Title: Minimization of Cost for an Instrumentation Production Line

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MINIMIZATION OF COST FOR AN INSTRUMENTATION PRODUCTION LINE

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Term Project

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Abstract

A single oscilloscope production line manufactures four types of products on a continuous basis. The workers and equipment are able to manufacture any of the four products flexibly, and do so regularly. The demand for these four types of products is forecasted a year in advance. Given expected demand combined with known capacity of the line, minimizing the cost of production becomes a primary goal.

As a project, this team extracted data from the company in which the production line operates and formulated a linear program. From the solution of this linear program the team suggested alternatives and generated a set of linear programs to match. Sensitivity analysis and obvious points of continued work are outlined for feedback to the company.

Executive Summary

This project was solved by the use of three linear programming models, each building upon knowledge gained by the last. The first of the three cases allowed the use of inventory, backlog, plus hiring and laying off of employees. Although this yielded an optimal solution, it was not seen to be congruent with the direction management desires to take. This first case did point out the number of people that could be considered excess on a period-by-period basis, and such information was used to formulate the second and third models. The LP recommended producing exactly the demand for each product each period and varying the work force to exactly match capacity to demand whenever possible. This makes sense in that it is cheaper to alter the work force number than it is to build to inventory or miss anticipated customer commitments. The only period in which the LP could not exactly match capacity to demand was period 2. Backlog of 2 units of product B was carried forward to period 3.

The second case was formulated to include the removal of four people from the line at the start of period 1 and the use of overtime up to the maximum allowable under management policy. This yielded a feasible solution at a slightly higher objective function value, but the solution was more sensitive to demand and manufacturing cost than in the first case. Overtime was used in just two periods and then less than half the available overtime was required. In period 5, the LP even comes 848 minutes shy of utilizing all the available

regular time. As before, this LP recommended producing exactly the demand for each product each period whenever possible. Inventory of 21 units of product A was carried from periods 5 to 6, and backlog of product B was carried from periods 3 to 4. It is generally cheaper to expend overtime pay than build to inventory or miss anticipated customer commitments.

The third case removed workers at the start of period 1 and prohibited the use of overtime. Removing 4 workers as in case 1 yielded an infeasible LP, so 3 workers were removed in case 3. The solution became feasible but even more sensitive to demand and manufacturing cost than was case 2. The intent was to see what change in objective function value would result from forcing the LP to recommend the use of backlog and inventory. The objective function came out slightly lower than either cases 1 or 2. Even prohibiting overtime did not significantly change the amount of backlog used. In case 3 the LP recommended backlog only on product B in period 3, but for 45 units. Period 3 becomes the only period in which all capacity is consumed.

It is interesting to note that as the second and third LP models began constraining the amount of capacity available, backlog on products A, C, and D were minimized while backlog on B was allowed to grow. This is due to product B having the highest manufacturing cost.

This case 3 strategy is consistent with policies the company wishes to use, namely long-term employment stability in the lines and minimization of overtime hours used, for the reasons of team-building, positive attitudes, aversion from burnout, and overall morale. Our final recommendation is to decrease the number of employees in the line by three, accept the lower objective function cost as being negligible, use no overtime, and keep a very close watch on production costs and demand estimates.

Problem Definition

A single production line employs fifteen people to produce four products on a continuous basis. All fifteen people have been trained in production of all four products, and equipment in the manufacture of the four products is usable on all four products. The line has been operating for more than 5 years with very low turnover in that time, resulting in a very stable and long-term work force in place. The skills of the workers are desired on other production lines, so

decreasing the number of workers on this line is estimated to be $100.00, essentially the cost of the line manager to negotiate a transfer. This number holds true for approximately 5 people, at which point no room would exist in other lines for transfers. Management policy is to maintain fixed numbers of people in the work force, and not to employ strategies that involve month-by-month fluctuations in the numbers of workers.

Because the average worker is so highly trained on all four products, the cost of bringing a new worker on board and training them to the point of average efficiency for the line is significantly higher than the cost of downsizing by one. This cost is estimated at $1000.00, and is directly related to the lower efficiency of the new worker for a week plus lower efficiency of an experienced worker training the new person. The salaries of the fifteen workers are reflected in the manufacturing cost for each instrument shown in table 1 below. Since the manufacturing cost includes material, burdened labor, overhead, and the cost of selling, distributing, advertising, and shipping the product, the salaries of workers are less than 1% of the total cost of the product. For this reason, the manufacturing cost can be assumed to be constant even if the number of production workers varies from fifteen.

