Title: A Linear Programming Model to Optimize the Production Schedule for a Compressor Manufacturer

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A LINEAR PROGRAMMING MODEL TO OPTIMIZE THE PRODUCTION SCHEDULE FOR A COMPRESSOR MANUFACTURER

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A Linear Programming Model to Optimize the Production Schedule for a Compressor Manufacturer

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ABSTRACT:

A linear programming model is utilized to determine the production scheduling requirements for an air compressor manufacturer's new product line which consists of eight compressors. The objective of the production scheduling is to maximize profit. The model included in this report is intended for use as a quarterly production scheduling tool. Broad opportunities exist for extensions of the model throughout all the company's product lines.

Table of Contents:

I) EXECUTIVE SUMMARY:

This paper analyzes the production scheduling requirements for an original equipment compressor manufacturer (OEM) using linear programming techniques.

The manufacturer's objective is to develop a quarterly production schedule which will maximize profit. The scope of the analysis is restricted to one of the manufacturer's product lines which consist of eight different compressor packages, ranging in size from five to ten horsepower.

This analysis was completed using "LINDO" software. The result of our analysis suggests a production schedule which matches product demand and will generate anticipated profit of $203,000.

The true value of this model is its ability to be used to maximize profitability of the company's other product lines.

II) INTRODUCTION:

The company's problem which is analyzed in this project, relates to manufacturing and inventory management. It involves determining a quarterly production schedule for a new line of eight compressors assembled by an OEM. The compressors range in size from five to ten horsepower with different configurations. The manufacturer currently produces larger custom built compressors up to 1000 horsepower on an order to order basis. In the past, the manufacturer's customer mix has been highly dependent upon the forest products industry. With the decline of this industry, the manufacturer is attempting to diversify their product line to reach the light industrial market.

Management's goal is to determine an optimal quarterly production mix to maximize the company's profit. A quarterly sales forecast has been made and the compressor prices have been determined. One of the company's key requirements is that all orders must be filled on a timely basis. If a sale of a given unit is expected in a certain quarter, it is obtained from inventory or that unit must be manufactured in that quarter. The company has established and wishes to maintain a reputation for being responsive to customer needs. Market studies have shown that most customers have no brand loyalty and would purchase a small compressor from a competitor if it was not immediately available.

III) LITERATURE SEARCH:

The production model, that has been designed here, is very widely used one. we have extensively used the prescribed class book.

-Introduction to mathematical programming - applications and algorithms

-Lindo user's manual

Besides the above, we have done some selective references of various books in introduction level of linear programming on demand-to-manufacturing-requirement model.

IV) **MODEL:**

When the model was initially formed our goal was to minimize the inventory for each type of compressor with various features such as horsepower (5 & 10), phase (single & three) and air receiver positioning (vertical or horizontal). However, through meetings with the company's management it was discovered that they do not mind keeping a reasonable amount of inventory on hand to ensure prompt supply to customers. Short lead times and dedication to customer satisfaction are key components of the company's philosophy.

Considering the conviction of the company to meet the customer's demand, we have changed our model to maximize profit. The model analyzes operations quarterly for the period of one year.

Our objective function has become :

MAX = Sales Price * S(i,j) - Straight time Cost * P(i,j)
- Overtime Cost * Q(i,j) - Inventory Cost * I(i,j)

Where

S(i,j) = Sales forecast quarterly in 'number of compressors'

P(i,j) = Production on regular time quarterly in 'number of compressors'

Q(i,j) = Production on overtime quarterly in 'number of compressors'

I(i,j) = Inventory costs in 'number of compressors'

i = Compressors model

1 = 5 HP, 1 phase, vertical air receiver

2 = 5 HP, 3 phase, vertical air receiver

3 = 5 HP, 1 phase, horizontal air receiver

4 = 5 HP, 3 phase, horizontal air receiver

5 = 10 HP, 3 phase, horizontal air receiver

6 = 5 HP, 1 phase, steel base

7 = 5 HP, 3 phase, steel base

8 = 10 HP, 3 phase, steel base

j = 1,2,3 & 4 quarters of the year

By satisfying this objective function, we are able to meet projected demand, even if it is necessary to maintain a small inventory.

The above objective function is subject to the following constraints,

Production constraint :

Σ[{Compressor assembly time} * P(i,j)] <= 264 Hours / Quarter

regular Production time is limited by the available working hours of the assembly department which must be less than 22 working days a month, 4 hours a day, 3 months a quarter.

As these products have not yet been produced, we had some problem in narrowing down the exact production time. Many components of the compressors are bought and assembled on the production line. We timed the production of comparable existing compressor's assembly to estimate the new line's assembly time. This information was reviewed in detail with the production manager. The estimated time used in the model is expected to deviate from the actual time by less than 15%. Moreover, the model constructed for this project can be kept as a reference model for other similar projects. The model may be modified for the exact production time & cost, sales demand, inventory cost and inventory space, as required.

When the model was initially run with the above production constraints, the upper bound was hit and that caused unmet demand in the market. To avoid this problem, overtime was introduced to increase available production time.

<u>Overtime production constraint</u> :

Σ[{Compressor assembly time} * Q(i,j)] <= 132 Hours / quarter

Overtime is limited by the available overtime working hours and can not be exceed 2 hours a day, 22 days a month and 3 months a quarter.

<u>Inventory storage space constraint</u> :

Σ[{Compressor Length} * I(i,j)] <= 1317 Inches

The inventory is restricted to warehouse space which is measured in linear inches of shelf storage space. All machines have comparable widths.

