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Abstract: This project models and optimizes operation of a heat treatment furnace department of a foundry. Heat treatment furnace operation involves different operational temperatures, durations and atmospheres. It is inherently a batch process, but may be modeled as a linear program. The objective is to develop a tool to allow a weekly operational plan providing operators with a cost optimal strategy. Two methods were developed, one was formulated using pounds of furnace capacity and one using furnace time as variables. Both methods provided solutions, but the furnace hours best represented the nature of the problem. The program solutions provide a method to lower operational costs while providing insight to the characteristics of the process. The program may be rerun for any changes such as equipment failure or new parts to run, and a new department operation plan developed.

OPTIMIZATION OF
HEAT TREATMENT FURNACE OPERATION

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EMP-P92~~19~~

Optimization
of
Heat Treatment Furnace Operation

By
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Heat Treatment Furnace Operation

Executive Summary

Donna Street, Steve Blaine, John Brunke, Murat Ayabakan

A tool was developed using linear programming techniques to improve operation in the Precision Castparts Heat Treatment Department.

The production schedule for a particular week was analyzed and the optimal strategy for running the department was found.

The program provides guidance to load furnaces in the most cost effective manner. Forecast of the weekly schedule for operations, labor (leave and overtime), and maintenance is enhanced. It also provides a tool for Engineering to study future equipment additions or modifications. Trade offs in operating cost can be predicted for changes in schedule and product mix. Impact of additional workload or downtime can be forecast.

Introduction

The heat treatment department at Precision Castparts Corporations Small Structures Plant (SSBO) processes thousands of stainless steel alloy parts each month. The parts made include artificial joints and turbine blades to mention a few. The properties of these materials are critical. To achieve these properties treatments are required at temperatures from 1100 to 2200 degrees Fahrenheit, for times from 1 to 20 hours, in either atmospheric air, argon, or vacuum.

The department has a number of furnaces available. Some are exclusively for vacuum processing and others are for either atmospheric air or argon processing. The cost of department operations varies with the type of processing required and which furnace is selected for each processing operation. The furnaces differ in capacity and efficiency. Meeting delivery schedule is the primary goal forcing parts to be moved through the department quickly. This often forces a furnace to be run with less than a full load. A methodology to assist operators in determining which furnace to run how much of each process has the potential to save the company money and improve department operations. This is the goal of this project.

Figure 1 shows the layout of the heat treat area. The department has seven different furnaces. Some of these furnaces are more costly to operate since they operate at higher temperatures and with argon or vacuum atmosphere. The furnaces have different capabilities, capacities, and efficiencies. The vacuum furnaces only had three processes scheduled and the problem became trivial. This report will therefore examine only the air/argon furnaces in detail. The vacuum furnace solution is included in Appendix III.

The Problem

The problem is to schedule the furnaces to reduce processing cost while meeting production schedule.

Furnaces C18 and C25 operate in air or argon and at temperatures from 1200 to 2100 degrees Fahrenheit. Furnaces C10 and C30 operate in vacuum over the same temperature range and can quench parts with argon.

In formulation of our problem the furnaces are identified as follows:

Furnace #1	C18	air or argon processing
Furnace #2	C25	air or argon processing
Furnace #3	C10	vacuum processing
Furnace #4	C30	vacuum processing

Appendix #1 shows the furnace capabilities, capacities, operational costs etc.

Methodology

Two different methodologies were employed for solving this problem. They were then compared. The problem formulations were all of a transportation type problem (1,2,3,4,5).

In the first method, the objective variables were pounds of material processed. The a_{ij} coefficients were in hours per pound and the C_{ij} coefficients were in dollars per pound.

In the second method the objective variables are in hours, the a_{ij} coefficients are in units of pounds per hour for the demand constraints, and the C_{ij} coefficients are in units of dollars per hour. This method is similar to that used by Chung (5, page 279) to create a transportation problem by using ratios of machine capacity.

Assumptions

Because of the batch nature of this process an initial difficulty was encountered.

The batch process lends itself to an integer solution. Reducing the number of batch runs will reduce cost. As an option we studied the calculation of a cost function of the pounds of material processed. This resulted in a non-linear problem. It was apparent that we would have to make certain assumptions to allow a linear program solution

Assumptions:

Method 1

The furnaces process material at a constant cost per pound regardless of percent capacity used. Therefore a half full batch costs the same per pound as a full batch to process. The time duration of each process was factored into the formulation.

Method 2

The assumption for this method is that batches are divisible at the same batch cost, but factors are included for partial batches in the derivation of the constraint coefficients.

Linear Program Conditions

It is important to verify that we have met the basic assumptions of linear programming:

1. Linearity and additivity. This assumption was key to simplify our program. For method 1, cost per pound is constant for each furnace and process. For method 2, cost per hour is constant. This is the key difference between the two methods.

2. Divisibility. The divisibility assumption applies because the problem was formulated in terms of pounds of material processed or hours of furnace operation, not batches. If this assumption could not be met the problem would have to be solved as an integer problem. If we had the tools to solve this type of problem in this manner we might be able to improve our solution.
3. Fitness or limited resources. There is finite number of furnaces and finite number of hours to process all the parts in the department, therefore this condition is met.
4. Deterministic. The variables do not change over the period of the study (one week).

Data Acquisition

Data for a typical production week was gathered. For furnaces 1 & 2, over 1200 parts were in the staging area waiting to be heat treated. There were 18 different heat treatment processes required for these parts. For furnaces 3 & 4, 135 parts required 3 different processes. Sample documentation showing process requirements and pounds per part are attached. See appendix II.

Parts mix varies from week to week. Studies could be run at the beginning of each week to give operators guidance to the most optimal method for the weeks operation.

Linear Program Formulation

Method 1

In order to avoid the integer/non-linear difficulties previously discussed, this problem was set up on a per pound basis.

The objective coefficients were computed for each of the 18 processes and for each furnace. Adjustments for different operating temperatures were then made to determine the furnace operating cost per hour (\$/hr) for each process. These are summarized in appendix I. The objective coefficients were then calculated in units of dollars per pound as follows:

$$C_{ij} = \frac{\text{Furnace operational cost per hour}(\$/\text{hr}) \times \text{Cycle time}(\text{hrs})}{\text{Furnace Capacity (lbs)}}$$

$$= \text{dollars per pound } (\$/\text{lb})$$

These coefficients are only accurate for full furnace loads. Making the coefficients change with percent capacity results in a nonlinear problem. In this problem this assumption is justified as almost all furnace loads are near capacity due to the large number of parts in each process batch.

The constraints were then formulated for the time required to process parts in each of the furnaces. A limit of 5 days (3 shifts per days) was selected to process all the parts in order to meet delivery schedule. Each furnace has a capacity limit and the constraint was formulated with the coefficients in terms of pounds per hour. These were calculated for each furnace and each process as follows:

$$a_{ij} = \frac{\text{Cycle Time (hrs)}}{\text{Furnace Capacity (lbs)}} = \text{hrs/lb}$$

The variables (X_{ij}) are the pounds of each process to be processed in each furnace. The first subscript (1,2,3,4) is the furnace number. The second subscript (1 to 18 for argon/air, and 1 to 3 for vacuum) is the process number. For example; X_{214} is the pounds processed in furnace 2, of process 14.

The constraint equations are therefore:

1. The total hours available in furnace number 1
2. The total hours available in furnace number 2
- 3-21. The requirements to complete each required component.

All variables are non-negative.

