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Abstract: CUSUM control charts may be used instead of Shewhart control charts to detect small changes in a process mean. The average run length (ARL) is generally less than the comparable Shewhart chart. CUSUM charts are underutilized within the current manufacturing environment. A literature review of current advances in their possible application is presented. Possible areas for use are identified.

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ABSTRACT

CUSUM control charts may be used instead of Shewhart control charts to detect small changes in a process mean. The average run length (ARL) is generally less than the comparable Shewhart chart. CUSUM charts are underutilized within the current manufacturing environment. A literature review of current advances in their possible application is presented. Possible areas for use are identified.

INTRODUCTION

Cumulative sum (CUSUM) control charts may be used instead of standard Shewhart charts when detection of small changes in a process parameter are important. For comparable average run lengths (ARLs), when the process is on target, CUSUM charts can be designed to give shorter ARLs than Shewhart charts for detecting certain small changes in process parameters [1]. CUSUM charts were proposed in 1954 by a British statistician, E. S. Page, and developed by him and other British statisticians [2]. The CUSUM chart is in reality a type of sequential analysis, since it relies upon past data for each decision [3].

A recent article by Hawkins, "Cumulative Sum Control Charting: An Underutilized SPC Tool" [4], draws attention to the fact that CUSUM charts are not as widely known or used as the typical Shewhart charts. This paper will present a literature review, discuss some of the pros and cons and look at application areas for CUSUM charts.

LITERATURE REVIEW

A literature review was performed to facilitate a better understanding of the CUSUM chart, its applications and recent developments. The volume of literature written about Shewhart type charts verses the literature for CUSUM charts definitely supports Hawkins statement, that CUSUM charts are an underutilized SPC tool. As manufacturing methods are moving toward smaller run sizes and just-in-time processes, the Shewhart charts have statistical limitations.

Many of the recent articles discuss applications relating to short and small manufacturing runs or continuous processes with individual data points. Nicholas Farnum [5] notes that the three main recommendations for monitoring short-run production are: "(1) to use a deviation control method (e.g., CUSUM or EWMA) with its increased power for early detection of small process shifts, (2) to use a chart based on individuals data to monitor control variables. (e.g., monitoring a process variable such as chemical plating thickness), or (3) to chart deviations from nominal (DNOM) when different part types are run through the same process." Hawkins [4] states that the CUSUM chart is a little better than the exponentially weighted moving range (EWMA). Both the charts are effective in detecting and diagnosing persistent shifts. A "persistent shift" or special cause is one which persists until action is taken to correct it. If this type of shift is not large, the Shewhart chart is not very good at detecting it. Duncan [2] developed a comparison model showing that for a one sigma shift in the process mean, the CUSUM chart would give a 40% savings in sampling costs, due to the earlier detection over a similar Xbar chart.

Bourke [6] has extended the use of the Poisson-based CUSUM chart to a run-length based CUSUM chart. He has proposed this for production processes where 100% inspection is done. Typically a p-chart or poisson-based CUSUM chart is used to monitor the lengths of runs of conforming items between successive nonconforming items. The run-length CUSUM chart was found to be more efficient than the poisson CUSUM chart. The average time to detect an upward shift was improved by 10% to 30%.

Pignatiello [1] has investigated to determine if the shorter ARLs for the univariate case of the CUSUM chart can be extended to the multivariate case. His models indicated that superior performance in the ARL could be obtained for shifts in the mean that are less than a distance of 3.0 from the target mean. For larger shifts in the process mean, the chi square chart offers better protection.

Quesenberry [9] has proposed a method for using "Q" charts for start-up processes. He presents a method for "constructing control charts for the process mean and variance when the measurements are from a normal distribution and when either the mean or the variance, or both, or neither, are assumed known." By charting in "real time" from the start-up of the process, he feels that, one can begin the task of identifying and removing assignable causes and thus bring the process into control at an earlier time.

DISCUSSION OF THE PROS AND CONS

PROS

The CUSUM chart is more effective than the Shewhart chart in detecting small changes in the process mean. The range for this increased effectiveness is for changes in the magnitude of 1/2 to 2 times the square root of the process mean. The CUSUM chart is based on an additive sequential analysis which makes small shift effects more pronounced.