Demand constraint :

$$S(i,j) \Rightarrow D(i,j)$$

The company has completed an annual sales forecast to estimate demand for the new product line. The estimated sales figures are shown in Table 1.

Table 1: Quarterly Sales Forecast

Demand (i,j)	Quarter 1	Quarter 2	Quarter 3	Quarter 4
D(1,j)	20	22	18	19
D(2,j)	16	17	14	15
D(3,j)	10	11	9	8
D(4,j)	10	9	9	8
D(5,j)	20	19	18	17
D(6,j)	3	5	4	2
D(7,j)	5	5	4	2
D(8,j)	5	10	8	8

Initially, the model was made with an equal to (=) the demand constraint. Later, it was enhanced to be greater than or equal to (=>) to include the possibility of increased demand.

Inventory equation constraint :

$$\Sigma[-I(i,j) + P(i,j) + Q(i,j) + I(i,j-1) - S(i,j)] = 0$$

In the model's preliminary runs, there was some inventory remaining at the end of the year. To ensure not having any inventory at the end of the year and to reduce inventory carrying costs, the year end inventory is made zero. This is discussed in the 'Discussion of Results'. The entire model is presented in the Appendix .

ASSUMPTIONS MADE WHILE DESIGNING THE MODEL :

The standard assumptions of linear programming apply.

The proportionality assumption :

1) The contribution to the objective function from each decision variable is proportional to the value of the decision variable.

2) The contribution of each variable to the left hand side of each constraint is proportional to the value of the variable.

The additivity assumption :

1) The contribution to the objective function for any variable is independent of the values of the other decision variables.

2) The contribution of a variable to the left hand side of each constraint is independent of the values of the other variables.

The divisibility assumption :

1) It is required that each variable be allowed to assume fraction values.

The certainty assumption :

1) It is assumed that each objective function coefficient, right hand side and technological coefficient is known with certainty.

The discount assumption :

1) It is assumed there will be no volume discount or price negotiations for the compressors. In reality this is not true. Negotiations do occur for compressors price. However, we can predict with reasonable accuracy the average unit selling price for each machine.

The demand assumption :

1) It is assumed that the sales forecast is exactly equal to demand. In addition, units demanded in a given quarter are assumed to be sold in that quarter.

The inventory assumption :

1) It is assumed that production is accounted for quarterly. Within the three months of the quarter, the stock of compressors are not consider as inventory.

The supply assumption :

1) It is assumed that the supply of raw materials and related products is made on time and do not cause any production delay.

Solution:

1) The solution to the model is presented in Appendix.

V) Discussion Of Results:

The sales forecast is totally met by production. When we changed the sales constraint to be greater than or equal to (=>) from an equal to constraint, there was no change in the value of the variables except for the 10 HP, 3 phase, steel base compressor for the first, second and third quarters. The recommend production for this compressor is more than demand, because of its higher selling price and corresponding higher contribution to profits as shown in Table 3. If any more units can be sold, it is preferred to sell the ones that result in the higher profits.

Table 2: Lindo Production Recommendations Vs. Demand

Compressor Model	Demand	L P Production values
10 HP, 3 phase, steel base, quarter-1	5	6.25
10 HP, 3 phase, steel base, quarter-2	10	18.5
10 HP, 3 phase, steel base, quarter-3	8	23.75

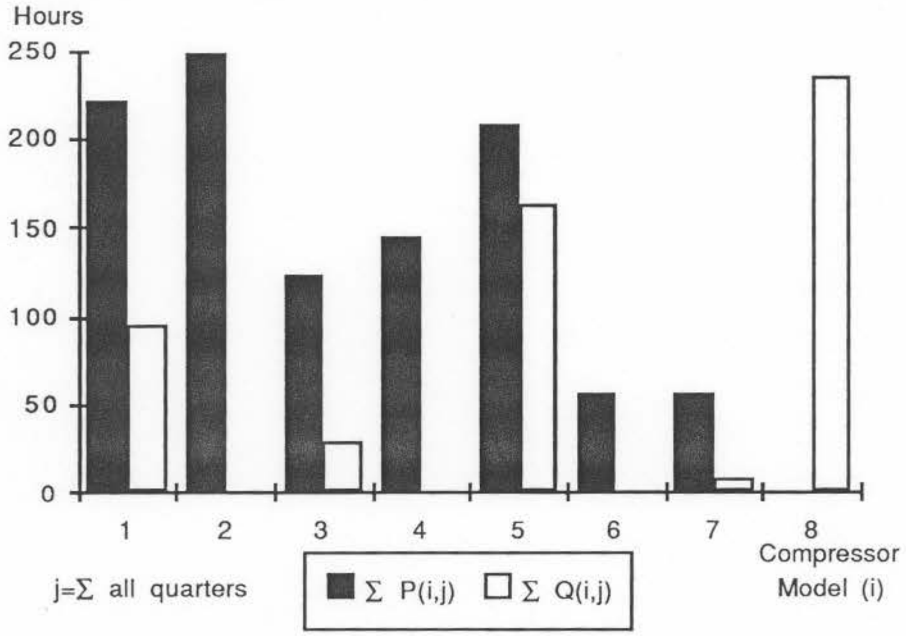
As the sales constraints are very tight, the accuracy of the forecast is very important for this model.

The regular and overtime labor production capacities are fully engaged. The company should consider adding regular time production labor if demand increases in the future. Figure 1 shows the regular time and overtime production requirements for each compressor model for all four quarters.