It is a process scheduling problem. It is also similar to a transportation problem. We have a demand for parts and a supply of a resource (furnace time). In researching this problem we found similar problems in a number of sources. In a book by Chung(ref, page 279) there is a problem which has a number of machines which operate at different efficiencies are scheduled to process material in the most efficient manner. Machines are related in terms of their hourly production rates to make the problem a transportation problem. This is the same approach which we have taken.

Method 2

For this solution the problem was set up in a per hour basis.

The objective coefficients were computed for each furnace, at each process operation temperature. These are included in Appendix II.

C_{ij} = operational cost in dollars per hour (@ operational temperature). = \$/lb

These coefficients are valid for either full loads or partial loads unlike method 1. The objective function totals operational costs from both furnaces.

The constraints are 1) total available furnace hours (supply), and 2) to for each process to total demand for each process.

The coefficients of the demand constraints were derived as follows:

1. The total pounds of each process was divided by the furnace capacity, then rounded up to make whole "batches".

2. The "batches" was multiplied by the hours per batch.
3. The total pounds to be processed was divided by total hours (2).

The units of the "batches" (lb/lb) is dimensionless. Then multiplied by hours (units hours). Total pounds divided by this number then yields pound per hour.

$d = \text{Integer}(\text{lbs matl to be processed} / \text{furnace capacity})$

$$a_{ij} = \frac{\text{total lbs to be processed in this process}}{d * \text{hours / batch}}$$

The variables (X_{ij}) are the hours of furnace time for each process, for each furnace. The first subscript (1,2,3,4) is the furnace number. The second subscript (1 to 18 for argon/air, and 1 to 3 for vacuum) is the process number. For example; X_{214} is the hours in furnace 2, of process 14.

The constraint equations are therefore:

1. The total hours available in furnace number 1
2. The total hours available in furnace number 2
- 3-21. The requirements to complete each required component.

All variables are non-negative.

Note: these are the same as in method 1.

LINDO

The linear programs were solved using LINDO (6). The solution for the method 1 problem was obtained in 6 iterations for the air/argon problem and in 6 iterations for the vacuum processed parts (see Appendix III). The method 2 solution was run for the air/argon processes only and gave a solution in 7 steps.

A warning message was given by LINDO for the method 1 air/argon processed parts. The two first constraint equations were multiplied on both sides by 10 to better scale the problem.

The Solution By Method 1

LINDO gave a solution to the problem as follows:

Furnace # 1

Process	Pounds
5	1943
6	459
7	462
8	3460
9	525
18	639

Furnace #2

1	14707
2	1318
3	10964
4	669
8	823
10	2333
11	2618
12	1314

Process	Pounds
13	2770
14	1046
15	2142
16	578
17	2003

The objective function value is \$3015.35.

Looking at the results we see that only process 8 is split between the two furnaces. As expected furnace #2 is preferred as it has the lower cost per pound. The capacity of furnace #2 is entirely used up, and furnace #1 has over 37 hours available. This indicates what reserve capacity this department has.

Solution by Method 2

LINDO produced the following solution for the problem when the problem was constructed on an hours basis and the total weekly available furnace hours was held to 130 each:

Furnace #1

Process	Hours
4	5
5	55.2
6	21
7	3.8
10	12.3
16	5.1
17	3.86
18	4.0

Furnace #2

Process	Hours
1	40
2	3.5
3	35.2
8	12
11	8.8
12	3.5
13	7.6
14	4.3
15	10
17	5

The value of the objective function is \$3595.08.

Sensitivity Analysis

METHOD 1

SHADOW PRICES. Furnace one has a surplus of 37.9 (379.4 in equation due to scaling) hours which can be used for processing. The shadow price for this constraint is therefore zero. Any additional processing should be done in Furnace one until the surplus is exhausted.

The shadow price for an additional hour of furnace two is \$9.40. This tells us that for each additional hour of use of furnace two should cost us less than \$9.40 to operate the furnace or adding the additional resources is not profitable.

The shadow prices of the remaining constraints (weight constraints for processes 1-18) are the cost per pound to process an additional pound of material. For example, the shadow price for process 6 from the LINDO run is \$0.315/lb.

RANGE ON THE OBJECTIVE FUNCTION COEFFICIENTS. The objective function ranges show how the price per pound can decrease or increase by the indicated amount without changing the solution. For instance, the cost per pound of furnace one, process 5 can increase \$0.054 before requiring a new solution to the problem. It can also decrease to zero.

RANGE ON RIGHTHAND SIDE COEFFICIENTS. For constraints one and two (time constraints on furnaces one and two respectively), the right hand side ranges show how much additional time could be made available for a particular furnace before the optimal solution would change. For furnace one, the amount of available time could increase to infinity (due to the fact that there is slack) and the available time could decrease 37.94 hours and the given solution would still remain optimal.

For furnace two, the amount of available time could increase 9.206 hours and decrease 2.189 hours and the given solution would remain optimal.

The right hand side ranges for the remaining constraints (weight constraints on processes 1 through 18) tell us the range of weights for which the given solution remains optimal. For example for process 15, the amount of material could range from 0 to 2799.474 lbs and the solution remains unchanged. Note that all processes except 1, 3, and 8 can decrease to zero and the solution does not change.

SENSITIVITY ANALYSIS FOR METHOD 2

The sensitivity analysis is very similar to that for method one. The slack in furnace 1 is 10.78 hours and the shadow price for furnace 2 is \$5.00 per hour. The objective coefficient ranges give an indication of the impact of a cost increase (perhaps due to labor or energy) and its affects on the solution. The right hand side coefficients show how either available furnace hours or pounds of each process can be varied without changing the solution. This is discussed above in the method 1 analysis.

Discussion

These results are for a "snapshot" of a particular week of data and can deviate significantly in other weeks. The value of this program is the insight that it gives in understanding the availability of resources and the impacts of various process\parts mixes on that.

Without this information operators might load parts into any available furnace. The solution specifies which process goes in which furnace. Without this planning tool parts might be loaded in a less cost effective manner or in a manner which won't meet production schedule.

Sensitivity analysis showed slack in furnace one for both methods. This would assist the shift supervisor in scheduling labor, knowing how much labor/furnace surplus is available for the week.

Scheduling maintenance on equipment could also be done using the slack. This would maintain optimal schedule and not require overtime for maintenance. If maintenance requirements exceed slack time a new solution could be run and new schedule created.

Both methods produced a solution with similar objective function values. The assumptions did drive processes into different furnaces. As method 1 assumed a continuous process it deviated more from the batch nature of the solution. It did not yield correct furnace hour impacts however. If the batch nature of the processes is reintroduced into the solution we find that the actual number of hour required on furnace one is 108 (vs 82) and 142 hours (vs 120). This is a significant deviation.

Method two tends to retain the batch nature in the solution and actually schedules total furnace resource time within a much closer margin. When time resources are reduced, more of the processes are forced into both furnaces, but with 130 hours per week available for each furnace(note method two showed that 120 hours per furnace per week was infeasible), only one process was forced into both furnaces. It did not correctly divide time resources between the furnaces, but it did schedule more time than needed so the work can be completed.

In terms of cost per pound, furnace 2 is much more efficient than furnace 1, so method 1 drove all the material towards that furnace. Without resource constraint all material goes to that furnace. Method 2 recognized that the smaller furnace 1 was more economical for batches within its capacity.

The more accurate method is method 2, however the most accurate would be to develop an integer solution which would fully recognize the batch characteristics of this process. Method two however provide a good solution of the problem.

Conclusions

A linear program can be developed to solve a batch type problem. Selection of the formulation can greatly affect the solution. Solutions found gave a reasonable plan for department operations. The process can be understood using this tool and decisions on department operational procedures, resource requirements, etc. can be better made.

