Title: Linear Programming Techniques Applied to A Paper Manufacturing Facility

Course:
Year: 1993
Author(s): T. Dugan, J. Gabriel and V. Scallesse

Report No: P93019

Abstract: We developed a linear program to run on LINDO, which provided a plan for optimum paper production allocation for each machine of three machines in a paper mill, to maximize contribution to profit through production allocation. Results indicate that one machine is under-utilized, and recommendations for use include machine shut-down and retrofit.

Linear Programming Techniques Applied to a Paper Manufacturing Facility

*

Tim Dugan
Vinny Scallesse
Joe Gabriel

EMP-P9319

P 9319.

LINEAR PROGRAMMING TECHNIQUES APPLIED TO A PAPER MANUFACTURING FACILITY

Emgt 540 - Linear Programming

Spring 1993

Tim Dugan

Vinny Scallesse

Joe Gabriel

ABSTRACT

The subject of this study is a paper mill which has 3 paper machines. All of the machines produce a wide variety of paper grades. A linear program is developed and run on LINDO, which provides a plan for optimum paper production allocation for each machine. The objective function of the model is to maximize contribution to profit through production allocation. Constraints include machine availabilities, minimum machine run time requirements, product demands, and machine efficiencies. Results indicate that one machine is under-utilized, and recommendations for utilization include machine shut-down and machine retrofit. Sensitivity analysis provides input for managers to evaluate new products and capital improvement proposals.

TABLE OF CONTENTS

EXECUTIVE SUMMARY

The purpose of this paper is to develop a linear programming model which can be used to assign production schedules to several paper machines in a typical multi-grade paper mill. The subject of this model is a paper mill which has 3 paper machines. All of the machines produce a wide variety of paper grades. The objective function of the model is to maximize contribution to profit through production allocation. In order to formulate the model, the following input was required:

Hourly Contribution - Each combination of a product with a paper machine results in a unique level of contribution for each hour of production. These contributions can be estimated from historical data and are effected largely by machine width and speed, as well as the product specifications and sales price.

Production Rates - Again unique to each machine/product combination, this figure varies widely with machine speed/width and paper weight.

Machine Availability - The number of hours per period of time that a machine is available for production is dictated by the dependability of the machinery. This is also estimated from historical data.

The output data from the model run showed an optimal allocation of products to machines based on total contribution. This model was set up for annual production planning but can easily be modified for monthly or even weekly schedules. The

sensitivity analysis indicates the penalty that is paid (reduced cost) when the optimal allocation is not used.

An alternate use of this model is to evaluate new products. By estimating the contribution and production rate for a new product, it can be added to the model. The output would illustrate whether or not the new grade would increase the mill's total contribution. If positive results are obtained, trials can be run to verify the estimates. A sensitivity analysis would yield acceptable ranges on these numbers.

INTRODUCTION

A local paper manufacturing company uses three separate machines to produce over 100 different fiber-based products. These products have unique characteristics defined in an extensive specification and are manufactured on a per-order basis. Each machine can produce only a certain number of the products based on the operating characteristics of the machine and the product specification. Many of the products are machine specific, meaning that they can be produced on a single machine. However, a number of the products, including some with very high demands, can be produced on multiple machines. This operational environment gives rise to a classic linear optimization problem in that each machine has different operating characteristics and costs which can drastically influence the overall success of the plant according to the production allocation for each machine. Realizing this, a study was undertaken to apply linear programming techniques as a means of determining an optimal product mix as well as applying sensitivity analysis to identify potential upgrades to increase efficiency.

The primary objectives of the plant study included four major steps. First, a problem description which defined the salient issues was formulated and analyzed to ensure that the problem was indeed a candidate for linear programming. The problem description was based in part on the actual 1992 production records and further supplemented with information from the personnel responsible for the day to day operation of the plant. A linear programming model which described an objective function and constraint set was then developed from this description to provide a mathematical form of the problem for analysis by LINDO, a computer aided linear programming software package. After the model was input, the optimal solution produced by the software was evaluated to identify the similarities as well as any major discrepancies between the optimal computer solution and the actual product mix selection. Finally, an in-depth sensitivity analysis was performed to put forth proposals for future study including avenues which might be pursued to increase plant effectiveness.