Table 3: Production Labor Utilization / Sales Price Summary For Year End

i	Compressor Model	Annual Total Prod. (HRS) Σ P(i,j)	Annual Total OT (HRS) Σ Q(i,j)	Sell Price
1	5 HP, 1 PHASE, VER. TANK	221	95	$2,192
2	5 HP, 3 PHASE, VER. TANK	248	0	$2,056
3	5 HP, 1 PHASE, HOR. TANK	123	29	$2,192
4	5 HP, 3 PHASE, HOR. TANK	144	0	$2,056
5	10 HP, 3 PHASE, HOR. TANK	208	162	$4,057
6	5 HP, 1 PHASE, STEEL BASE	56	0	$1,970
7	5 HP, 3 PHASE, STEEL BASE	56	8	$1,833
8	10 HP, 3 PHASE, STEEL BASE	0	234	$3,730

Figure 1: Annual Labor Utilization



The 10 HP, 3 phase,compressors are the two products which are produced during overtime extensively. This occurs because these two compressors contribute more profit than the other units.

INVENTORY STOCKING :

This model has shot out excellent control of inventory. Only in one quarter out of four is inventory carried over. In the 1st quarter 3.75 extra units of the 5 HP, 3 phase, steel base are carried over to 2nd quarter.

SLACK (SURPLUS) & SHADOW PRICES (DUAL PRICES) :

There is no slack in regular time or overtime production. The second quarter of production is more important than the other quarters, since the dual price is the maximum in that quarter. The production department should bear this in mind while planning their schedule. All the supporting departments such as maintenance and purchasing need to be aware of this.

Warehouse space for inventory is only required at the end of the first quarter. However it is important to remember we have made an assumption that the stocks will be counted as inventory only when they are produced in one quarter and not sold within that quarter. Of course, some warehouse space is required during the quarter for production and preparation for shipping .

The demand constraints project negative dual prices because of the greater than or equal to constraints. If the production department is forced to produce more than the forecasted demand, producing the 10 HP, 3 phase, horizontal air receiver compressor is the best choice among all compressors in the first , third and forth quarters . The 10 HP, 3 phase, steel base compressor is the best choice in the second quarter. This is because they have the highest dual prices in each respective quarter.

RANGE ANALYSIS :

The selling price for all of the compressors have an upper bound except the 10 HP, 3 phase, steel base in first quarter. These ranges do not consider the effect of a price change on demand.

The regular time production cost of the following compressors have no range at all. They are tightly bound and efforts must be made to have a control there

5 HP, 1 phase, vertical air receiver	- 1st quarter
5 HP, 3 phase, vertical air receiver	- 2nd quarter
5 HP, 1 phase, vertical air receiver	- 3rd quarter
10 HP, 3 phase, horizontal air receiver	- 4th quarter

The overtime production cost on the following compressors have an upper bound.

10 HP, 3 phase, horizontal air receiver	- 1st quarter
10 HP, 3 phase, steel base	- 1st quarter
5 HP, 1 phase, Vertical air receiver	- 2nd quarter
10 HP, 3 phase, steel base	- 2nd quarter
10 HP, 3 phase, horizontal air receiver	- 3rd quarter
10 HP, 3 phase, steel base	- 3rd quarter
5 HP, 3 phase, steel base	- 4th quarter
10 HP, 3 phase, steel base	- 4th quarter

<u>RANGES ON RIGHT HAND SIDE</u> :

It is interesting to look at the upper and lower bounds of production time. The company's interest will be to reduce production time for the benefit of labour cost, machine running cost etc. During third quarter production time has the widest range giving more flexibility to the production manager.

The warehouse capacity is not a immediate concern to the company at all.

The sales demand ranges are very tight on some compressors. The viability of the optimal solution depends on the accuracy of the demand forecast. Even if the demand increases by one unit on these compressors, the optimal solution changes. In addition, the company will not be able to meet the demand due to the production capacity constraints. Knowing company's goal to meet customer's demand completely, we suggest that they hold 10% of extra inventory for these compressors.

VI) EXTENSIONS:

One way the model could be extended would be to use a monthly model for the first quarter, and then a quarterly model for the remaining three quarters. This would provide more detailed information for the immediate quarter, which is of greatest value to management. Each quarter the model could be updated to do a monthly analysis of the current quarter. If a monthly analysis for the entire year was completed it would be cumbersome due to the large number of variables and constraints. In turn this could cloud immediate issues.

Another extension of the model would be to apply it to other compressor lines within the organization. Further extensions would be to integrate all the company's production into one large model. This would provide the greatest long term value.

VII) CONCLUSIONS:

This type of model lends itself to being periodically updated as sales prices, forecasts, production requirements, etc. change. The changes will result in different optimal solutions.

The model chooses to meet compressor demand by first producing compressors on regular time, then by overtime, and at a last resort by stock inventory. This is because inventory stocking charges are substantially larger than overtime charges. However, in assembly of other compressor lines labor is a greater proportion of the total cost. Then inventory could be favored in lieu of overtime.

If conditions were to change and require a significant amount of inventory we would suggest developing a constraint for minimizing inventory to reduce inventory costs. This would be developed by marketing and address the cost of a lost sale.

For this specific compressor line the assembly labor is a small fraction of cost. For this reasons the model selects overtime production in lieu of inventory. The greatest value of this model lies in the potential for extensions to other areas in the business and the integration of the entire manufacturing operation for all compressor lines.