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Precision Castparts Corporation
SSBO Heat Treat Department

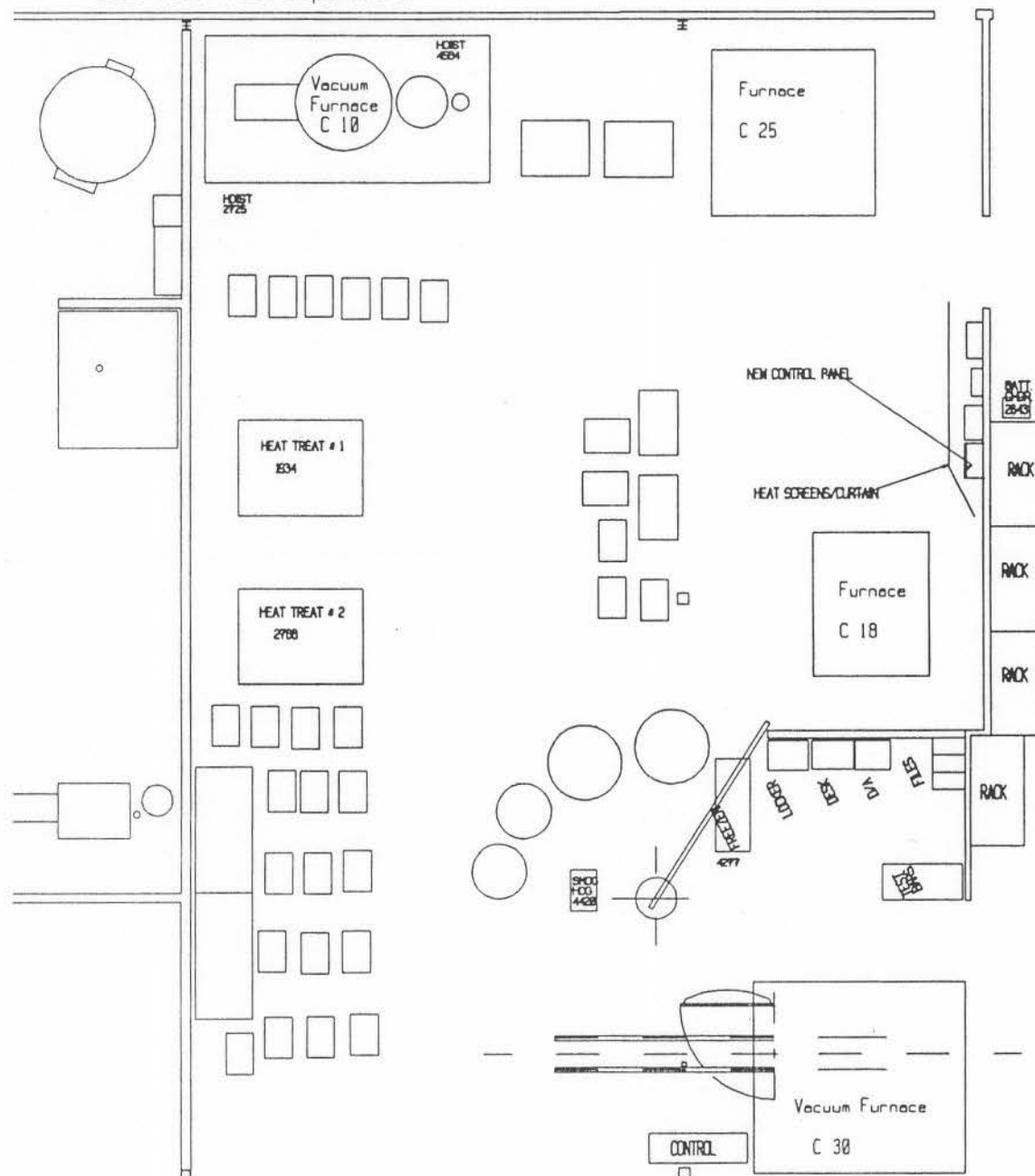


Figure 1 Heat Treatment Department Layout

	Solutions for the Air/Argon Furnaces			
Process	M1 F1	M1 F2	M2 F1	M2 F2
	lbs	lbs	hrs	hrs
1	0	14707	0	40
2	0	1318	0	3.5
3	0	10964	0	35.2
4	0	669	4.79	0
5	1943	0	55.2	0
6	459	0	20.96	0
7	462	0	3.8	0
8	3460	823	0	12
9	525	0	4	0
10	0	2333	12.2	0
11	0	2618	0	8.8
12	0	1314	0	3.5
13	0	2770	0	7.6
14	0	1046	0	4.3
15	0	2142	0	10
16	0	578	5.1	0
17	0	2003	3.86	5.03
18	639	0	4	0
	7488	43285	113.91	129.93

Figure 2 Comparison of Method 1 and Method 2 Results

Appendix I
Heat Treatment Furnace
Operational and Cost Data

EMGT 540 - Heat Treat Project Data - Furnace Cost Summary - 1700 (f)

Furnace	Furnace Name	\$/hr Labor	\$/hr Supplies	\$/hr Argon	\$/hr Repairs	\$/hr Electric	\$/hr Gas	\$/hr Total	Pounds Capacity
1	C18	2	1	4	1	0	6	14	800
2	C25	2	2	5	1	0	7	17	1500
3	C10	1	1	0	4	3	0	9	900
4	C30	2	1	1	4	4	0	12	1000

EMGT 540 - Heat Treat Project Data

Process #	REF #	TEMP (deg F)	CYCLE TIME (HRS)	ATMOSPHE	PART #	# OF PIECES	POUNDS/PIECE	POUNDS	Furn 1 \$/hr	Furn 2 \$/hr	Furn 1 \$/pound	Furn 2 \$/pound	Furn 1 hr/pound	Furn 2 hr/pound
1	1050	1750	4	argon	3154	1	93	93	14	17	0.07	0.045	0.005	0.00298
					3396	28	25	725						
					4493	12	30	360						
					5420	119	70	8,330						
					5882	18	150	2,700						
					6331	1	94	94						
					6511	1	140	140						
					7241	3	185	495						
					7268	1	38	38						
					7874	18	55	990						
					7856	2	195	390						
					8071	4	88	352						
					TOTAL POUNDS			14,707						
2	1051	1900	3.5	argon	3025	2	33	66	15	18	0.098	0.042	0.00438	0.00233
					7438	2	44	88						
					7439	22	17	374						
					10309	6	45	270						
					10339	4	130	520						
					TOTAL POUNDS			1,318						
3	1052	2000	4.4	argon	3154	1	93	93	16	19	0.088	0.056	0.0055	0.00293
					3396	28	25	700						
					3797	2	63	126						
					5420	117	70	8,190						
					5640	8	50	400						
					5882	4	150	600						
					7241	3	165	495						
					7680	3	120	360						
					TOTAL POUNDS			10,964						