The increased sensitivity of the CUSUM reduces the ARL needed to detect an out of control process. Since the ARL or number of samples needed to detect a process shift is less, the cost of the sampling and also the potential lost product is less. The CUSUM sensitivity is not improved with larger subgroup samples, whereas the Shewhart chart increases its sensitivity with larger subgroups. Sampling costs are again less.

CONS

The CUSUM chart is not well known. It may require more training for people to set up. Likewise, it may be harder for people to interpret the readings. The increased sensitivity may give more "out of control" errors which are not out of control.

The CUSUM is an additive chart and will not give the same historical performance patterns as the Shewhart. The CUSUM chart is designed to detect small changes, it may be slow to detect large changes in process parameters [10].

CUSUM data analysis and plotting routines are not common in many of the current software QC packages [12]. Only one CUSUM plotting procedure for means is contained in the extensive RS/1 software package. A sample of this chart is included in the appendix. Manual calculating and plotting of the data may not be cost effective.

SUGGESTION FOR UTILIZING THE CUSUM CHART

The CUSUM chart is most useful when small changes from an expected value need to be detected for in control processes. An excellent application for this is checking standards for chemical analysis tests. When conducting analysis tests on unknown samples, a standard control sample is included. This control sample is divided in half. The first half is processed through the chemical analysis test. The unknown samples are then analyzed. The second half of the control sample is then processed through the test. The value for the first control sample is compared to the value for the second sample. The difference of the two values is calculated and then squared. This squared value is then plotted on a CUSUM chart with similar standard control sets to determine if a unacceptable shift in the analysis test has occurred. If it has, then the test results are invalid. The analysis procedure is evaluated to determine the cause. After the problem is corrected, a new set of control points are checked against the established limits. If acceptable, then normal testing is resumed. This procedure is recommended by the EPA for control of chemical analysis tests [11]. A copy of this procedure is contained in the appendix.

POSSIBLE OTHER AREAS FOR USING CUSUM CONTROL CHARTS

The fabrication of integrated circuits consists of 30 to 60 sequential processing steps. Direct measurement of the process variables at each step on the IC wafer can not be done. The various process layers and topography differences interfere with measurement techniques. To maintain control of the individual process steps, test wafers are included with the actual product wafers. The test wafers do not have the topography or previous process steps. They are designed for measurement of the individual process parameters for that process sequence. Currently the results of these measurements are plotted on Shewhart Xbar and R type control charts. The use of CUSUM control charts in addition to the Shewhart charts would allow tighter control of these processes, with fewer test samples required.

The photoresist processing sequence is critical for maintaining line width control of the small features on the IC circuits. Numerous material and equipment variables, such as resist viscosity, exposure lamp intensity, developer strength, etc., all interact to determine the final size of the exposed circuit feature. The combined alignment and size variations of all the individual circuit features has a major effect on the yield for the wafer. Earlier detection of shifts in this processing sequence would be very beneficial to increasing yields and decreasing costs.

SUGGESTED IMPROVEMENTS AND TRENDS

The trend, as noted in the literature review, is that more technical analysis of the capabilities and applications of the CUSUM charting techniques is being done. Most of this work, however, is published in the academic journals which are not routinely read by manufacturing engineers and managers. The present push in industry to achieve greater levels of quality and more flexibility in manufacturing operations needs to apply the increased control levels offered by the application of these CUSUM techniques. Improving the communication of these applications to the manufacturing industry is needed.

The QC software developers need to incorporate the CUSUM charting routines in their packages. Without the use of computer driven analysis software, the techniques will not be widely evaluated and accepted.

CONCLUSIONS

CUSUM control charting techniques offer reduced ARLs for detecting small deviations from the expected mean. Various new techniques for the basic CUSUM chart have been developed to broaden the applications. The use of CUSUM style charts is very low compared to the Shewhart style charts. Increased visibility of these techniques to the manufacturing environment is needed to evaluate their effectiveness in industry.