Compressor Worksheet

i	COMPRESSOR	Sales Price S (i,j)	Material Cost	Hours to Build	Labor Cost $14/hr	Overtime Cost $21/hr	Standard Total Cost P (i,j)	Overtime Total Cost Q (i,j)	Lenght Inches	Inventory Cost 2.5% investment I (i,j)
1	5 HP, 1 PHASE, 110/220 VOLT, VERTICAL TANK	$2,192	$1,698	4	$56	$84	$1,754	$1,782	36	$44
2	5 HP, 3 PHASE, 208/230/460 VOLT, VERTICAL TANK	$2,056	$1,589	4	$56	$84	$1,645	$1,673	36	$41
3	5 HP, 1 PHASE, 110/220 VOLT, HORIZONTAL TANK	$2,192	$1,698	4	$56	$84	$1,754	$1,782	63	$44
4	5 HP, 3 PHASE, 208/230/460 VOLT, HORIZONTAL TANK	$2,056	$1,589	4	$56	$84	$1,645	$1,673	63	$41
5	10 HP, 3 PHASE, 208/230/460 VOLT, HORIZONTAL TANK	$4,057	$3,176	5	$70	$105	$3,246	$3,281	69	$81
6	5 HP, 1 PHASE, 110/220 VOLT, STEEL BASE	$1,970	$1,520	4	$56	$84	$1,576	$1,604	36	$39
7	5 HP, 3 PHASE, 208/230/460 VOLT, STEEL BASE	$1,833	$1,410	4	$56	$84	$1,466	$1,494	36	$37
8	10 HP, 3 PHASE, 208/230/460 VOLT, STEEL BASE	$3,730	$2,928	4	$56	$84	$2,984	$3,012	42	$75

Width of all new compressors is 36 inches, for storage allow 6 inches on each side, therefore total width is 48 inches or 4 feet

Warehouse available to store all company's inventory is 97 ft long by 51 ft wide

The lineary feet avaiable in the warehouse is (51/4)+97 = 109 ft (or 1317 lineary inches)

Compressor Worksheet

i	COMPRESSOR	Sales Forecast Winter S (i,1)	Sales Forecast Spring S (i,2)	Sales Forecast Summer S (i,3)	Sales Forecast Fall S (i,4)
1	5 HP, 1 PHASE, 110/220 VOLT, VERTICAL TANK	20	22	18	19
2	5 HP, 3 PHASE, 208/230/460 VOLT, VERTICAL TANK	16	17	14	15
3	5 HP, 1 PHASE, 110/220 VOLT, HORIZONTAL TANK	10	11	9	8
4	5 HP, 3 PHASE, 208/230/460 VOLT, HORIZONTAL TANK	10	9	9	8
5	10 HP, 3 PHASE, 208/230/460 VOLT, HORIZONTAL TANK	20	19	18	17
6	5 HP, 1 PHASE, 110/220 VOLT, STEEL BASE	3	5	4	2
7	5 HP, 3 PHASE, 208/230/460 VOLT, STEEL BASE	3	5	4	2
8	10 HP, 3 PHASE, 208/230/460 VOLT, STEEL BASE	5	10	8	8