EMGT 540 - Heat Treat Project Data

Process #	REF #	TEMP (deg F)	CYCLE TIME (HRS)	ATMOSPHE	PART #	# OF PIECES	POUNDS/PIECE	POUNDS	Furn 1 \$/hr	Furn 2 \$/hr	Furn 1 \$/pound	Furn 2 \$/pound	Furn 1 hr/pound	Furn 2 hr/pound
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
4	1053	2100	5	argon	4730	2	120	240	17	20	0.106	0.066	0.00625	0.00333
					5126	2	120	240						
					5730	3	13	39						
					10036	10	15	150						
TOTAL POUNDS							669							
5	1162	1400	18.4	air	10435	29	67	1,943	8	10	0.184	0.123	0.023	0.01226
								1,943						
TOTAL POUNDS							1,943							
6	1205	1325	21	argon	5655	17	27	459	12	15	0.315	0.21	0.02625	0.014
								459						
TOTAL POUNDS							459							
7	1208	1400	3.8	argon	4973	2	71	142	12	15	0.057	0.036	0.00475	0.00253
					5662	4	80	320						
TOTAL POUNDS							462							
8	1225	1750	4	argon	3293	252	17	4,284	14	17	0.07	0.045	0.005	0.00266
								4,284						
TOTAL POUNDS							4,284							
9	1227	1750	4	argon	7084	5	105	525	14	17	0.07	0.045	0.005	0.00266
								525						
TOTAL POUNDS							525							
10	1250	1900	4.1	argon	746	109	11	1,199	15	18	0.077	0.049	0.00512	0.00273
					4730	1	120	120						
					5802	4	120	480						
					5126	2	120	240						
					5730	3	13	39						
					4730	1	120	120						
					10265	1	135	135						
TOTAL POUNDS							2,333							

EMGT 540 - Heat Treat Project Data

Process #	REF #	TEMP (deg F)	CYCLE TIME (HRS)	ATMOSPHE	PART #	# OF PIECES	POUNDS/PIECE	POUNDS	Furn 1 \$/hr	Furn 2 \$/hr	Furn 1 \$/pound	Furn 2 \$/pound	Furn 1 hr/pound	Furn 2 hr/pound
11	1251	1900	4.4	argon	8128	22	119	2,618	15	18	0.063	0.053	0.0055	0.00293
							TOTAL POUNDS	2,618						
12	1264	1900	3.5	argon	746	109	11	1,199	15	18	0.066	0.042	0.00437	0.00233
					10506	1	115	115						
							TOTAL POUNDS	1,314						
13	1268	1925	3.8	argon	7680	4	120	480	15	18	0.071	0.048	0.00475	0.00253
					7481	19	110	2,090						
					7855	1	200	200						
							TOTAL POUNDS	2,770						
14	1280	2000	4.3	argon	90201	1	18	18	16	19	0.066	0.055	0.00537	0.00296
					7240	2	140	280						
					7084	6	105	630						
					7088	1	118	118						
							TOTAL POUNDS	1,046						
15	1282	2000	5	argon	3293	128	17	2,142	16	19	0.1	0.063	0.00625	0.00333
							TOTAL POUNDS	2,142						
16	1294	2100	5.1	argon	7457	1	131	131	17	20	0.108	0.068	0.00637	0.0034
					10339	2	130	260						
					7439	11	17	187						
							TOTAL POUNDS	578						

							POUNDS/PIECE	POUNDS	Furn 1 \$/hr	Furn 2 \$/hr	Furn 1 \$/pound	Furn 2 \$/pound	Furn 1 hr/pound	Furn 2 hr/pound
	1432	1975	3.8	argon	10435	24	67	1,608	18	19	0.078	0.048	0.00475	0.00253
					10436	5	79	395						
							TOTAL POUNDS	2,003						
18	1433	1750	4	air	10407	11	42	462	10	12	0.05	0.032	0.005	0.00266
					10353	1	177	177						
							TOTAL POUNDS	639						

EMGT 540 - Heat Treat Project Data

Process #	REF #	TEMP (deg F)	CYCLE TIME (HRS)	ATMOSPHE	PART #	# OF PIECES	POUNDS/PIECE	POUNDS	Furn 3 \$/hr	Furn 4 \$/hr	Furn 3 \$/pound	Furn 4 \$/pound	Furn 3 hr/pound	Furn 4 hr/pound
1	1422	1750	9	vacuum	3688	9	47	423	9	12	0.09	0.108	0.01	0.009
					7443	8	45	360						
					4535	20	42	840						
							TOTAL POUNDS	1,623						
2	1436	1400	10.8	vacuum	10506	2	115	230	7	10	0.062	0.108	0.0117	0.0108
					7481	21	110	2,310						
					90201	1	18	18						
							TOTAL POUNDS	2,558						
3	1437	1925	9.8	vacuum	8964	8	290	1,740	10	13	0.109	0.127	0.011	0.0098
					7481	41	110	4,510						
					10084	9	205	1,845						
					10308	14	200	2,800						
					8071	4	88	352						
							TOTAL POUNDS	11,247						

LABOR BREAKDOWN

Furnace #1 & #2

Time To Load/Unload & Do Paperwork = 1 Hour

Avg Time/Furnace Cycle = 4.47

Labor Hrs/Furn Hr = 0.22

Furnace #8

Unload/Load = 0.5

Time/Furn Cycle = 4.3

Labor Hrs/Furn Hr = 0.12

<u>Furn</u>	<u>Labor Hrs/Furn Hr</u>	<u>Ratio</u>
#1	0.220	0.69
#2	0.220	0.69
#8	0.116	0.36
#10	0.185	0.58
#18	0.266	0.83
#25	0.319	1.00
New	0.313	1.00

Furnace #10

Unload/Load = 1.5

Avg Time/Furn Cycle = 8.1

Labor Hrs/Furn Hr = 0.185

Furnace #18

Unload/Load = 1.25

Avg Time/Furn Cycle = 4.7

Labor Hrs/Furn Hr = 0.266

Furnace #25

Unload/Load = 1.5

Time/Furn Cycle = 4.7

Labor Hrs/Furn Hr = 0.319

New Furnace #430

Unload/Load = 1.5

Time/Furn Cycle = 4.8

Labor Hrs/Furn Hr = 0.313

FURNACE COST DATA

Pg 1 OF 5

EXPECTED MAINTENANCE COSTS

<u>FURN #</u>	<u>COST</u>	<u>RATIO</u>
#1	\$ 4,000	0.33
#2	4,000	0.33
#8	1,000	0.08
#10	50,000	4.17
#18	12,000	1.00
#25	12,000	1.00
NEW	24,000	2.00

FURNACE COST DATA
pg 2 OF 5

ELECTRICAL CONSUMPTION CALC.

Furnace #1 & #2 Heater Capacity = 120KW Motors = 4 HP
or 3KW/Hour

	<u>Time/Cycle</u>	<u>KW</u>
To Temp Furnace at 70% or 84 + 3 KW/H	1.53	133.1
At Temp Furnace at 25% or 30 + 3 KW/H	2.94	97.0
Avg KW/Hour = 51.5	4.47	230.1

Furnace #18 Blowers = 4HP or 3KW/Hour
" #25 " = 5HP or 3.7KW/Hour

Furnace #10

To Temp 25v x 3400 amps x 1.73 = 147.1KW/Hour
At Temp (Avg) 15v x 1700 amps x 1.73 = 44.1KW/Hour

Pumps Operate Through Cycle (21HP) or 15.7KW/Hour

		<u>Time/Cycle</u>	<u>KW</u>
Pump Down	15.7 KW/Hour	1.8	28.3
Heat Up	147.1 + 15.7	1.8*	293.0
At Temp	44.1 + 15.7	2.8**	167.0
Cool Down	15.7	2.2	34.5
Avg = 60.8 KW/Hour		8.6	523.0

* Furn reaches temp in avg. of 1.8 hours load takes longer
** Includes some load heat up time

Electrical Consumption

Furnace #1	51.5 KW/Hour	13.91
#2	51.5 KW/Hour	13.91
#10	60.8 KW/Hour	16.40
#18	3.0 KW/Hour	0.81
#25	3.7 KW/Hour	1.00
EST. NEW	75.0 KW/Hour	20.20