PROBLEM DESCRIPTION

In 1992 the plant manufactured a total of 202,577 tons of fiber based products utilizing 24,653 hours on three machines. The breakdown of the number of grades produced on each machine is listed in Appendix 1: 1992 Plant Production. The appendix also contains data on the tonnage for each grade on the respective machines as well as the tons per hour and associated contribution for each product. In order to formulate a linear programming model to analyze the production allocation for each machine, it was first necessary to describe the pertinent facts with regard to the products, machines, and objective of the optimization.

Products: Although the specifications for the products manufactured at the plant are extremely exhaustive, only three specifications affect the determination of which machines could manufacture or "run" each product. Therefore, for the purposes of the study, segregation according to grade, color, and basis weight was used to distinguish the various products. There are well in excess of 100 unique grades, which group the various products according to general usage or product category. If applicable, a color designation is assigned to further subdivide a grade according to various available colors. Finally, the basis weight (BW) is a specification based on the weight in pounds of a unit sized sheet of product, and can range anywhere from 10 to 150 lbs. Since there is limited storage or inventory holding capacity, grades are produced on actual order only. Additionally, as a result of the inherent inefficiencies within the process, a certain amount of paper is produced during the product run which does not meet specification. This product is labelled "Joblot" and is sold as sub-grade product.

Both net sales price and variable cost of production vary among grades depending on such factors as demand, characteristics of the specific grades, and the particular machine on which the grade is produced. As a result, there is a unique contribution for each product/machine combination which gives rise to a multitude of decision variables in the objective function. In order to consolidate the scope of the model, products of similar characteristics are grouped for each machine which pared down the product field to 32 different types and are listed in Appendix 2: Consolidated Product Groupings, along with the respective averaged contribution per hour and production per hour for each product.

Machines: Each of the three machines is capable of producing specific grades of product for only a certain range of BW. Machine production is measured in tons per hour (TPH), which varies according to the BW of the respective grade. Additionally, each machine has an associated efficiency factor which is a measure of how much Joblot is produced before the grade specification is met. Finally, due to maintenance factors, each machine is required to run for a specified minimum number of hours annually to avoid costly shutdowns and startup procedures, as well as excessive training expenses.

Constraints: The ability to maximize hourly contribution as defined in the objective function is constrained by machine availability, machine efficiency, and minimum run time. The machine availability constraint defines the number of hours each machine is available annually to manufacture product. As shown in Appendix 2, a number of the products are machine specific and cannot be manufactured on multiple machines. In effect, they are constants which cannot be adjusted or re-allocated. Therefore, these products are accounted for by subtracting the amount of time required to produce the orders from the total available hours for the particular machine, leaving the remaining hours to be allocated for production of grades which can be run on multiple machines. The machine efficiency constraints were arrived at through conversations with plant personnel regarding the amount of Joblot produced during product runs and grade changes. It was determined that the factor varies between machines and is derived by applying an efficiency factor of 7% to the amount of each grade produced on machine 1, 3% for machine 2, and 4% for machine 3. Finally, in order to maintain a full crew for each machine and avoid the operational and maintenance difficulties associated with shutting down a machine due to under-utilization, it was determined that each machine must operate for at least 75% of the annual hours available.

LITERATURE SEARCH

Linear programming techniques were not found to be in wide use for production planning in the paper industry. The industry more heavily employs these techniques in other areas such as forest management, sawmill cutting operations, and paper machine roll trim. However, these applications could not be adapted to the paper production scheduling problem. Only two models were found which were closely related to the production scheduling problem of this project.

The first model [1] dealt with production scheduling in a paperboard mill. Although some of the assumptions of this model, such as machine production rates, machine/product combinations, multiple customers, nothing produced for inventory, were valid, others did not fit the requirements of this application. Struve used only about a dozen different grades. Also, grade results were similar and could be scaled or normalized based on machine speed. Since Struve was dealing with product being shipped from several locations, freight was considered a variable cost. In the case of a single mill location, freight costs can be deducted from sales price and a net sales price can be used. Either approach should yield the same results.

In the second model [2], heuristic methods and a statistical approach are utilized to formulate the program input from a "large integrated management information system." The historical data used in this project's model formulation was derived from a similar database.

MODEL FORMULATION

The objective of the paper mill is to maximize "contribution" toward profit, not net profit, which is calculated as;

$$C = P - S - V$$

Where: C = total contribution, \$

P = total sales volume, \$

S = shipping costs, \$

V = total variable cost, \$

Presently, paper mill management policy is to make a product on a particular machine if the contribution is positive. Thus, they are actually losing money on products where contribution per unit is less than fixed cost per unit.