Lindo Computer Input

```
!
!                    Term Project
!
!
! For a packaged compressor manufacturer determine the quarterly
! production mix for 8 different products to maximize profit.
! Subject to:
!      1) Quarterly sales forecast  (S)
!      2) Production Capacity (P), (Q)
!      3) Inventory costs (I)
!      4) Inventory space
!
!      Variable (i,j) where
!            j = quarter of the year
!            i = compressor model
!                where
!              1 =  5 hp, 1 phase, 110/220 volt, vertical air tank
!              2 =  5 hp, 3 phase, 208/230/460 volt, vertical tank
!              3 =  5 hp, 1 phase, 110/220 volt, horizontal tank
!              4 =  5 hp, 3 phase, 208/230/460 volt, horizontal tank
!              5 = 10 hp, 3 phase, 208/230/460 volt, horizontal tank
!              6 =  5 hp, 1 phase, 110/220 volt, steel base
!              7 =  5 hp, 3 phase, 208/230/460 volt, steel base
!              8 = 10 hp, 3 phase, 208/230/460 volt, steel base
!
!
MAX
2192S11 + 2056S21 + 2192S31 + 2056S41 + 4057S51 + 1970S61 +
1833S71 +3730S81 + 2192S12 + 2056S22 + 2192S32 + 2056S42 +
4057S52 + 1970S62 + 1833S72 +3730S82 + 2192S13 + 2056S23 +
2192S33 + 2056S43 + 4057S53 + 1970S63 + 1833S73 + 3730S83 +
2192S14 + 2056S24 + 2192S34 + 2056S44 + 4057S54 + 1970S64 +
1833S74 + 3730S84 - 1754P11 - 1645P21 - 1754P31 -1645P41 -
3246P51 - 1576P61  - 1466P71 - 2984P81 - 1754P12 - 1645P22 -
```

```
1754P32 -1645P42 - 3246P52 - 1576P62 - 1466P72 - 2984P82 -
1754P13 - 1645P23 - 1754P33 -1645P43 - 3246P53 -1576P63 -
1466P73 - 2984P83- 1754P14 - 1645P24 - 1754P34 -1645P44 -
3246P54 - 1576P64 - 1466P74 - 2984P84 - 1782Q11 - 1673Q21 -
1782Q31 - 1673Q41 - 3281Q51 - 1604Q61 - 1494Q71 - 3012Q81-
1782Q12 - 1673Q22 - 1782Q32 - 1673Q42 - 3281Q52 - 1604Q62 -
1494Q72 - 3012Q82- 1782Q13 - 1673Q23 - 1782Q33 - 1673Q43 -
3281Q53 - 1604Q63 - 1494Q73 - 3012Q83- 1782Q14 - 1673Q24 -
1782Q34 - 1673Q44 - 3281Q54 - 1604Q64 - 1494Q74 - 3012Q84-
44I11 - 41I21 - 44I31 - 41I41 - 81I51- 39I61 - 37I71 - 75I81 -
44I12 - 41I22 - 44I32 - 41I42 - 81I52 - 39I62 - 37I72 - 75I82 -
44I13 - 41I23 - 44I33 - 41I43 - 81I53 - 39I63 - 37I73 - 75I83 -
44I14 - 41I24 - 44I34 - 41I44 - 81I54 - 39I64 - 37I74 - 75I84
!
ST
!
! Production limitation, hours to assemble each machine must be
!lessthan 22 working days a !month, 4 hours a day, 3 months a
!quarter
!1st quart
4P11 + 4P21 + 4P31 + 4P41 + 5P51 + 4P61 + 4P71 + 4P81 <= 264
!2nd quart
4P12 + 4P22 + 4P32 + 4P42 + 5P52 + 4P62 + 4P72 + 4P82 <= 264
!3rd quart
4P13 + 4P23 + 4P33 + 4P43 + 5P53 + 4P63 + 4P73 + 4P83 <= 264
!4th quart
4P14 + 4P24 + 4P34 + 4P44 + 5P54 + 4P64 + 4P74 + 4P84 <= 264
! Overtime limitation, must be less than 2 hours a day, 22 days a
month, 3 months a quarter
!1st quart
4Q11 + 4Q21 + 4Q31 + 4Q41 + 5Q51 + 4Q61 + 4Q71 + 4Q81 <= 132
!2nd quart
4Q12 + 4Q22 + 4Q32 + 4Q42 + 5Q52 + 4Q62 + 4Q72 + 4Q82 <= 132
!3rd quart
4Q13 + 4Q23 + 4Q33 + 4Q43 + 5Q53 + 4Q63 + 4Q73 + 4Q83 <= 132
```

```
!4th quart
4Q14 + 4Q24 + 4Q34 + 4Q44 + 5Q54 + 4Q64 + 4Q74 + 4Q84 <= 132
! Inventory of compressors is limited to warehouse space of 1317
linear inches
36I11 + 36I21 + 63I31 + 63I41 + 69I51 + 36I61 + 36I71 + 42I81 <=
1317
36I12 + 36I22 + 63I32 + 63I42 + 69I52 + 36I62 + 36I72 + 42I82 <=
1317
36I13 + 36I23 + 63I33 + 63I43 + 69I53 + 36I63 + 36I73 + 42I83 <=
1317
36I14 + 36I24 + 63I34 + 63I44 + 69I54 + 36I64 + 36I74 + 42I84 <=
1317
!


! Sales forecasts 1st quarter
S11 > 20
S21 > 16
S31 > 10
S41 > 10
S51 > 20
S61 > 3
S71 > 5
S81 > 5
!
! Saleas forecasts 2nd quarter
S12 > 22
S22 > 17
S32 > 11
S42 > 9
S52 >19
S62 >5
S72 >5
S82 >10
!
```

```
! Sales forecasts 3rd quarter
S13 >18
S23 >14
S33 > 9
S43 > 9
S53 > 18
S63 > 4
S73 > 4
S83 > 8
!
! Sales forecasts 4th quarter
S14 > 19
S24 > 15
S34 > 8
S44 > 8
S54 >17
S64 > 2
S74 > 2
S84 > 8
!
! Inventory equation first quarter
-I11 + P11  + Q11 - S11 = 0
-I21 + P21  + Q21 - S21 = 0
-I31 + P31  + Q31 - S31 = 0
-I41 + P41  + Q41 - S41 = 0
-I51 + P51  + Q51 - S51 = 0
-I61 + P61  + Q61 - S61 = 0
-I71 + P71  + Q71 - S71 = 0
-I81 + P81  + Q81 - S81 = 0
!
! Inventory equation second quarter
-I12 + P12  + I11 + Q12 - S12 = 0
-I22 + P22  + I21 + Q22 - S22 = 0
-I32 + P32  + I31 + Q32 - S32 = 0
-I42 + P42  + I41 + Q42 - S42 = 0
-I52 + P52  + I51 + Q52 - S52 = 0
```

```
-I62 + P62  + I61 + Q62 - S62 = 0
-I72 + P72  + I71 + Q72 - S72 = 0
-I82 + P82  + I81 + Q82 - S82 = 0
!
! Inventory equation third quarter
-I13 + P13  + I12 + Q13 - S13 = 0
-I23 + P23  + I22 + Q23 - S23 = 0
-I33 + P33  + I32 + Q33 - S33 = 0
-I43 + P43  + I42 + Q43 - S43 = 0
-I53 + P53  + I52 + Q53 - S53 = 0
-I63 + P63  + I62 + Q63 - S63 = 0
-I73 + P73  + I72 + Q73 - S73 = 0
-I83 + P83  + I82 + Q83 - S83 = 0
!
! Inventory equation forth quarter
 + P14  + I13 + Q14 - S14 = 0
 + P24  + I23 + Q24 - S24 = 0
 + P34  + I33 + Q34 - S34 = 0
 + P44  + I43 + Q44 - S44 = 0
 + P54  + I53 + Q54 - S54 = 0
 + P64  + I63 + Q64 - S64 = 0
 + P74  + I73 + Q74 - S74 = 0
 + P84  + I83 + Q84 - S84 = 0
!
END
```