FURNACE COST DATA
Pg 3 of 5

ESTIMATED SUPPLY COST/YR

<u>FURN #</u>	<u>FIXTURES BASKETS</u>	<u>TC</u>	<u>ARGON*</u>	<u>TOTAL</u>	<u>RATIO TO #25</u>
#1	2,500	11,000		13,500	0.23
#2	2,500	11,000		13,500	0.23
#8	-----	1,000		1,000	0.017
#10	7,000	11,000		18,000	0.31
#18	31,200	11,000		42,200	0.73
#25	46,800	11,000		57,800	1.00
NEW #C30	7,000	11,000		18,000	0.31

*See Derivation Below Will Be Separate

ARGON USE/CYCLE

#10 Backfill to -5 in Hg then maintain level while cooling. Before unloading, equalize to atmosphere pressure with Argon. This procedure uses 3 argon equivalent to the chamber volume, or 57.75 ft^3 . On an operating basis this is $7.1 \text{ ft}^3/\text{furn hour}$ ($57.75/8.1$)

#18 Purge for one hour at 150 scfh, run in furnace for an average of 4.7 hours at 120 scfh, and cool for an avg. of 0.5 hours at 120 scfh. Total Argon used/cycle is 774 scf . On a furnace hour basis this is $774/4.7$ or $165 \text{ ft}^3/\text{furn hour}$.

#25 Purge for two hours at 150 scfh, run for 4.7 hours at 120 scfh, and cool for an avg. of 0.5 hours at 120 3 scfh. Total Argon used/cycle is 924 scf . or $197 \text{ ft}^3/\text{furn hour}$.

NEW Estimated use is $100 \text{ ft}^3/\text{cycle}$ or $21.3 \text{ ft}^3/\text{furn hour}$.

<u>FURN #</u>	<u>FT³/FURN HOUR</u>	<u>RATIO TO #25</u>
#10	7.1	0.04
#18	165.0	0.84
#25	197.0	1.00
NEW	21.3	0.11

FURNACE COST DATA

Pg 4 OF 5

GAS CONSUMPTION

Only furnaces #18 & #25 use gas. BTU rating of #18 is 1,800,000 BTU/HR & #25 is 2,000,000/HR

$$1,800,000/2,000,000 = 0.90$$

<u>FURNACE</u>	<u>RATIO TO #25</u>
#18	0.90
#25	1.00

RENT

Rent is for fork trucks. Fork trucks are not used for the vacuum furnaces or freezer. Fork truck rent should be divided equally between remaining furnaces (furnace hours will determine apportionment).

SALARY & FRINGE

Divided equally between all furnaces (furnace hours will determine apportionment).

cc: Art Greenwood
George Harriman
Bob McClelland
Bob McGinley
Terry Spaulding

MM/tlg
Clack/x505

FURNACE COST DATA

pg 5 of 5

Appendix II
Sample Week
Heat Treatment Department
Production Schedule

SFS0B1.

JOB DBERNT29

P R E C I S I O N C A P A R T S C O R P .

04/22/92

11:07

8

MSGNUM DISPATCH LIST
HI TMP HEAT TREATPLANT: SSBO
DEPARTMENT: 107

DUE OUT OPER	PART	JOB	MSGNUM /CYCLE	EXPEDITE PRIORITY	JOB STATUS	PART DESC	PART ENG	NBR PCS	DAYS IN W/C	JOB DUE DATE	JOB STD HRS	SETUP HRS	CURR/WC
05/14/92													
14017	10436	0006-02	1162	20-SMP	21-SCHED	STRUT END	CURT RUND	4	1	05/15/92	1.3		HTHTP
TOTALS FOR DATE								4			1.3	0.0	
05/21/92													
14017	10435	0388-03	1162		21-SCHED	STRUT END	CURT RUND	8	1	05/26/92	2.7		HTHTP
TOTALS FOR DATE								8			2.7	0.0	
03/19/92													
14013	05655	0087-07	1205		25-R	BRACKET	C. MCCLUNG	4	1	04/10/92	0.1		HTHTP
TOTALS FOR DATE								4			0.1	0.0	
04/22/92													
14013	05655	0087-02	1205		24-F	BRACKET	C. MCCLUNG	13	1	05/14/92	0.4		HTHTP
TOTALS FOR DATE								13			0.4	0.0	
04/15/92													
14013	04973	0004-02	1208	20-SMP	24-F	IMPELLER	T. MCGINNI	2	2	04/24/92	5.3		HTHTP
TOTALS FOR DATE								2			5.3	0.0	
05/12/92													
14011	05662	0231-01	1222		21-SCHED	HUB	M. MCCARTH	2	1	05/15/92	1.0		HTHTP
	05662	0231-03	1222		21-SCHED	HUB	M. MCCARTH	1	1	05/15/92	0.5		HTHTP
	05662	0232-03	1222		21-SCHED	HUB	M. MCCARTH	1	1	05/18/92	0.5		HTHTP
TOTALS FOR DATE								4			2.0	0.0	
04/16/92													
14011	03293	0035-01	1225		21-SCHED	LUG	K. KRUEGER	126	1	05/07/92	0.0		HTHTP
TOTALS FOR DATE								126			0.0	0.0	

SCHEDULE DATA
EXAMPLE 1 OF 1

PLANT CODE : 001

ACTIVE PARTS

PART	DESC	BU	CT	OS	HD	PROD	PART	PLANNER	CUSTOMER	PCS/ HOLD	PCS/ LOT	YIELD %	DIP CODE	SHELL MT	ALLOY	POUR MT	SHIP MT	FIXED LT	XRY EN
P01001	ARM	01																	
P01002	ARM	01																	
P01003	ARM	01																	
P01004	ARM	01																	
P01005	ARM	01																	
P01006	ARM	01																	
P01007	ARM	01																	
P01008	ARM	01																	
P01009	ARM	01																	
P01010	ARM	01																	
P01011	ARM	01																	
P01012	ARM	01																	
P01013	ARM	01																	
P01014	ARM	01																	
P01015	ARM	01																	
P01016	ARM	01																	
P01017	ARM	01																	
P01018	ARM	01																	
P01019	ARM	01																	
P01020	ARM	01																	
P01021	ARM	01																	
P01022	ARM	01																	
P01023	ARM	01																	
P01024	ARM	01																	
P01025	ARM	01																	
P01026	ARM	01																	
P01027	ARM	01																	
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P01029	ARM	01																	
P01030	ARM	01																	
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P01037	ARM	01																	
P01038	ARM	01																	
P01039	ARM	01																	
P01040	ARM	01																	
P01041	ARM	01																	
P01042	ARM	01																	
P01043	ARM	01																	
P01044	ARM	01																	
P01045	ARM	01																	
P01046	ARM	01																	
P01047	ARM	01																	
P01048	ARM	01																	
P01049	ARM	01																	
P01050	ARM	01																	
P01051	ARM	01																	
P01052	ARM	01																	
P01053	ARM	01																	
P01054	ARM	01																	
P01055	ARM	01																	
P01056	ARM	01																	
P01057	ARM	01																	
P01058	ARM	01																	
P01059	ARM	01																	
P01060	ARM	01																	
P01061	ARM	01																	
P01062	ARM	01																	
P01063	ARM	01																	
P01064	ARM	01																	
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P01076	ARM	01																	
P01077	ARM	01																	
P01078	ARM	01																	
P01079	ARM	01																	
P01080	ARM	01																	
P01081	ARM	01																	
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P01090	ARM	01																	
P01091	ARM	01																	
P01092	ARM	01																	
P01093	ARM	01																	
P01094	ARM	01																	
P01095	ARM	01																	
P01096	ARM	01																	
P01097	ARM	01																	
P01098	ARM	01																	
P01099	ARM	01																	
P01100	ARM	01																	