A linear program was developed to schedule which products would be made on which paper machine for the year of 1993, using 1992 data as a basis. The objective function of this linear programming model is to maximize the sum of the optimum product/machine contributions:

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

where Cij = contribution for product i on machine j, \$/hr

xij = hours/yr that product i is produced on machine j

The four types of constraints in the model are maximum machine time available, product demands, Joblot percentages, and minimum machine utilization.

Maximum machine hours were determined from actual data of 1992 run time, which reflects the reliability of the machines. Machine 3 is the newest, with over 99% utilization, and #1 is the oldest, with under 88% use rate. Based on mill experience for 1992, the under-utilization was mostly a result of down-time for maintenance, not lack of demand, so these numbers should be appropriate for this analysis. The mill has a policy of keeping the paper machines running as much as possible, so minimum hours were assumed to be 75 percent of the total possible. This is to maintain staffing levels and to avoid costly shut-down and start-up. This analysis also compared the loss in efficiency and the resulting loss in total contribution with the costs associated with a lengthy shut-down. Initially, total machine hour constraints are written as;

$$.75Hmin <= \sum_{i=1}^{m} xij <= Hmax$$

Joblot is a function of the machine capabilities, and is expressed as the sum of the total number of hours actually run on the machine. Joblot can be sold, and has a contribution toward profit. The newer machine makes a lower percentage of Joblot with a lower contribution, while the older machine makes a higher percentage with a higher contribution. Joblot is expressed as;

$$x33j = Aj \sum_{i=1}^{m} xij \quad \text{for machine } j$$

where $x33j$ = joblot for machine j (joblot is the 33rd product)

Aj = percentage joblot for machine j

Demands for the various product groups were taken from actual 1992 historical data. As discussed earlier, many of the products can be made on more than one machine, with different variable costs, contributions, and production rates in tons/hr. This linear program calculates the optimum product/machine combinations. Since there is virtually no inventory in the paper mill studied, the mill only produces product based on orders. However, it can produce less if there is no machine time available when an optimum allocation exists. In this case, slack on demand constraints would indicate when backorders are required.
Demand constraints are written as;

$$Di <= \sum_{j=1}^{n} Tijxij \quad \text{for product group } i$$

where T_{ij} = production rate for product group i on machine j, tons/hr.

A complete listing of the model is contained in Appendix 3: Paper Mill Optimization Model.

SOLUTION

As shown in Appendix 4: LINDO Output For Paper Mill Optimization, all demands were met (no slack in rows 7 to 42). As shown in row 2, Machine 1 is underutilized by 1926 hours, at its lower bound of 75 percent. Machine 2 is used 100 percent of the time, and Machine 3 is underutilized by 1459 hours.

Some products that could possibly be made on more than one machine were not split by the code, and others were. For instance, product 3, which can be made on all three machines, is only scheduled for machine 1. This is due to the much higher contribution per hour for that product/machine mix than for the other two combinations. Product 23 was split between machines 2 and 3. More run time was allocated to machine 3, due to its higher relative contribution.

Product 17 is scheduled for both Machines 1 and 2. However, one would guess that the code would have scheduled that product on Machine 3, because it has the highest contribution per hour, and there are slack hours on machine 3. It appears that the minimum run requirement for machine 1, constraint 3, forced the code to schedule that product even though it wasn't the most profitable choice. As shown by the "dual price" for constraint 3, the mill could save $704.67 for every hour it reduced the minimum run time constraint for Machine 1.

SENSITIVITY ANALYSIS

The primary problems and opportunities for the mill are shown by the "dual prices" that are the most positive and most negative. The greatest problem appears to be the underutilization of Machine 1, shown by the very large negative shadow price on the lower bound constraint. This is mainly caused by the lack of capability for Machine 1 to make highest demand products, 23, 22, and 21, all of which are in the 15 to 20 basis weight range. It is also caused by the low contribution of making high demand products 9, 10, and 17 on Machine 1. Machine 1 minimum run time can be decreased by 1929 hours before the solution is changed, as shown in the "allowable decrease" for row 2. If mill management allowed Machine 1 to shut down for 1929 hours, the mill would gain an overall contribution of 1929 x $704.67 or $1,359,308.