During any given period, each of the fifteen people work four weeks, five days a week, eight hours a day, at 65% efficiency. Overtime is available paid at 1.5 times the normal rate, which is $28 2/3 per hour. Overtime is limited to two hours per day plus one weekend day, every two weeks. This translates to 36 hours overtime available per person in a period. Efficiency is assumed to be 80% during overtime, since no meetings or other distractions are occurring. Use of overtime is discouraged by management and unpopular with workers.

It is the desire of management to ship products to customers as soon as possible after an order is placed, but the products enjoy status in the marketplace that allow some delay in shipping to occur without great dissatisfaction. Inventory can be carried forward to allow the company to carry buffer stock, but all inventory carried from one period to the next is assessed a holding charge. The charge is 11% annual percentage rate of the sales price, $^1/_{12}$ of which is assessed against products in inventory at the end of a period. The company can choose to carry a backlog from one period to the next,

but incurs costs by so doing. The estimated cost of not meeting demand is estimated to be $75.00 on average for all products and consists of additional scheduling, expediting, sales expense plus an occasional lost sale. The amount of backlog accumulated on a product is allowed to be determined by the LP, however the backlog on any product at the completion of this six-period analysis must be zero for all products. The inventory and backlog figures for all products are assumed to be zero at the beginning of the first period.

Table 1 below shows data for each product with all figures expressed per unit. As shown in Table 1A the manufacturing cost is made up of material cost plus burdened overhead, which includes costs of material, production, overhead, corporate allocations, manufacturing engineering, distribution, sales, and warranties. In all cases the salaries of the production workers are such a small percentage of the manufacturing cost that adding or subtracting workers from the line has negligible impact on the manfacturing costs.

Product	Mfg. Cost	Mfg. Minutes	Sales Price	Inventory Cost
A	$2,619	203	$6,460	$59
B	$4,394	263	$14,950	$137
C	$3,587	135	$6,950	$63
D	$4,145	190	$7,900	$99

Table 1

Product	Material Costs	Burdened Overhead	Total Mfg. Cost
A	$2,163	$456	$2,619
B	$3,938	$456	$4,394
C	$3,206	$381	$3,587
D	$3,684	$461	$4,145

Table 1A.

Table 2 shows the anticipated demand for the next six periods for each of the products and the total demand to be filled by the line for each period.

	Period					
Product	1	2	3	4	5	6
A	157	157	196	157	156	196
B	48	59	49	47	60	47
C	45	40	102	28	31	42
D	95	95	107	78	62	80
Totals	345	351	454	310	309	365

Table 2

Literature Search

To formulate this problem into a linear program, we referred to the methodologies of production line scheduling in examples found in the class text and handouts.

Winston(1) uses the Mondo case to introduce production smoothing during a discussion on unrestricted-in-sign variables. The concept of allowing inventories to take on positive or negative values gives us a method of allowing the linear program to flex the monthly build rate to be unequal to demand in finding an optimal minimal solution. Mondo addresses the simple single-product case and assumes that people can be hired and fired monthly to meet demand.

Another useful example from class is found in Anderson, Sweeney and Williams(2) in their section on production scheduling. This example deals with multiple products on a single production line but, like the Mondo case, looks at the variation in production cost as a function of the increase or decrease from last month's production.

Given that the company policy in our case discouraged monthly fluctuations in work force, our formulation did not employ the approach of these two examples where the variation in production is a key variable. Our LP focused in case 1 on the variation the number of workers needed in order to identify possible one-time adjustments to the work force. In case 2 our LP focused on the amount of overtime required versus backlogs and inventories to minimize the objective function. In case 3 the focus was on meeting demand with fewer workers and no overtime.

Case 1

Model

As described in the executive summary, our solution required three related formulations which are presented here in three cases. The presentation of these three models will be done in sequence, with the formulation, solution, sensitivity analysis, and discussion presented on one before the next.