PLANT CODE : 001

ACTIVE PARTS

PART	DESC	BU	CT	OS	HD	PROD	PART	PLANNER	CUSTOMER	PCS/ HOLD	PCS/ LOT	YIELD %	DIP CODE	SHELL WT	ALLOY	POUR WT	SHIP WT	FIXED LT	XRY SM
P01597	NOZZLE	01								1	16	98.00	1812	15	6010A	40	24.75	44	X
P01640	SUPPORT	04								4	80	90.00	1812	12	5010A	28	2.75	70	
P01684	PART	04								4	80	90.00	1812	12	5010A	28	1.97	32	
P01707	IMPELLER	01								4	80	90.00	1812	28	5010A	100	43.00	45	
P01722	IMPELLER	01								10	100	90.00	1812	28	5010A	100	12.00	22	
P01740	END BELL	00								12	120	90.00	1812	12	5010A	12	0.19	42	X
P01761	END BELL	00								40	400	90.00	1812	12	5010A	18	0.14	49	X
P01762	GIMBAL	00								10	250	90.00	1812	10	5010A	18	0.28	46	X
P01763	NOZZLE	01								10	250	90.00	1812	10	5010A	18	7.00	45	X
P01764	STEM	04								10	100	90.00	1812	10	5010A	18	1.00	32	
P01765	STEM	04								6	60	85.00	1711	6	1012A	14	0.50	29	
P01794	STEM	04								6	60	85.00	1711	6	1012A	14	0.50	29	
P01802	BOOY	01								4	80	97.80	1812	13	5010A	30	3.50	45	X
P01803	BOOY	01								4	80	97.80	1812	13	5010A	30	3.50	45	X
P01834	MOUNT	00								4	80	97.80	1812	13	5010A	30	3.50	45	X
P01835	VALVE	04								16	160	90.00	1812	14	5010A	30	2.25	38	X
P01879	VALVE	01								20	200	90.00	1812	14	5010A	18	4.63	34	X
P01917	HOUSING	01								10	100	93.70	1812	10	5010A	30	8.00	40	X
P01925	PAD	00								4	40	89.00	1819	5	6020A	20	1.19	31	
P01930	NOZZLE	00								20	200	93.80	1812	14	5010A	36	15.50	49	X
P01932	HOUSING	00								12	120	96.40	1812	12	5010A	28	9.00	38	
P01940	IMPELLER	01								20	200	94.00	1812	8	5010A	12	8.00	44	X
P01943	IMPELLER	01								4	40	94.00	1812	8	5010A	12	8.00	44	X
P01954	JACK PAD	00								24	240	95.00	1812	15	5010A	31	0.06	46	
P01961	NOZZLE	01								1	16	92.70	1812	10	3470A	25	10.00	36	X
P01977	SEAL	00								4	40	92.70	1812	29	2622A	57	12.25	51	
P01979	BEAD	01								1	8	94.10	1812	13	3470A	48	3.06	53	X
P02031	LINK	00								44	440	90.00	1819	13	6030A	18	0.13	25	
P02032	LINK	00								44	440	90.00	1819	13	6030A	18	0.13	25	
P02033	HINGE	00								24	480	88.20	1819	12	6030A	20	0.50	64	
P02045	NOZZLE	00								1	16	90.00	1812	14	1050A	50	28.00	38	X
P02046	HOUSING	00								1	1	93.80	1812	48	8010G	115	16.50	49	
P02067	BRACKET	00								2	40	97.30	1812	3	5010A	35	4.40	46	
P02074	IMPELLER	00								2	2	95.00	1812	30	4660G	150	70.00	40	
P02079	GUIDE	00								2	20	97.00	1812	2	5010A	1	1.20	41	X
P02103	FLITING	00								2	20	97.00	1812	25	5010A	58	5.00	51	
P02132	VALVE	04								4	40	93.00	1812	10	3470A	24	1.56	38	X
P02169	IMPELLER	01								1	4	94.00	1812	35	8010G	100	61.00	45	
P02187	PLATE	00								4	88	89.00	1812	13	5010A	22	1.50	41	X
P02193	SUPPORT	00								15	80	85.30	1812	17	3100A	35	1.50	47	X
P02202	BOOY	01								15	150	89.00	1812	17	3100A	38	1.42	40	X
P02229	SHAFT	00								4	88	93.40	1812	24	5010A	70	2.50	42	X
P02231	SEAL	00								1	1	94.00	1812	13	5010G	38	7.63	40	
P02241	SEAL	00								1	1	94.00	1812	13	5010G	37	5.00	42	
P02274	HOOK	00								8	80	75.00	1812	14	5010A	25	0.28	60	X
P02365	SEAL	00								1	20	94.50	1812	16	2622A	35	6.50	41	

REF#	TEMP	HRS	SAT	TEMP	ATMOS	QCH- MED1	COOLRT1	TOTEMP1	ATTEMP1	QCH- MED2	COOLRT2	TOTEMP2	ATTEMP2	QCH- MED3	COOLRT3	TOTEMP3	ATTEMP3	DEWPT
1000	-1	0.50			AIR AIR		0	0	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1001	900	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1002	925	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1003	1000	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1004	1050	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1005	1100	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1006	1125	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1007	1150	1.50			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1008	1150	4.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1009	1150	16.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1010	1200	2.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1011	1325	8.00			AIR FUR		0	1150	8.00	AIR	0	120	0.00		0	0	0.00	0
					COMMENTS													
1050	1750	1.00			ARG FCR		0	120	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1051	1900	0.50			ARG FCR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1052	2000	1.00			ARG FCR		0	120	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1053	2100	1.50			ARG FCR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1080	1350	2.00			VAC APV		0	120	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1100	450	1.50			AIR OIL		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1101	500	1.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1102	625	3.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1103	900	1.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1104	900	2.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1105	900	3.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1106	925	1.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1107	925	2.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													
1108	925	4.00			AIR AIR		0	70	0.00		0	0	0.00		0	0	0.00	0
					COMMENTS													

PROCESS DATA
EXAMPLE 1 OF 1

Appendix III
Vacuum Furnace Solution

MIN 0.01 X31 + 0.008 X32 + 0.012 X33 + 0.012 X41 + 0.01 X42
 + 0.014 X43

SUBJECT TO

- 2) 0.01 X31 + 0.0117 X32 + 0.011 X33 <= 120
- 3) 0.009 X41 + 0.0106 X42 + 0.0098 X43 <= 120
- 4) X31 + X41 = 1623
- 5) X32 + X42 = 2558
- 6) X33 + X43 = 11247

END

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

- 1) 180.40070

VARIABLE	VALUE	REDUCED COST
X31	1623.000000	.000000
X32	.000000	.000127
X33	9433.637000	.000000
X41	.000000	.000182
X42	2558.000000	.000000
X43	1813.364000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.181818
3)	75.114230	.000000
4)	.000000	-.011818
5)	.000000	-.010000
6)	.000000	-.014000

NO. ITERATIONS= 6

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT	ALLOWABLE	ALLOWABLE
	COEF	INCREASE	DECREASE
X31	.010000	.000182	INFINITY
X32	.008000	INFINITY	.000127
X33	.012000	.000120	.000200
X41	.012000	INFINITY	.000182
X42	.010000	.000127	INFINITY
X43	.014000	.000200	.000120

ROW	RIGHTHAND SIDE RANGES		
	CURRENT	ALLOWABLE	ALLOWABLE
	RHS	INCREASE	DECREASE
2	120.000000	19.947000	84.311890
3	120.000000	INFINITY	75.114230
4	1623.000000	8431.190000	1623.000000
5	2558.000000	7086.249000	2558.000000
6	11247.000000	7664.718000	1813.364000