BIBLIOGRAPHY

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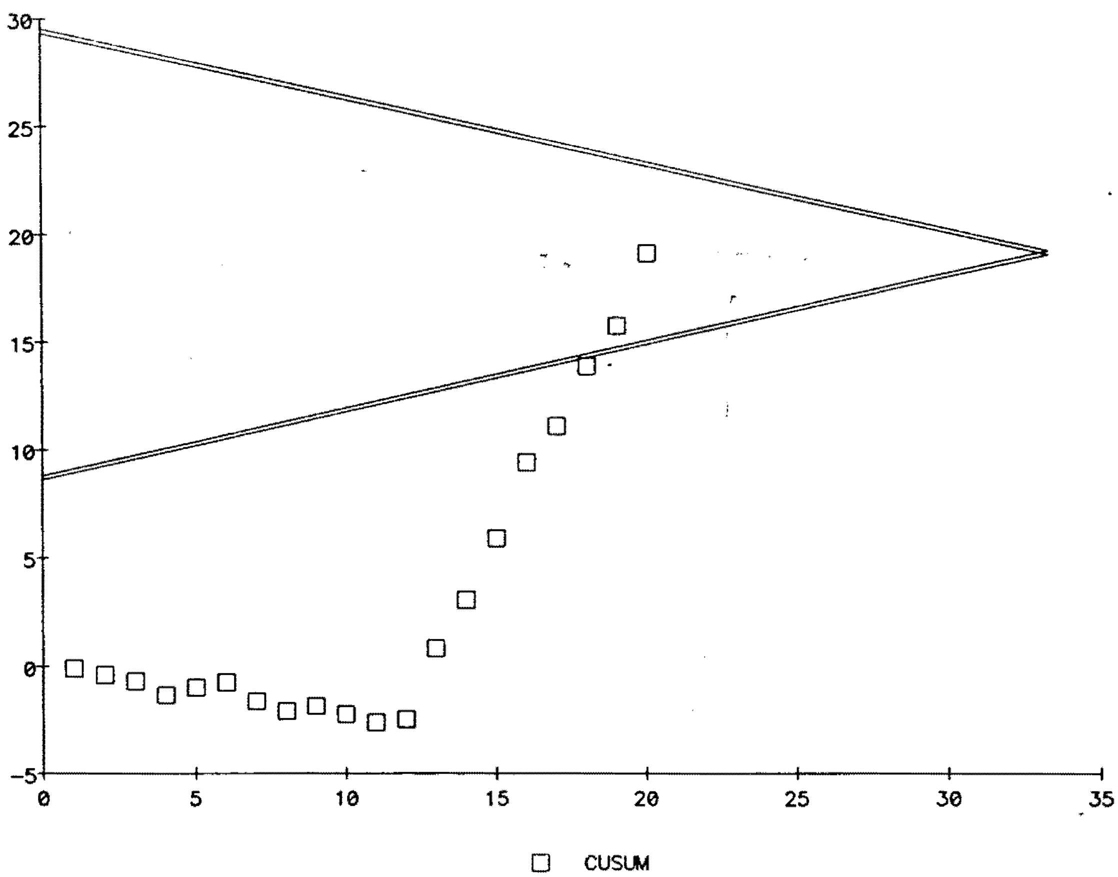
[10] Richard Deckro, "Cumulative Sum Control Charts," Class notes partly based on Mitro p 215-219, May 1993

[11] "Handbook for Analytical Quality Control in Water and Wastewater Laboratories," U.S. Environmental Protection Agency, Cincinnati, p 6-6, 6-11, Jun 1972.

[12] "OCA User's Guide," BBN Software Products Corporation, Cambridge, p 3-50, 3-90, 1984.

APPENDIX

```
# CALL PUBLIC $CUSUM <RET>
Tableportion containing subgroup MEAN values: COL "MEAN" OF VOLTS <RET>
Tableportion containing subgroup STDEV values: COL "STDEV" OF VOLTS <RET>
Subgroup size: 4 <RET>
Name of CUSUM chart: VOLTSCUSUM <RET>
(Creating new CUSUM chart)
Target value for process mean: [325.95375] 325 <RET>
Size of the shift you wish to detect: [1] <RET>
Significance level (alpha) of the test: [0.00135] <RET>
(Computing CUSUM statistics ... Done)
Display CUSUM chart? [Yes] <RET>
```

Theta = 14.036255, Lead Distance = 13.215301, Kscale = 1.248382

Figure 3-17. Sample $CUSUM Dialogue.