In the formulation of the models, our goal is to minimize cost of production over a six period time frame. If we define our variables in the form Vpt, where p is the product designator (a,b,c,d) and t is the period (1,2,3,4,5,6), our set of variables for the first model become:

P_{at} = number of product A produced during period t.
P_{bt} = number of product B produced during period t.
P_{ct} = number of product C produced during period t.
P_{dt} = number of product D produced during period t.
i_{at}^{+} = inventory of product A at end of period t.
i_{bt}^{+} = inventory of product B at end of period t.
i_{ct}^{+} = inventory of product C at end of period t.
i_{dt}^{+} = inventory of product D at end of period t.
i_{at}^{-} = backlog of product A at end of period t.
i_{bt}^{-} = backlog of product B at end of period t.
i_{ct}^{-} = backlog of product C at end of period t.
i_{dt}^{-} = backlog of product D at end of period t.
w_{t+} = increase in number of workers in period t
w_{t-} = decrease in number of workers in period t.

for periods t = 1,2,3,4,5,6 and products p = a,b,c,d

Our objective function is defined as: the manufacturing cost of the product + inventory cost + cost of missed opportunity + cost of increasing the work force + cost of decreasing the work force. We constrain the problem in the following manner:

1. For each product,the production in any period must equal the demand for that period, minus the backlog, plus the inventory for that period.

2. Total production minutes available per period for the 15 workers is 93,600 minutes. Minutes per worker added or subtracted due to increase or decrease in number or workers is 2,630 per period.

3. Inventories and backlogs at the end of period 6 must be zero for all products.

This formulation allows the inventory or backlog figures for periods 1 through 5 to be unconstrained, and allows the LP to show us whether it will be preferable to flex in build rate (employing inventory and backlog) or to flex in numbers of workers.