Appendix IV
LINDO Solution Method 1

MIN $0.07 X_{11} + 0.066 X_{12} + 0.088 X_{13} + 0.106 X_{14} + 0.184 X_{15}$
 $+ 0.315 X_{16} + 0.057 X_{17} + 0.07 X_{18} + 0.07 X_{19} + 0.077 X_{110}$
 $+ 0.08299999 X_{111} + 0.066 X_{112} + 0.071 X_{113} + 0.086 X_{114} + 0.1 X_{115}$
 $+ 0.108 X_{116} + 0.07599999 X_{117} + 0.05 X_{118} + 0.045 X_{21} + 0.042 X_{22}$
 $+ 0.056 X_{23} + 0.066 X_{24} + 0.123 X_{25} + 0.21 X_{26} + 0.038 X_{27} + 0.045 X_{28}$
 $+ 0.045 X_{29} + 0.049 X_{210} + 0.053 X_{211} + 0.042 X_{212} + 0.046 X_{213}$
 $+ 0.055 X_{214} + 0.063 X_{215} + 0.068 X_{216} + 0.048 X_{217} + 0.032 X_{218}$

SUBJECT TO

- 2) $0.05 X_{11} + 0.0438 X_{12} + 0.055 X_{13} + 0.0625 X_{14} + 0.23 X_{15}$
 $+ 0.2625 X_{16} + 0.0475 X_{17} + 0.05 X_{18} + 0.05 X_{19} + 0.0512 X_{110}$
 $+ 0.055 X_{111} + 0.0437 X_{112} + 0.0475 X_{113} + 0.0537 X_{114} + 0.0625 X_{115}$
 $+ 0.06369999 X_{116} + 0.0475 X_{117} + 0.05 X_{118} \leq 1200$
- 3) $0.0266 X_{21} + 0.0233 X_{22} + 0.0293 X_{23} + 0.0333 X_{24} + 0.1226 X_{25}$
 $+ 0.14 X_{26} + 0.0253 X_{27} + 0.0266 X_{28} + 0.0266 X_{29} + 0.0273 X_{210}$
 $+ 0.0293 X_{211} + 0.0233 X_{212} + 0.0253 X_{213} + 0.0286 X_{214} + 0.0333 X_{215}$
 $+ 0.034 X_{216} + 0.0253 X_{217} + 0.0266 X_{218} \leq 1200$
- 4) $X_{11} + X_{21} = 14707$
- 5) $X_{12} + X_{22} = 1318$
- 6) $X_{13} + X_{23} = 10964$
- 7) $X_{14} + X_{24} = 669$
- 8) $X_{15} + X_{25} = 1943$
- 9) $X_{16} + X_{26} = 459$
- 10) $X_{17} + X_{27} = 462$
- 11) $X_{18} + X_{28} = 4284$
- 12) $X_{19} + X_{29} = 525$
- 13) $X_{110} + X_{210} = 2333$
- 14) $X_{111} + X_{211} = 2618$
- 15) $X_{112} + X_{212} = 1314$
- 16) $X_{113} + X_{213} = 2770$
- 17) $X_{114} + X_{214} = 1046$
- 18) $X_{115} + X_{215} = 2142$
- 19) $X_{116} + X_{216} = 578$
- 20) $X_{117} + X_{217} = 2003$
- 21) $X_{118} + X_{218} = 639$

END

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

1) 3015.3460

VARIABLE	VALUE	REDUCED COST
X11	.000000	.000000
X12	.000000	.002102
X13	.000000	.004462
X14	.000000	.008703
X15	1943.000000	.000000
X16	459.000000	.000000
X17	462.000000	.000000
X18	3460.921000	.000000
X19	525.000000	.000000
X110	.000000	.002342
X111	.000000	.002462
X112	.000000	.002102
X113	.000000	.001222
X114	.000000	.004120
X115	.000000	.005703
X116	.000000	.008045
X117	.000000	.004222
X118	639.000000	.000000
X21	14707.000000	.000000
X22	1318.000000	.000000
X23	10964.000000	.000000
X24	669.000000	.000000
X25	.000000	.054226
X26	.000000	.026579
X27	.000000	.004778
X28	823.078900	.000000
X29	.000000	.000000
X210	2333.000000	.000000
X211	2618.000000	.000000
X212	1314.000000	.000000
X213	2770.000000	.000000
X214	1046.000000	.000000
X215	2142.000000	.000000
X216	578.000000	.000000
X217	2003.000000	.000000
X218	.000000	.007000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	379.431400	.000000
3)	.000000	.939850
4)	.000000	-.070000
5)	.000000	-.063898
6)	.000000	-.083538
7)	.000000	-.097297
8)	.000000	-.184000
9)	.000000	-.315000
10)	.000000	-.057000
11)	.000000	-.070000
12)	.000000	-.070000
13)	.000000	-.074658
14)	.000000	-.080538
15)	.000000	-.063898
16)	.000000	-.069778
17)	.000000	-.081880
18)	.000000	-.094297
19)	.000000	-.099955
20)	.000000	-.071778
21)	.000000	-.050000

NO. ITERATIONS= 6

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X11	.070000	INFINITY	.000000
X12	.066000	INFINITY	.002102
X13	.088000	INFINITY	.004462
X14	.106000	INFINITY	.008703
X15	.184000	.054226	INFINITY
X16	.315000	.026579	INFINITY
X17	.057000	.004778	INFINITY
X18	.070000	.000000	.000000
X19	.070000	.000000	INFINITY
X110	.077000	INFINITY	.002342
X111	.083000	INFINITY	.002462
X112	.066000	INFINITY	.002102
X113	.071000	INFINITY	.001222
X114	.086000	INFINITY	.004120
X115	.100000	INFINITY	.005703
X116	.108000	INFINITY	.008045
X117	.076000	INFINITY	.004222
X118	.050000	.007000	INFINITY
X21	.045000	.000000	INFINITY
X22	.042000	.002102	INFINITY
X23	.056000	.004462	INFINITY
X24	.066000	.008703	INFINITY
X25	.123000	INFINITY	.054226
X26	.210000	INFINITY	.026579
X27	.038000	INFINITY	.004778
X28	.045000	.000000	.000000
X29	.045000	INFINITY	.000000
X210	.049000	.002342	INFINITY
X211	.053000	.002462	INFINITY
X212	.042000	.002102	INFINITY
X213	.046000	.001222	INFINITY
X214	.055000	.004120	INFINITY
X215	.063000	.005703	INFINITY
X216	.068000	.008045	INFINITY
X217	.048000	.004222	INFINITY
X218	.032000	INFINITY	.007000

RIGHTHAND SIDE RANGES			
ROW	CURRENT	ALLOWABLE	ALLOWABLE
	RHS	INCREASE	DECREASE
2	1200.000000	INFINITY	379.431400
3	1200.000000	92.060500	21.893900
4	14707.000000	823.078900	3460.921000
5	1318.000000	939.652300	1318.000000
6	10964.000000	747.231900	3141.996000
7	669.000000	657.474400	669.000000
8	1943.000000	1649.702000	1943.000000
9	459.000000	1445.453000	459.000000
10	462.000000	7988.030000	462.000000
11	4284.000000	7588.628000	3460.921000
12	525.000000	7588.628000	525.000000
13	2333.000000	801.974200	2333.000000
14	2618.000000	747.231900	2618.000000
15	1314.000000	939.652300	1314.000000
16	2770.000000	865.371500	2770.000000
17	1046.000000	765.520900	1046.000000
18	2142.000000	657.474400	2142.000000
19	578.000000	643.938100	578.000000
20	2003.000000	865.371500	2003.000000
21	639.000000	7588.628000	639.000000