Computer Output

LP OPTIMUM FOUND AT STEP 61

OBJECTIVE FUNCTION VALUE

1) 202732.240

VARIABLE	VALUE	REDUCED COST
S11	20.000000	.000000
S21	16.000000	.000000
S31	10.000000	.000000
S41	10.000000	.000000
S51	20.000000	.000000
S61	3.000000	.000000
S71	5.000000	.000000
S81	6.250000	.000000
S12	22.000000	.000000
S22	17.000000	.000000
S32	11.000000	.000000
S42	9.000000	.000000
S52	19.000000	.000000
S62	5.000000	.000000
S72	5.000000	.000000
S82	10.000000	.000000
S13	18.000000	.000000
S23	14.000000	.000000
S33	9.000000	.000000
S43	9.000000	.000000
S53	18.000000	.000000
S63	4.000000	.000000
S73	4.000000	.000000
S83	18.500000	.000000

S14	19.000000	.000000
S24	15.000000	.000000
S34	8.000000	.000000
S44	8.000000	.000000
S54	17.000000	.000000
S64	2.000000	.000000
S74	2.000000	.000000
S84	23.750000	.000000
P11	18.250000	.000000
P21	16.000000	.000000
P31	10.000000	.000000
P41	10.000000	.000000
P51	.000000	.000000
P61	3.000000	.000000
P71	8.750000	.000000
P81	.000000	186.500000
P12	.000000	.000000
P22	17.000000	.000000
P32	11.000000	.000000
P42	9.000000	.000000
P52	18.200000	.000000
P62	5.000000	.000000
P72	1.250000	.000000
P82	.000000	195.750000
P13	18.000000	.000000
P23	14.000000	.000000
P33	9.000000	.000000
P43	9.000000	.000000
P53	6.400000	.000000
P63	4.000000	.000000
P73	4.000000	.000000
P83	.000000	186.500000
P14	19.000000	.000000
P24	15.000000	.000000
P34	.750000	.000000
P44	8.000000	.000000

P54	17.000000	.000000
P64	2.000000	.000000
P74	.000000	.000000
P84	.000000	186.500000
Q11	1.750000	.000000
Q21	.000000	.000000
Q31	.000000	.000000
Q41	.000000	.000000
Q51	20.000000	.000000
Q61	.000000	.000000
Q71	.000000	.000000
Q81	6.250000	.000000
Q12	22.000000	.000000
Q22	.000000	.000000
Q32	.000000	.000000
Q42	.000000	.000000
Q52	.800000	.000000
Q62	.000000	.000000
Q72	.000000	.000000
Q82	10.000000	.000000
Q13	.000000	.000000
Q23	.000000	.000000
Q33	.000000	.000000
Q43	.000000	.000000
Q53	11.600000	.000000
Q63	.000000	.000000
Q73	.000000	.000000
Q83	18.500000	.000000
Q14	.000000	.000000
Q24	.000000	.000000
Q34	7.250000	.000000
Q44	.000000	.000000
Q54	.000000	.000000
Q64	.000000	.000000
Q74	2.000000	.000000
Q84	23.750000	.000000

I11	.000000	7.000000
I21	.000000	4.000000
I31	.000000	7.000000
I41	.000000	4.000000
I51	.000000	34.750000
I61	.000000	2.000000
I71	3.750000	.000000
I81	.000000	38.000000
I12	.000000	81.000000
I22	.000000	78.000000
I32	.000000	81.000000
I42	.000000	78.000000
I52	.000000	127.250000
I62	.000000	76.000000
I72	.000000	74.000000
I82	.000000	112.000000
I13	.000000	44.000000
I23	.000000	41.000000
I33	.000000	44.000000
I43	.000000	41.000000
I53	.000000	81.000000
I63	.000000	39.000000
I73	.000000	37.000000
I83	.000000	75.000000
I14	.000000	44.000000
I24	.000000	41.000000
I34	.000000	44.000000
I44	.000000	41.000000
I54	.000000	81.000000
I64	.000000	39.000000
I74	.000000	37.000000
I84	.000000	75.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	186.500000

3)	.000000	195.750000
4)	.000000	186.500000
5)	.000000	186.500000
6)	.000000	179.500000
7)	.000000	188.750000
8)	.000000	179.500000
9)	.000000	179.500000
10)	1182.000000	.000000
11)	1317.000000	.000000
12)	1317.000000	.000000
13)	1317.000000	.000000
14)	.000000	-308.000000
15)	.000000	-335.000000
16)	.000000	-308.000000
17)	.000000	-335.000000
18)	.000000	-121.500000
19)	.000000	-352.000000
20)	.000000	-379.000000
21)	1.250000	.000000
22)	.000000	-345.000000
23)	.000000	-372.000000
24)	.000000	-345.000000
25)	.000000	-372.000000
26)	.000000	-167.750000
27)	.000000	-389.000000
28)	.000000	-416.000000
29)	.000000	-37.000000
30)	.000000	-308.000000
31)	.000000	-335.000000
32)	.000000	-308.000000
33)	.000000	-335.000000
34)	.000000	-121.500000
35)	.000000	-352.000000
36)	.000000	-379.000000
37)	10.500000	.000000
38)	.000000	-308.000000