6.4.1.1 Construction of CuSum Quality Control Charts

The control charts are derived from three basic calculations:

a. Standard deviations (S_d) of the differences between duplicates or, in the case of spiked or standard samples, between the known quantity and the quantity obtained.

b. The upper control limit (UL)

c. The lower control limit (LL)

Prior to these calculations, two decisions must be made:

a. The α and β levels

b. The allowable variability levels

<u>Mathematical Equations</u>

$$S_d^2 = \frac{\sum_{i=1}^{n} di^2 - \frac{(\sum_{1}^{n} di)^2}{N}}{N-1} = \text{Variance of the differences}$$

$$S_d = \sqrt{S_d^2} = \text{Standard deviation of the differences} \qquad \text{(a)}$$

$$S_0^2 = (.8S_d)^2 = .64\, S_d^2 \text{ (estimates } \sigma_0^2\text{)}$$

$$S_1^2 = (1.2S_d)^2 = 1.44\, S_d^2 \text{ (estimates } \sigma_1^2\text{)}$$

$$UL(M) = \frac{2\log_e\left[\frac{1-\beta}{\alpha}\right]}{\frac{1}{S_0^2} - \frac{1}{S_1^2}} + M\,\frac{\log_e\left[\frac{S_1^2}{S_0^2}\right]}{\frac{1}{S_0^2} - \frac{1}{S_1^2}}$$

$$LL(M) = \frac{2\log_e\left[\frac{\beta}{1-\alpha}\right]}{\frac{1}{S_0^2} - \frac{1}{S_1^2}} + M\,\frac{\log_e\left[\frac{S_1^2}{S_0^2}\right]}{\frac{1}{S_0^2} - \frac{1}{S_1^2}}$$

Where: UL(M) = upper limit at M sets of samples

LL(M) = lower limit at M sets of samples

di = the difference between the ith set of duplicates or spiked samples

N = the total number of sets of duplicates or spiked samples used to construct the control charts

S_0^2 = minimum amount of variation allowed in the system

S_1^2 = maximum amount of variation allowed in the system

α = percent (decimal fraction) of time you are willing to judge the procedure out of control when it is in control

β = percent (decimal fraction) of time you are willing to judge the procedure in control when it is out of control

M = number of sets of duplicates or spiked samples used in calculating the value to be plotted on the chart

By definition, α is the probability of judging the process to be out of control when in fact, it is in control. It is recommended that α be chosen to lie between the boundaries of .05 and .15, that is, the laboratory personnel are willing to stop the laboratory process somewhere between 5 and 15% of the time, judging it to be out of control, when in fact, it is in control. If the cost of examining a process to determine the reason or reasons for being out of control is considerable, then it may be desirable to choose a low α. Likewise, if the cost is negligible, it may be desirable to choose a larger α value, and thus stop the process more frequently. (See Figure 6-2)

On the other hand, β is defined as the probability of judging the process to be in control when it is not. Again, it is recommended that β be chosen to lie between the values of .05 and .15; thus, the laboratory personnel are willing to accept out of control data somewhere between 5 and 15% of the time. The economic considerations used for choosing α are also applicable to the choice of B. (See Figure 6-2.)

It is also essential to set maximum and minimum allowable variability levels. It is necessary to specify a value for the minimum and maximum amount of variation that will be allowable in the system. These minimum and maximum amounts are referred to as σ_0^2 and σ_1^2 respectively. The values used should be based on a knowledge of the variation in the procedure under consideration. However, if such knowledge is not available, the values may be arbitrarily set at $\sigma_0^2 = (\sigma - .20\sigma)^2$ and $\sigma_1^2 = (\sigma + .20\sigma)^2$.