The formulation as input to Lindo is as follows:

```
MIN      2619 PA1 + 2619 PA2 + 2619 PA3 + 2619 PA4 + 2619 PA5 + 2619 PA6
     + 59 IA1PLUS + 59 IA2PLUS + 59 IA3PLUS + 59 IA4PLUS + 59 IA5PLUS
     + 59 IA6PLUS + 75 IA1MIN + 75 IA2MIN + 75 IA3MIN + 75 IA4MIN + 75 IA5MIN
     + 75 IA6MIN + 4394 PB1 + 4394 PB2 + 4394 PB3 + 4394 PB4 + 4394 PB5
     + 4394 PB6 + 137 IB1PLUS + 137 IB2PLUS + 137 IB3PLUS + 137 IB4PLUS
     + 137 IB5PLUS + 137 IB6PLUS + 75 IB1MIN + 75 IB2MIN + 75 IB3MIN
     + 75 IB4MIN + 75 IB5MIN + 75 IB6MIN + 3587 PC1 + 3587 PC2 + 3587 PC3
     + 3587 PC4 + 3587 PC5 + 3587 PC6 + 63 IC1PLUS + 63 IC2PLUS + 63 IC3PLUS
     + 63 IC4PLUS + 63 IC5PLUS + 63 IC6PLUS + 75 IC1MIN + 75 IC2MIN
     + 75 IC3MIN + 75 IC4MIN + 75 IC5MIN + 75 IC6MIN + 4145 PD1 + 4145 PD2
     + 4145 PD3 + 4145 PD4 + 4145 PD5 + 4145 PD6 + 99 ID1PLUS + 99 ID2PLUS
     + 99 ID3PLUS + 99 ID4PLUS + 99 ID5PLUS + 99 ID6PLUS + 75 ID1MIN
     + 75 ID2MIN + 75 ID3MIN + 75 ID4MIN + 75 ID5MIN + 75 ID6MIN
     + 1000 W1PLUS + 1000 W2PLUS + 1000 W3PLUS + 1000 W4PLUS + 1000 W5PLUS
     + 1000 W6PLUS + 100 W1MIN + 100 W2MIN + 100 W3MIN + 100 W4MIN
     + 100 W5MIN + 100 W6MIN
SUBJECT TO
! Production, inventory, and backlog balance constraints
     2)  PA1 - IA1PLUS + IA1MIN =    157
     3)  PA2 + IA1PLUS - IA2PLUS - IA1MIN + IA2MIN =    157
     4)  PA3 + IA2PLUS - IA3PLUS - IA2MIN + IA3MIN =    196
     5)  PA4 + IA3PLUS - IA4PLUS - IA3MIN + IA4MIN =    157
     6)  PA5 + IA4PLUS - IA5PLUS - IA4MIN + IA5MIN =    156
     7)  PA6 + IA5PLUS - IA6PLUS - IA5MIN + IA6MIN =    196
     8)  PB1 - IB1PLUS + IB1MIN =    48
     9)  PB2 + IB1PLUS - IB2PLUS - IB1MIN + IB2MIN =    59
    10)  PB3 + IB2PLUS - IB3PLUS - IB2MIN + IB3MIN =    49
    11)  PB4 + IB3PLUS - IB4PLUS - IB3MIN + IB4MIN =    47
    12)  PB5 + IB4PLUS - IB5PLUS - IB4MIN + IB5MIN =    60
    13)  PB6 + IB5PLUS - IB6PLUS - IB5MIN + IB6MIN =    47
    14)  PC1 - IC1PLUS + IC1MIN =    45
    15)  PC2 + IC1PLUS - IC2PLUS - IC1MIN + IC2MIN =    40
    16)  PC3 + IC2PLUS - IC3PLUS - IC2MIN + IC3MIN =    102
    17)  PC4 + IC3PLUS - IC4PLUS - IC3MIN + IC4MIN =    28
    18)  PC5 + IC4PLUS - IC5PLUS - IC4MIN + IC5MIN =    31
```

```
  19)  PC6 + IC5PLUS - IC6PLUS - IC5MIN + IC6MIN =    42
  20)  PD1 - ID1PLUS + ID1MIN =    95
  21)  PD2 + ID1PLUS - ID2PLUS - ID1MIN + ID2MIN =    95
  22)  PD3 + ID2PLUS - ID3PLUS - ID2MIN + ID3MIN =    107
  23)  PD4 + ID3PLUS - ID4PLUS - ID3MIN + ID4MIN =    78
  24)  PD5 + ID4PLUS - ID5PLUS - ID4MIN + ID5MIN =    62
  25)  PD6 + ID5PLUS - ID6PLUS - ID5MIN + ID6MIN =    80
!  Production capacity constraints
  26)  203 PA1 + 263 PB1 + 135 PC1 + 190 PD1 - 6240 W1PLUS + 6240 W1MIN
           =    93600
  27)  203 PA2 + 263 PB2 + 135 PC2 + 190 PD2 - 6240 W1PLUS - 6240 W2PLUS
       + 6240 W1MIN + 6240 W2MIN =    93600
  28)  203 PA3 + 263 PB3 + 135 PC3 + 190 PD3 - 6240 W1PLUS - 6240 W2PLUS
       - 6240 W3PLUS + 6240 W1MIN + 6240 W2MIN + 6240 W3MIN =    93600
  29)  203 PA4 + 263 PB4 + 135 PC4 + 190 PD4 - 6240 W1PLUS - 6240 W2PLUS
       - 6240 W3PLUS - 6240 W4PLUS + 6240 W1MIN + 6240 W2MIN + 6240 W3MIN
       + 6240 W4MIN =    93600
  30)  203 PA5 + 263 PB5 + 135 PC5 + 190 PD5 - 6240 W1PLUS - 6240 W2PLUS
       - 6240 W3PLUS - 6240 W4PLUS - 6240 W5PLUS + 6240 W1MIN
       + 6240 W2MIN + 6240 W3MIN + 6240 W4MIN + 6240 W5MIN =    93600
  31)  203 PA6 + 263 PB6 + 135 PC6 + 190 PD6 - 6240 W1PLUS - 6240 W2PLUS
       - 6240 W3PLUS - 6240 W4PLUS - 6240 W5PLUS - 6240 W6PLUS + 6240 W1MIN
       + 6240 W2MIN + 6240 W3MIN + 6240 W4MIN + 6240 W5MIN + 6240 W6MIN
       =    93600
!  Ending inventory and backlog constraints
  32)  IA6MIN + IB6MIN + IC6MIN + ID6MIN =    0
  33)  IA6PLUS + IB6PLUS + IC6PLUS + ID6PLUS =    0
```

Solution

This LP yields an objective function value of $7,212,209. As can be seen in figures 1 and 2, the formulation finds adding and subtracting people from the production line is better for the objective function than carrying inventory or backlog, and prescribes maintaining enough production capacity to meet demand in all cases but one.