39)	.000000	-335.000000
40)	.000000	-308.000000
41)	.000000	-335.000000
42)	.000000	-121.500000
43)	.000000	-352.000000
44)	.000000	-379.000000
45)	15.750000	.000000
46)	.000000	-2500.000000
47)	.000000	-2391.000000
48)	.000000	-2500.000000
49)	.000000	-2391.000000
50)	.000000	-4178.500000
51)	.000000	-2322.000000
52)	.000000	-2212.000000
53)	.000000	-3730.000000
54)	.000000	-2537.000000
55)	.000000	-2428.000000
56)	.000000	-2537.000000
57)	.000000	-2428.000000
58)	.000000	-4224.750000
59)	.000000	-2359.000000
60)	.000000	-2249.000000
61)	.000000	-3767.000000
62)	.000000	-2500.000000
63)	.000000	-2391.000000
64)	.000000	-2500.000000
65)	.000000	-2391.000000
66)	.000000	-4178.500000
67)	.000000	-2322.000000
68)	.000000	-2212.000000
69)	.000000	-3730.000000
70)	.000000	-2500.000000
71)	.000000	-2391.000000
72)	.000000	-2500.000000
73)	.000000	-2391.000000
74)	.000000	-4178.500000

75)	.000000	-2322.000000
76)	.000000	-2212.000000
77)	.000000	-3730.000000

NO. ITERATIONS= 61

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
S11	2192.000000	308.000000	INFINITY
S21	2056.000000	335.000000	INFINITY
S31	2192.000000	308.000000	INFINITY
S41	2056.000000	335.000000	INFINITY
S51	4057.000000	121.500000	INFINITY
S61	1970.000000	352.000000	INFINITY
S71	1833.000000	379.000000	INFINITY
S81	3730.000000	INFINITY	37.000000
S12	2192.000000	345.000000	INFINITY
S22	2056.000000	372.000000	INFINITY
S32	2192.000000	345.000000	INFINITY
S42	2056.000000	372.000000	INFINITY
S52	4057.000000	167.750000	INFINITY
S62	1970.000000	389.000000	INFINITY
S72	1833.000000	416.000000	INFINITY
S82	3730.000000	37.000000	INFINITY
S13	2192.000000	308.000000	INFINITY
S23	2056.000000	335.000000	INFINITY
S33	2192.000000	308.000000	INFINITY
S43	2056.000000	335.000000	INFINITY
S53	4057.000000	121.500000	INFINITY
S63	1970.000000	352.000000	INFINITY
S73	1833.000000	379.000000	INFINITY
S83	3730.000000	74.000000	37.000000
S14	2192.000000	308.000000	INFINITY

S24	2056.000000	335.000000	INFINITY
S34	2192.000000	308.000000	INFINITY
S44	2056.000000	335.000000	INFINITY
S54	4057.000000	121.500000	INFINITY
S64	1970.000000	352.000000	INFINITY
S74	1833.000000	379.000000	INFINITY
S84	3730.000000	37.000000	97.200000
P11	-1754.000000	.000000	.000000
P21	-1645.000000	4.000000	.000000
P31	-1754.000000	7.000000	.000000
P41	-1645.000000	4.000000	.000000
P51	-3246.000000	.000000	INFINITY
P61	-1576.000000	2.000000	.000000
P71	-1466.000000	37.000000	.000000
P81	-2984.000000	186.500000	INFINITY
P12	-1754.000000	.000000	INFINITY
P22	-1645.000000	78.000000	.000000
P32	-1754.000000	81.000000	.000000
P42	-1645.000000	78.000000	.000000
P52	-3246.000000	.000000	.000000
P62	-1576.000000	76.000000	.000000
P72	-1466.000000	2.000000	.000000
P82	-2984.000000	195.750000	INFINITY
P13	-1754.000000	44.000000	.000000
P23	-1645.000000	41.000000	.000000
P33	-1754.000000	44.000000	.000000
P43	-1645.000000	41.000000	.000000
P53	-3246.000000	.000000	46.250000
P63	-1576.000000	39.000000	.000000
P73	-1466.000000	37.000000	.000000
P83	-2984.000000	186.500000	INFINITY
P14	-1754.000000	308.000000	.000000
P24	-1645.000000	335.000000	.000000
P34	-1754.000000	.000000	.000000
P44	-1645.000000	335.000000	.000000
P54	-3246.000000	121.500000	.000000

P64	-1576.000000	352.000000	.000000
P74	-1466.000000	.000000	INFINITY
P84	-2984.000000	186.500000	INFINITY
Q11	-1782.000000	.000000	.000000
Q21	-1673.000000	.000000	INFINITY
Q31	-1782.000000	.000000	INFINITY
Q41	-1673.000000	.000000	INFINITY
Q51	-3281.000000	34.750000	.000000
Q61	-1604.000000	.000000	INFINITY
Q71	-1494.000000	.000000	INFINITY
Q81	-3012.000000	38.000000	37.000000
Q12	-1782.000000	81.000000	.000000
Q22	-1673.000000	.000000	INFINITY
Q32	-1782.000000	.000000	INFINITY
Q42	-1673.000000	.000000	INFINITY
Q52	-3281.000000	.000000	.000000
Q62	-1604.000000	.000000	INFINITY
Q72	-1494.000000	.000000	INFINITY
Q82	-3012.000000	37.000000	38.000000
Q13	-1782.000000	.000000	INFINITY
Q23	-1673.000000	.000000	INFINITY
Q33	-1782.000000	.000000	INFINITY
Q43	-1673.000000	.000000	INFINITY
Q53	-3281.000000	46.250000	.000000
Q63	-1604.000000	.000000	INFINITY
Q73	-1494.000000	.000000	INFINITY
Q83	-3012.000000	74.000000	37.000000
Q14	-1782.000000	.000000	INFINITY
Q24	-1673.000000	.000000	INFINITY
Q34	-1782.000000	.000000	.000000
Q44	-1673.000000	.000000	INFINITY
Q54	-3281.000000	.000000	INFINITY
Q64	-1604.000000	.000000	INFINITY
Q74	-1494.000000	379.000000	.000000
Q84	-3012.000000	37.000000	97.200000
I11	-44.000000	7.000000	INFINITY