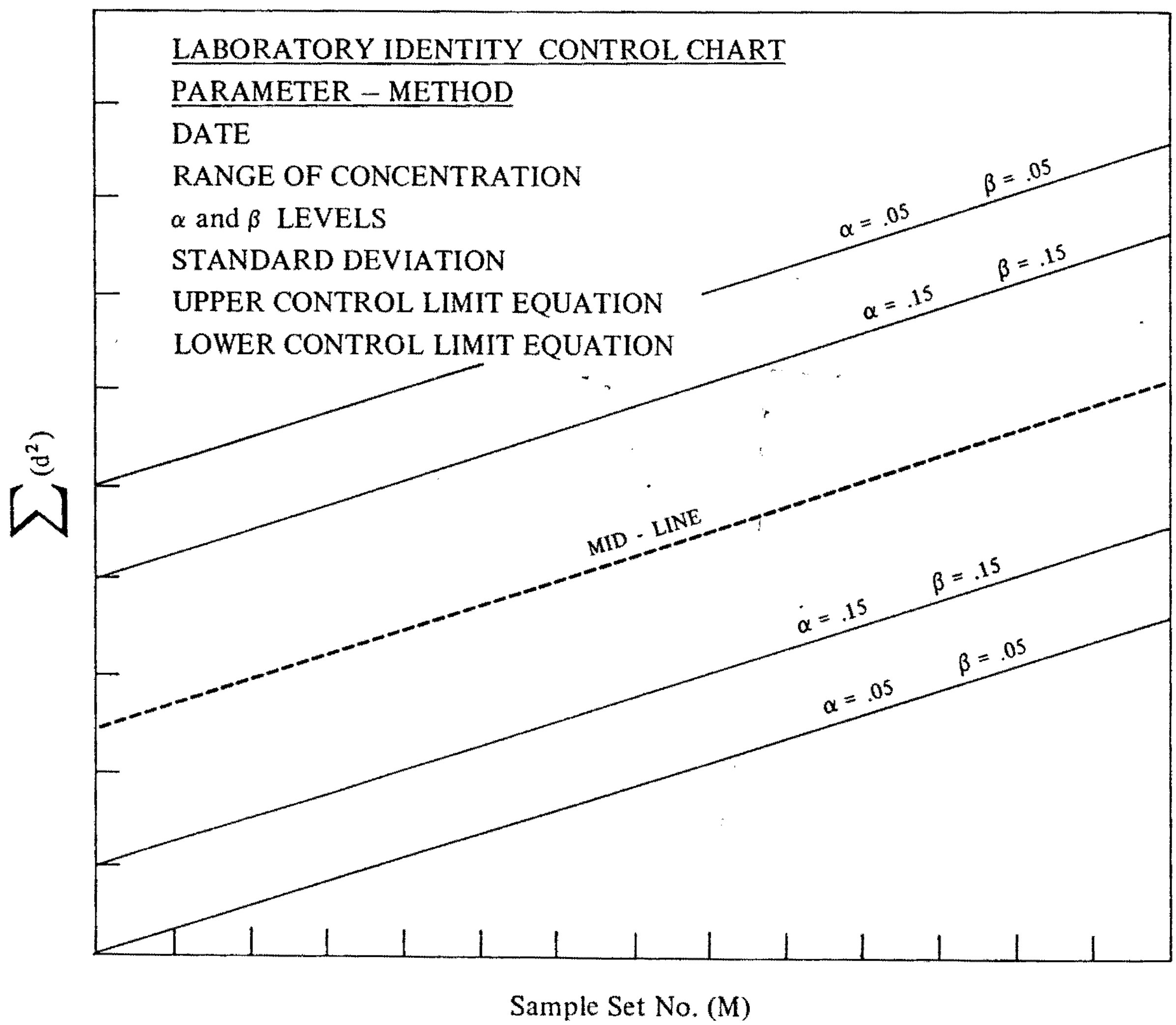


Figure 6-2. EFFECT OF α AND β LEVELS ON STANDARD CONTROL CHART

6.4.1.2 Use of CuSum Control Charts

Once the control charts are constructed, and prior to their use, consideration must be given to the number of duplicate analyses to be conducted during a series of samples; likewise, the same decision must be made on spiked or standard samples.

In considering the number of duplicate and spiked sample analyses to be conducted in a series of samples, it is necessary to weight the consequences when the data go out of control. The consequences of this situation are reanalyzing a series of samples or discarding the questionable data obtained. The samples to be reanalyzed are those lying between the last in-control point and the present out-of-control point. A realistic frequency for running duplicate and spiked samples would be every fifth sample; however, economic consideration and experience may require more or less frequent duplicate and spiked sample analyses.

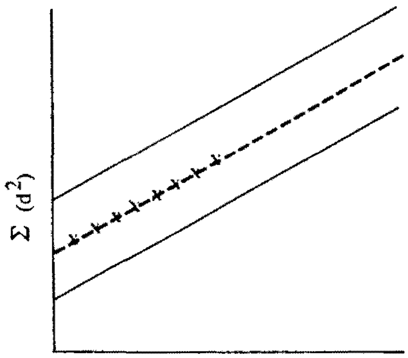
Once the frequency of duplicate and spiked samples has been determined, it is then necessary to prepare spiked or standard samples in concentrations relative to the concentration of the control charts, which should be similar to those of the environmental samples. These spiked or standard samples must be intermittently dispersed among the series of samples to be analyzed and without the analyst's knowledge of concentration. Similarly, duplicate samples must be intermittently dispersed throughout the series of samples to be analyzed, and ideally, without the analyst's knowledge; however, this is sometimes very difficult to accomplish.