Total demand during period 3 as shown in Table 2 is expected to appreciably higher than periods 2 and 4. The LP strikes a balance between having backlog at the end of period 3 and increasing capacity during that period. The backlog is concentrated in product B, since it has the highest manufacturing cost. Note that the actual cost of placing a product in inventory is more than the holding cost, but includes the cost of producing that extra unit as well.

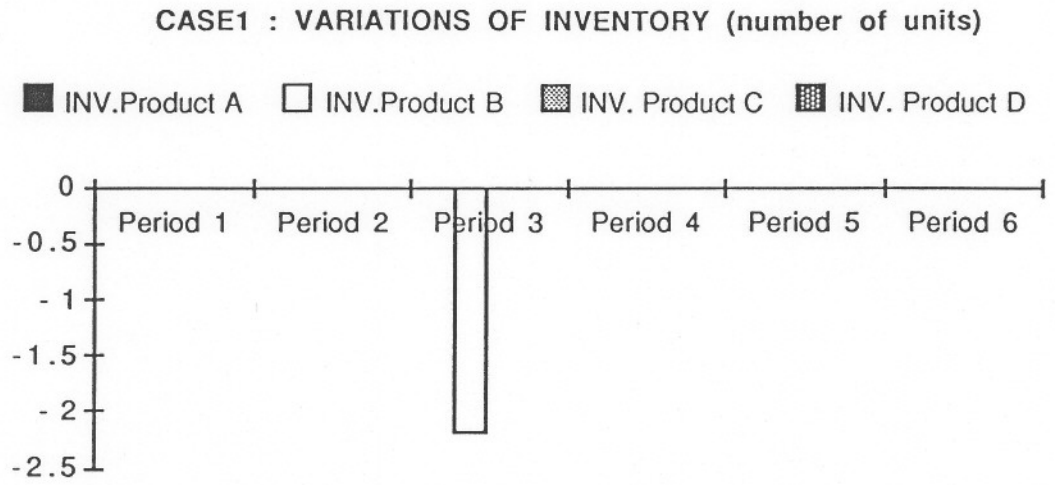

Figure 1.

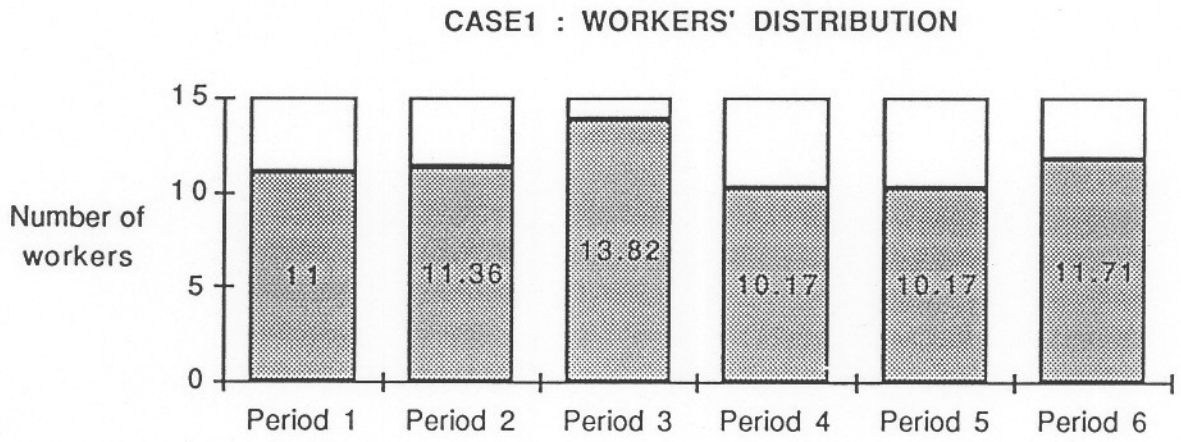

Figure 2.

Turning our attention to w_t+ and w_t- we find that the LP suggests removing four people from the line at the start of period 1, then adjusting the work force as shown in Figure 2. This is counter to company policy, but gives us a good estimate of how large our one-time adjustment to the work force should be. Our next case will make use of this data.