The primary opportunity for sales to make the most money with existing capability is with product no. 1, whose demand has a shadow price of $827.55, the highest of all constraints. As shown in the "allowable increase", the mill is able to produce 12,178 more tons of that product before the solution changes, a potential increase in contribution of $10,077,904.

The product that is the closest to having a change in schedule is 21. Since the non-basic variable x212 has the lowest "reduced cost" of $54.13, it will be the first to enter the basis if a parameter is changed. The product that is least likely to be rescheduled is product three, with the highest reduced costs for the non-basic variables of x32 an x33.

Product 23 has the tightest allowable range for contribution of C232 and C233. Since it is the highest demand product, small changes in the variable costs for these products will significantly effect total mill contribution.

DISCUSSION OF RESULTS

The data used in the model formulation was based on a year when product demand was low. As a result, the solution was constrained by product demand, and the machines, especially Machine 1, were under-utilized. Since the demand constraints were inequalities, as demand increases, production will fill available machine hours until the solution is constrained by machine hours. Then, backorder situations will occur and must be evaluated by management to determine which orders will be filled first. These decisions could be added to the model in the form of additional constraints.

The LP solution of Appendix 1 provides information that can be used for operational decisions. Most obviously, it provides a production schedule, or plan, which optimizes contribution to profit. However, while this plan is being implemented, it must be monitored to verify the accuracy of the estimated contributions and production rates. The sensitivity analysis will indicate when these estimates are outside acceptable limits. When this occurs, a revised production schedule is required.

As discussed in the sensitivity analysis, the mill could gain $1.3 million in contribution by allowing Machine 1 to shut down, rather than forcing it to run for the minimum 75 percent requirement. Mill management can compare this with the total shut-down, start-up, and training costs associated with an extended shut-down, as well as the possible political consequences of temporarily laying off personnel.

The solution can also be used to evaluate maintenance decisions. For example, if a machine is running at a reduced production rate, the maintenance time required to bring it back up to full capacity would be considered in light of slack hours in that machine's schedule, or lost contribution when that product is made on another machine.

EXTENSIONS

Strategic Planning/Capital Investment can be enhanced if the model formulation is slightly altered. A target grade can be evaluated for its addition to contribution. This increased contribution can be used as a benefit to justify the use of capital funding for necessary machine upgrades so that the target grade can be produced. For example, in the production plan shown in Appendix 4, Machine 1 was very under-utilized due to its inability to produce the high-demand, low weight range products, 21, 22, and 23. Contributions and production rates were estimated for x211, x221, and x231, and the model was re-run. The overall mill contribution rose by approximately $3 million for the year. Assuming demand remains constant for these products, mill management can justify the cost of upgrading the machine using the $3 million per year payback. This program run will also indicate which grades can be dropped when the target grade is produced, thereby showing which capabilities need not be retained.

New paper grades can be evaluated by estimating product coefficients and entering them into the model formulation. If the product is attractive to produce based on these estimates, the coefficients must be verified using the results of an actual pilot production run. During the pilot run, the coefficients would be monitored. If they are not within the sensitivity ranges, a new production schedule can be produced with actual coefficients, and the model would show if it is profitable enough to run the new product.

Corporate-wide production scheduling can be achieved by adding additional machines and locations to the model formulation. This use of the model would necessitate the cost of freight to be handled as a variable cost of the machine or the site where production occurs. In the current model formulation, only one location is considered, therefore freight costs were based solely on the customer location, not on which machine produced the product.

Timing of orders was not considered in this project. The model was formulated to provide an annual production plan. Monthly or weekly schedules can be determined by adjusting the product demand and the machine availability.

On-line model formulation would greatly enhance this model. Techniques used by Adkins can be employed to make the greatest use of the corporate database. In this project, output from a corporate database was used but was processed manually to produce the required coefficients for the model. This process was time consuming and tedious and necessitated large product groupings rather than the evaluation of individual paper grades. On-line formulation would reduce the variability in the schedule based on calculation errors, personal judgement, or bias.

<u>BIBLIOGRAPHY</u>

1. Struve, D. L., "Operations Research and the Design of Management Information Systems", TAPPI Special Technical Association Publication, no. 4, March 1966: 293-307.

2. Adkins, S. R., "A Model for Production Scheduling", TAPPI Journal, 53, no. 3, March 1970: 448-52.