The results of the duplicate and spiked sample analyses should be calculated immediately upon analyzing the samples to allow for early detection of problems that may exist in the laboratory. An example of these calculations follows:

Duplicate Sample No. M	Results No. 1	No. 2	Difference (di)	di^2	$\Sigma (di^2)$
1	5.4	5.2	.2	.04	.04
2	4.8	4.7	.1	.01	.05
3	6.1	5.8	.3	.09	.14

Upon plotting the summation or $\Sigma(di^2)$, one of three possibilities can occur (See Figure 6-3):

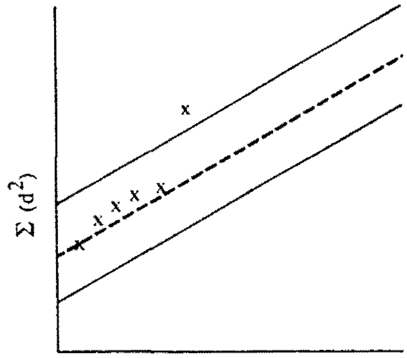
a. Out of control on the upper limit
   When data goes out of control on the upper limit the following steps should be taken:
   1. Stop work immediately
   2. Determine problems
      (a) Precision control chart
         (1) The analyst
         (2) Nature of the sample
         (3) Glassware contamination



SAMPLE SET NO.

ANALYSIS IN CONTROL

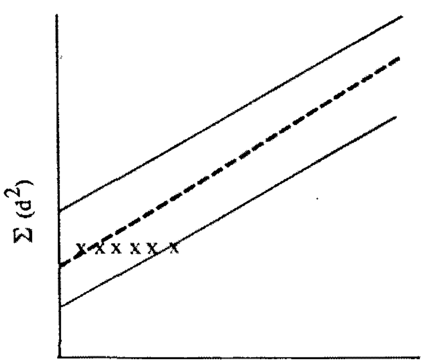
NO PROBLEMS:
CONTINUE ANALYSIS



SAMPLE SET NO.

ANALYSIS OUT OF CONTROL
UPPER LIMIT

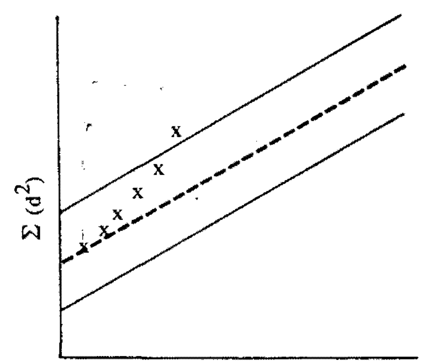
PROCEDURES:
1. STOP ANALYSIS
2. LOCATE PROBLEM
3. CORRECT PROBLEM
4. RERUN SAMPLES
5. START CHART AT SAMPLE SET NO. 1.



SAMPLE SET NO.

ANALYSIS OUT OF CONTROL
LOWER LIMIT

INCREASED EFFICIENCY OR FALSE REPORTING
PROCEDURES:
1. CONTINUE ANALYSIS
2. CONSTRUCT NEW CHART WITH RECENT DATA
3. OBSERVE ANALYST



SAMPLE SET NO.

ANALYSIS OUT OF CONTROL
UPPER LIMIT

CONTINUOUS ERROR TREND
PROCEDURES:
SAME AS ABOVE BUT STOP ANALYSIS WHEN TREND IS DETECTED.

Figure 6-3. LABORATORY QUALITY CONTROL CHARTS

(b) Accuracy control chart
    (1) The analyst
    (2) Glassware contamination
    (3) Contaminated reagents
    (4) Instrument problems
    (5) Sample interference with the spiked material

3. Rerun samples represented by that sample set number, including additional duplicate and spiked samples.
4. Begin plotting at sample No. 1 on chart.

b. In control within the upper and lower limit lines

When data continuously fall in between the upper and lower control limits, the analyses should be continued until an out-of-control trend is detected.

c. Out of control on the lower limit

When data fall out of control on the lower limit, the following steps should be taken:

1. Continue analyses unless trend changes
2. Construct new control charts on recent data
3. Check analyst's reporting of data