Sensitivity Analysis

Looking first at the Right-Hand Sides of the constraints, we see that this solution is quite sensitive to the demand estimates in periods 4 and 5. All four products are most sensitive to an increase in demand during period 4, and a decrease in period 5. The Table 3 shows the amount of increase/decrease in periods 4 and 5 that is permissible to allow the tableau to remain optimal. An increase change in demand of only two units of product B will affect optimality.

Product	Demand Variance
A	2.9
B	2.0
C	4.0
D	3.0

Table 3

Examining the capacity constraint we find that the most sensitive situation occurs during period 5, where a decrease in available capacity by 581 minutes will cause the tableau to change basis. This is less than one day's work for one worker, which indicates that any worker taking time off during period 5 will affect optimality. In looking at the RHS of the last two constraints, we find that a build plan that allows up to 36 units of backlog at the end of period 6 will retain the basis, but the range on inventory is zero.

Examining the range on the objective function coefficients, we see that the cost figures must be tightly controlled for all products. The Table 4 shows the minimum cost variance for each product that would allow the basis to remain optimal. Such small numbers suggest that it will be difficult in practice to maintain this basis due to normal manufacturing variations. One could question whether the raw data for the entire project is accurate to the degree of the

variance shown. From a management perspective, this underscores the importance of keeping a close watch on the cost of manufacturing.

Product	A	B	C	D
Cost Variance	$12.88	$17.72	$32.27	$20.82

Table 4

Finally, examining the dual values for each constraint confirms two intuitive facts. First is that one additional unit of demand for any product will increase the objective function value by roughly the cost of that unit, because this LP attempts to match production and demand for each product. Second is that the ability to carry backlog at the end of the sixth period will decrease the objective function value by the cost of the most expensive product to manufacture.

Discussion

This formulation shows us that we should reduce the number of workers by four in the first period, then vary from there. Given the desire by management to keep the work force as constant as possible, we search for alternatives to what the case 1 LP suggests. The most obvious next step is to decrease the work force as the LP indicates, then allow the use of overtime work to be factored into the LP.

Case 2

Model

In case 2 the LP is modified to include the use of overtime, and to prevent the monthly fluctuation in the work force. As before, our goal is to minimize cost of production over a six period time frame. Using the same basic variable set from case 1, we formulate an LP that uses four fewer workers than in case 1, and drop W_t^+ and W_t^-. A pair of overtime variables is then added that allows the LP to show the amount of time above or below capacity the production of each month requires. Sticking with the variable form Vpt, where p is the product designator (a,b,c,d) and t is the period (1,2,3,4,5,6), our set of variables for the second model become:

P_{at} = number of product A produced during period t.
P_{bt} = number of product B produced during period t.
P_{ct} = number of product C produced during period t.
P_{dt} = number of product D produced during period t.
i_{at}^+ = inventory of product A at end of period t.
i_{bt}^+ = inventory of product B at end of period t.
i_{ct}^+ = inventory of product C at end of period t.
i_{dt}^+ = inventory of product D at end of period t.
i_{at}^- = backlog of product A at end of period t.
i_{bt}^- = backlog of product B at end of period t.
i_{ct}^- = backlog of product C at end of period t.
i_{dt}^- = backlog of product D at end of period t.
O_{t+} = increase in number of workers in period t
O_{t-} = decrease in number of workers in period t.

for periods t = 1,2,3,4,5,6 and products P = a,b,c,d

Our objective function is defined as: the manufacturing cost of the product + inventory cost + cost of missed opportunity + cost of overtime work. No cost is associated with having negative overtime. We constrain the problem in the following manner:

1. For each product,the production in any period must equal the demand for that period, minus the backlog, plus the inventory for that period.

2. Total production minutes available per period for the 11 workers is 68,640 minutes.

3. Overtime minutes in each period must be less than or equal to 25,920 minutes.

4. Inventories and backlogs at the end of period 6 must be zero for all products.

This formulation allows the inventory or backlog figures for periods 1 through 5 to be unconstrained, and allows the LP to show us whether it will be preferable to flex in build rate (employing inventory and backlog) or to flex number of minutes worked.