I21	-41.000000	4.000000	INFINITY
I31	-44.000000	7.000000	INFINITY
I41	-41.000000	4.000000	INFINITY
I51	-81.000000	34.750000	INFINITY
I61	-39.000000	2.000000	INFINITY
I71	-37.000000	37.000000	2.000000
I81	-75.000000	38.000000	INFINITY
I12	-44.000000	81.000000	INFINITY
I22	-41.000000	78.000000	INFINITY
I32	-44.000000	81.000000	INFINITY
I42	-41.000000	78.000000	INFINITY
I52	-81.000000	127.250000	INFINITY
I62	-39.000000	76.000000	INFINITY
I72	-37.000000	74.000000	INFINITY
I82	-75.000000	112.000000	INFINITY
I13	-44.000000	44.000000	INFINITY
I23	-41.000000	41.000000	INFINITY
I33	-44.000000	44.000000	INFINITY
I43	-41.000000	41.000000	INFINITY
I53	-81.000000	81.000000	INFINITY
I63	-39.000000	39.000000	INFINITY
I73	-37.000000	37.000000	INFINITY
I83	-75.000000	75.000000	INFINITY
I14	-44.000000	44.000000	INFINITY
I24	-41.000000	41.000000	INFINITY
I34	-44.000000	44.000000	INFINITY
I44	-41.000000	41.000000	INFINITY
I54	-81.000000	81.000000	INFINITY
I64	-39.000000	39.000000	INFINITY
I74	-37.000000	37.000000	INFINITY
I84	-75.000000	75.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	264.000000	7.000000	5.000000

3	264.000000	7.000000	5.000000
4	264.000000	58.000000	32.000000
5	264.000000	29.000000	3.000000
6	132.000000	INFINITY	5.000000
7	132.000000	7.000000	4.000000
8	132.000000	INFINITY	42.000000
9	132.000000	INFINITY	63.000000
10	1317.000000	INFINITY	1182.000000
11	1317.000000	INFINITY	1317.000000
12	1317.000000	INFINITY	1317.000000
13	1317.000000	INFINITY	1317.000000
14	20.000000	1.250000	1.750000
15	16.000000	1.250000	1.750000
16	10.000000	1.250000	1.750000
17	10.000000	1.250000	1.750000
18	20.000000	1.000000	20.000000
19	3.000000	1.250000	1.750000
20	5.000000	1.250000	1.750000
21	5.000000	1.250000	INFINITY
22	22.000000	1.000000	1.750000
23	17.000000	1.250000	1.750000
24	11.000000	1.250000	1.750000
25	9.000000	1.250000	1.750000
26	19.000000	1.000000	1.400000
27	5.000000	1.250000	1.750000
28	5.000000	1.250000	1.750000
29	10.000000	1.000000	1.750000
30	18.000000	8.000000	14.500000
31	14.000000	8.000000	14.000000
32	9.000000	8.000000	9.000000
33	9.000000	8.000000	9.000000
34	18.000000	8.400000	11.600000
35	4.000000	8.000000	4.000000
36	4.000000	8.000000	4.000000
37	8.000000	10.500000	INFINITY
38	19.000000	.750000	7.250000

39	15.000000	.750000	7.250000
40	8.000000	15.750000	7.250000
41	8.000000	.750000	7.250000
42	17.000000	.600000	5.800000
43	2.000000	.750000	2.000000
44	2.000000	15.750000	2.000000
45	8.000000	15.750000	INFINITY
46	.000000	1.250000	1.750000
47	.000000	1.250000	1.750000
48	.000000	1.250000	1.750000
49	.000000	1.250000	1.750000
50	.000000	1.000000	20.000000
51	.000000	1.250000	1.750000
52	.000000	1.250000	1.750000
53	.000000	1.250000	INFINITY
54	.000000	1.000000	1.750000
55	.000000	1.250000	1.750000
56	.000000	1.250000	1.750000
57	.000000	1.250000	1.750000
58	.000000	1.000000	1.400000
59	.000000	1.250000	1.750000
60	.000000	1.250000	1.750000
61	.000000	1.000000	1.750000
62	.000000	8.000000	14.500000
63	.000000	8.000000	14.000000
64	.000000	8.000000	9.000000
65	.000000	8.000000	9.000000
66	.000000	8.400000	11.600000
67	.000000	8.000000	4.000000
68	.000000	8.000000	4.000000
69	.000000	10.500000	INFINITY
70	.000000	.750000	7.250000
71	.000000	.750000	7.250000
72	.000000	15.750000	7.250000
73	.000000	.750000	7.250000
74	.000000	.600000	5.800000

75	.000000	.750000	2.000000
76	.000000	15.750000	2.000000
77	.000000	15.750000	INFINITY