on the first eight hour run, the table on the following page summarizes the constraint changes and relaxation of other parameters. As can be seen from Table 3, the minimum value of total hours is obtained for the first eight hours by decreasing the second or fifth hour agent requirement by one agent. The third and fourth hour agent requirement are sensitive to any changes. Releasing the two hour minimum working time requirement, does not change the total number of hours needed. However, releasing the continuity of hours constraint will reduce the total hours needed by twenty-four hours. Also having one more agent available for work decreases the total hours needed by two hours.

PSENSI.XLS

SENSITIVITY ANALYSIS
(For the first 8 hours)

I. Bj (Bj=~~Nj~~ s_j: number of workers needed in jth hour)

	Bj	ΔB_j	Z	Δ Z	ALLOWABLE CHANGE(not changing Z) ALLOW. DECREASE	ALLOW. INCREASE
B1	12	1	94	1	0	0
B1	12	-1	92	-1		
B2	15	-1	90	-3	0	0
B3	3	12	93	0	3	12
B3	3	-3	93	0		
B4	3	12	93	0	3	12
B4	3	-3	93	0		
B5	15	-1	90	-3	0	0
B6	12	1	94	1	0	0
B6	12	-1	92	-1		
B7	6	1	94	1	0	0
B7	6	-1	92	-1		
B8	3	1	94	1	0	0
B8	3	-1	92	-1		

II. RELEASING 2 HOUR MINIMUM WORKING TIME REQUIREMENT

Z= 93

Change= 0

III. RELEASING CONTINUOUSLY WORKING REQUIREMENT

Z= 69

Change= -24

IV. STAFFING CHANGE(PLUS ONE WORKER)

Z= 91

Change -2

Due to late completion of the full day run, full sensitivity analysis for the run is not available at the time of publication. However, the following table shows the allowable changes, if any, in the RHS values without changing the current solution:

B_i	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8	j=9
Δ	0	0	+9/-3	+9/-3	0	0	0	0	0
B_i	j=10	j=11	j=12	j=13	j=14	j=15	j=16	j=17	j=18
Δ	0	+1/-0	0	+4/-3	0	0	0	0	0

Table 4

CONCLUSIONS

In developing these models, we discovered several problems and advantages about employee scheduling. The most prominent of these is that problems in this area get very big very fast. For example, in the first model, each additional employee needed implies that additional four equations are needed. If Alaska Airlines adds one more flight, three more agents are needed meaning twelve equations. As the literature indicated the growth is, in general, exponential. The second model presents an even greater challenge in a change in the number of agents needed since each agent has an associated fourteen equations.

These problems and their solutions are very sensitive to changes. In the first model, for example,changes in the number of agents needed by even one for only one hour almost always changes the solution.

We found that due to the nature of the growth of these problems, a linear programming approach is not efficient for small-scale problems. The entry time of the equations exceeds that for the usual bar chart methods used by small companies for employee scheduling. However, it should be pointed out that the bar chart solutions are not, in general, optimal.

On the other hand for very large numbers of employees, this is a good starting point for scheduling staff. It will find an optimal solution provided that the constraints form a consistent set. In further developing the models a more in-depth literature survey will help in avoiding duplicating work already established.

As the models are they can be used for day to day scheduling especially the second model since it has been tested over a full day. They are particularly effective if the changes from day to day are minimal or if a model generating program is being used. None the less, we repeat here that further development is needed for weekly and larger period planing.

Both models allow for tailoring the schedule to specific needs of employees. This is accomplished by designating each set cumulative hours decision variable to a specific employee and then constraining the available hours of that employee as needed.

Overall, while we have "re-invented the wheel" for this study, it has made us very aware of the limitations of this type of problem and its formulation. This allows the above conclusions to be summarized as follows:

Use the model for small scheduling problems associated with the scenario given in the study if optimality is a concern. For larger problems a more fully developed model will be needed.

EXTENSIONS

Some possible extensions for this problem and model are as follows:

i) by using a generator program, a larger size of these models could be built and run on Hyper Lindo or the full scale mainframe version of Lindo.

ii) the idea of cost can easily be associated to the objective function by the methods mentioned previously in this paper.

iii) further constraints can be added for items such as seniority, limiting daily employee hours, or designating employees as part-time or full-time.

iv) the nature of the formulation in this study did not permit employees to work a split shift. However, splitting shifts is a common practice in many businesses. The continuity of hours constraints given in either model could be changed to allow for this possibility