The formulation as input to Lindo is as follows:

```
MIN     2619 PA1 + 2619 PA2 + 2619 PA3 + 2619 PA4 + 2619 PA5 + 2619 PA6
    + 59 IA1PLUS + 59 IA2PLUS + 59 IA3PLUS + 59 IA4PLUS + 59 IA5PLUS
    + 59 IA6PLUS + 75 IA1MIN + 75 IA2MIN + 75 IA3MIN + 75 IA4MIN + 75 IA5MIN
    + 75 IA6MIN + 4394 PB1 + 4394 PB2 + 4394 PB3 + 4394 PB4 + 4394 PB5
    + 4394 PB6 + 137 IB1PLUS + 137 IB2PLUS + 137 IB3PLUS + 137 IB4PLUS
    + 137 IB5PLUS + 137 IB6PLUS + 75 IB1MIN + 75 IB2MIN + 75 IB3MIN
    + 75 IB4MIN + 75 IB5MIN + 75 IB6MIN + 3587 PC1 + 3587 PC2 + 3587 PC3
    + 3587 PC4 + 3587 PC5 + 3587 PC6 + 63 IC1PLUS + 63 IC2PLUS + 63 IC3PLUS
    + 63 IC4PLUS + 63 IC5PLUS + 63 IC6PLUS + 75 IC1MIN + 75 IC2MIN
    + 75 IC3MIN + 75 IC4MIN + 75 IC5MIN + 75 IC6MIN + 4145 PD1 + 4145 PD2
    + 4145 PD3 + 4145 PD4 + 4145 PD5 + 4145 PD6 + 99 ID1PLUS + 99 ID2PLUS
    + 99 ID3PLUS + 99 ID4PLUS + 99 ID5PLUS + 99 ID6PLUS + 75 ID1MIN
    + 75 ID2MIN + 75 ID3MIN + 75 ID4MIN + 75 ID5MIN + 75 ID6MIN
    + 0.43 O1PLUS + 0.43 O2PLUS + 0.43 O3PLUS + 0.43 O4PLUS + 0.43 O5PLUS
    + 0.43 O6PLUS
SUBJECT TO
! Production, inventory, and backlog balance constraints
   2)  PA1 - IA1PLUS + IA1MIN =  157
   3)  PA2 + IA1PLUS - IA2PLUS - IA1MIN + IA2MIN =  157
   4)  PA3 + IA2PLUS - IA3PLUS - IA2MIN + IA3MIN =  196
   5)  PA4 + IA3PLUS - IA4PLUS - IA3MIN + IA4MIN =  157
   6)  PA5 + IA4PLUS - IA5PLUS - IA4MIN + IA5MIN =  156
   7)  PA6 + IA5PLUS - IA6PLUS - IA5MIN + IA6MIN =  196
   8)  PB1 - IB1PLUS + IB1MIN =  48
   9)  PB2 + IB1PLUS - IB2PLUS - IB1MIN + IB2MIN =  59
  10)  PB3 + IB2PLUS - IB3PLUS - IB2MIN + IB3MIN =  49
  11)  PB4 + IB3PLUS - IB4PLUS - IB3MIN + IB4MIN =  47
  12)  PB5 + IB4PLUS - IB5PLUS - IB4MIN + IB5MIN =  60
  13)  PB6 + IB5PLUS - IB6PLUS - IB5MIN + IB6MIN =  47
  14)  PC1 - IC1PLUS + IC1MIN =  45
  15)  PC2 + IC1PLUS - IC2PLUS - IC1MIN + IC2MIN =  40
  16)  PC3 + IC2PLUS - IC3PLUS - IC2MIN + IC3MIN =  102
  17)  PC4 + IC3PLUS - IC4PLUS - IC3MIN + IC4MIN =  28
  18)  PC5 + IC4PLUS - IC5PLUS - IC4MIN + IC5MIN =  31
  19)  PC6 + IC5PLUS - IC6PLUS - IC5MIN + IC6MIN =  42
  20)  PD1 - ID1PLUS + ID1MIN =  95
  21)  PD2 + ID1PLUS - ID2PLUS - ID1MIN + ID2MIN =  95
  22)  PD3 + ID2PLUS - ID3PLUS - ID2MIN + ID3MIN =  107
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  23)  PD4 + ID3PLUS - ID4PLUS - ID3MIN + ID4MIN =    78
  24)  PD5 + ID4PLUS - ID5PLUS - ID4MIN + ID5MIN =    62
  25)  PD6 + ID5PLUS - ID6PLUS - ID5MIN + ID6MIN =    80
! Production capacity constraints
  26)  203 PA1 + 263 PB1 + 135 PC1 + 190 PD1 - O1PLUS + O1MIN =    68640
  27)  203 PA2 + 263 PB2 + 135 PC2 + 190 PD2 - O2PLUS + O2MIN =    68640
  28)  203 PA3 + 263 PB3 + 135 PC3 + 190 PD3 - O3PLUS + O3MIN =    68640
  29)  203 PA4 + 263 PB4 + 135 PC4 + 190 PD4 - O4PLUS + O4MIN =    68640
  30)  203 PA5 + 263 PB5 + 135 PC5 + 190 PD5 - O5PLUS + O5MIN =    68640
  31)  203 PA6 + 263 PB6 + 135 PC6 + 190 PD6 - O6PLUS + O6MIN =    68640
! Overtime constraints
  32)  O1PLUS <=  25920
  33)  O2PLUS <=  25920
  34)  O3PLUS <=  25920
  35)  O4PLUS <=  25920
  36)  O5PLUS <=  25920
  37)  O6PLUS <=  25920
! Ending inventory and backlog constraints
  38)  IA6MIN + IB6MIN + IC6MIN + ID6MIN =    0

  39)  IA6PLUS + IB6PLUS + IC6PLUS + ID6PLUS =    0
```