Appendix V
LINDO Solution Method 2

MIN $14 X_{11} + 17 X_{21} + 15 X_{12} + 18 X_{22} + 16 X_{13} + 19 X_{23} + 17 X_{14}$
 $+ 20 X_{24} + 8 X_{15} + 10 X_{25} + 12 X_{16} + 15 X_{26} + 12 X_{17} + 15 X_{27} + 14 X_{18}$
 $+ 17 X_{28} + 14 X_{19} + 17 X_{29} + 15 X_{110} + 18 X_{210} + 15 X_{111} + 18 X_{211}$
 $+ 15 X_{112} + 18 X_{212} + 15 X_{113} + 18 X_{213} + 16 X_{114} + 19 X_{214} + 16 X_{115}$
 $+ 19 X_{215} + 17 X_{116} + 20 X_{216} + 16 X_{117} + 19 X_{217} + 10 X_{118} + 12 X_{218}$

SUBJECT TO

- 2) $X_{11} + X_{12} + X_{13} + 2 X_{14} + X_{15} + X_{16} + X_{17} + X_{18} + X_{19} + X_{110}$
 $+ X_{111} + X_{112} + X_{113} + X_{114} + X_{115} + X_{116} + X_{117} + X_{118} \leq 130$
- 3) $X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} + X_{27} + X_{28} + X_{29} + X_{210}$
 $+ X_{211} + X_{212} + X_{213} + X_{214} + X_{215} + X_{216} + X_{217} + X_{218} \leq 130$
- 4) $193 X_{11} + 367 X_{21} = 14707$
- 5) $188.3 X_{12} + 376.6 X_{22} = 1318$
- 6) $178 X_{13} + 311.5 X_{23} = 10964$
- 7) $133.8 X_{14} + 133.8 X_{24} = 669$
- 8) $35.2 X_{15} + 52.8 X_{25} = 1943$
- 9) $21.9 X_{16} + 21.9 X_{26} = 459$
- 10) $121.57 X_{17} + 121.57 X_{27} = 462$
- 11) $178.5 X_{18} + 357 X_{28} = 4284$
- 12) $131.25 X_{19} + 131.25 X_{29} = 525$
- 13) $189.67 X_{110} + 284.51 X_{210} = 2333$
- 14) $148.75 X_{111} + 297.5 X_{211} = 2618$
- 15) $187.7 X_{112} + 375.4 X_{212} = 1314$
- 16) $182.2 X_{113} + 364.5 X_{213} = 2770$
- 17) $121.6 X_{114} + 243.3 X_{214} = 1046$
- 18) $142.6 X_{115} + 214.2 X_{215} = 2142$
- 19) $113.3 X_{116} + 113.3 X_{216} = 578$
- 20) $175.7 X_{117} + 263.5 X_{217} = 2003$
- 21) $159.75 X_{118} + 159.75 X_{218} = 639$

END

LP OPTIMUM FOUND AT STEP 9

OBJECTIVE FUNCTION VALUE

1) 3595.0820

VARIABLE	VALUE	REDUCED COST
X11	.000000	2.432912
X21	40.073570	.000000
X12	.000000	3.502276
X22	3.499734	.000000
X13	.000000	2.288316
X23	35.197430	.000000
X14	5.000000	.000000
X24	.000000	7.995447
X15	55.198860	.000000
X25	.000000	2.995448
X16	20.958900	.000000
X26	.000000	7.995447
X17	3.800280	.000000
X27	.000000	7.995447
X18	.000000	3.002276
X28	12.000000	.000000
X19	4.000000	.000000
X29	.000000	7.995447
X110	12.300310	.000000
X210	.000000	.495052
X111	.000000	3.502276
X211	8.800000	.000000
X112	.000000	3.502276
X212	3.500267	.000000
X113	.000000	3.505430
X213	7.599451	.000000
X114	.000000	4.007208
X214	4.299219	.000000
X115	.000000	.025439
X215	10.000000	.000000
X116	5.101501	.000000
X216	.000000	7.995447
X117	3.856054	.000000
X217	5.030327	.000000
X118	4.000000	.000000
X218	.000000	6.995447

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	10.784090	.000000
3)	.000000	4.995447
4)	.000000	-.059933
5)	.000000	-.061061
6)	.000000	-.077032
7)	.000000	-.127055
8)	.000000	-.227273
9)	.000000	-.547945
10)	.000000	-.098709
11)	.000000	-.061612
12)	.000000	-.106667
13)	.000000	-.079085
14)	.000000	-.077296
15)	.000000	-.061256
16)	.000000	-.063088
17)	.000000	-.098625
18)	.000000	-.112024
19)	.000000	-.150044
20)	.000000	-.091064
21)	.000000	-.062598

NO. ITERATIONS= 9

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGE		
	CURRENT	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X11	14.000000	INFINITY	2.432912
X21	17.000000	4.626315	INFINITY
X12	15.000000	INFINITY	3.502276
X22	18.000000	7.004553	INFINITY
X13	16.000000	INFINITY	2.288316
X23	19.000000	4.004552	INFINITY
X14	17.000000	7.995447	INFINITY
X24	20.000000	INFINITY	7.995447
X15	8.000000	1.996966	INFINITY
X25	10.000000	INFINITY	2.995448
X16	12.000000	7.995447	INFINITY
X26	15.000000	INFINITY	7.995447
X17	12.000000	7.995447	INFINITY
X27	15.000000	INFINITY	7.995447
X18	14.000000	INFINITY	3.002276
X28	17.000000	6.004553	INFINITY
X19	14.000000	7.995447	INFINITY
X29	17.000000	INFINITY	7.995447
X110	15.000000	.330029	INFINITY
X210	18.000000	INFINITY	.495052
X111	15.000000	INFINITY	3.502276
X211	18.000000	7.004553	INFINITY
X112	15.000000	INFINITY	3.502276
X212	18.000000	7.004553	INFINITY
X113	15.000000	INFINITY	3.505430
X213	18.000000	7.012784	INFINITY
X114	16.000000	INFINITY	4.007208
X214	19.000000	8.017712	INFINITY
X115	16.000000	INFINITY	.025439
X215	19.000000	.038212	INFINITY
X116	17.000000	7.995447	INFINITY
X216	20.000000	INFINITY	7.995447
X117	16.000000	.025480	.330098
X217	19.000000	.495052	.038212
X118	10.000000	6.995447	INFINITY
X218	12.000000	INFINITY	6.995447

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	130.000000	INFINITY	10.784090
3	130.000000	2.571191	5.030327
4	14707.000000	1846.130000	943.627000
5	1318.000000	1894.421000	968.310400
6	10964.000000	1566.947000	800.925900
7	669.000000	721.455500	669.000000
8	1943.000000	379.599900	1943.000000
9	459.000000	236.171500	459.000000
10	462.000000	1311.022000	462.000000
11	4284.000000	1795.827000	917.915000
12	525.000000	1415.412000	525.000000
13	2333.000000	2045.418000	2333.000000
14	2618.000000	1496.522000	764.929200
15	1314.000000	1888.385000	965.225000
16	2770.000000	1833.554000	937.199000
17	1046.000000	1223.879000	625.570700
18	2142.000000	1077.496000	550.749000
19	578.000000	1221.837000	578.000000
20	2003.000000	1894.764000	677.508700
21	639.000000	1722.758000	639.000000