Solution

This LP yields an objective function value of $7,216,094 versus $7,212,209 from case 1. The formulation finds using overtime minutes is generally less expensive than carrying inventory or backlog. The value of the objective function increases by only $3,885 out of over seven million dollars, which is negligible. As shown in Figures 3 and 4, the LP suggests using overtime in periods 2 and 3 but having small excess capacity in period 5, while carrying backlog of product B from periods 3 to 4 and inventory of product A from periods 5 to 6.

The maximum overtime required in period 3 is 12,327 minutes, less than half the overtime available. The LP has found that it is cheaper to carry a backlog of product B out of period 3 than to utilize more overtime minutes to meet the large increase in period 3 demand.

In period 5, the choice to inventory product A is driven by the need to have zero inventory and backlog at the end of period 6, plus an increase in demand for products A and D from periods 5 to 6. Since the manufacturing and inventory costs of product A are much lower than those of product D, the LP chooses to inventory product A.

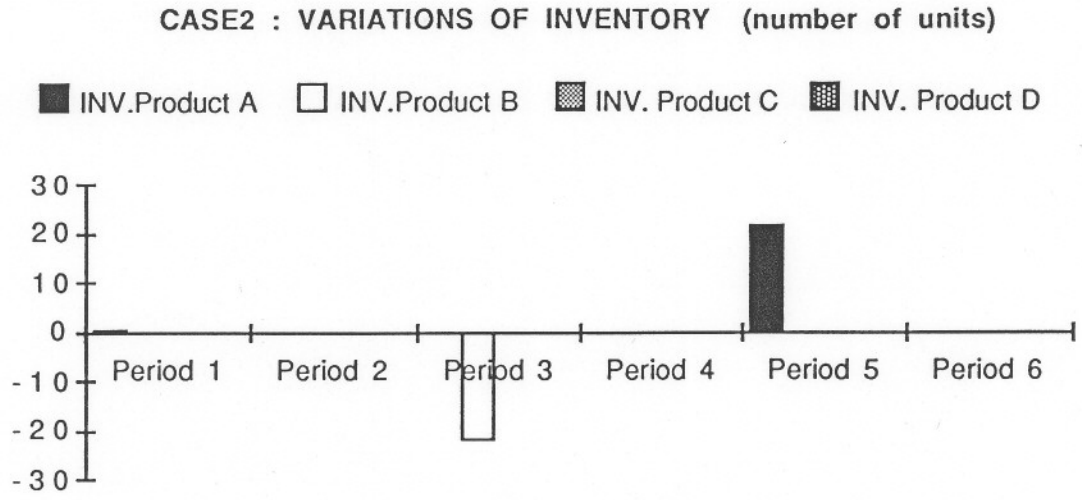

Figure 3.

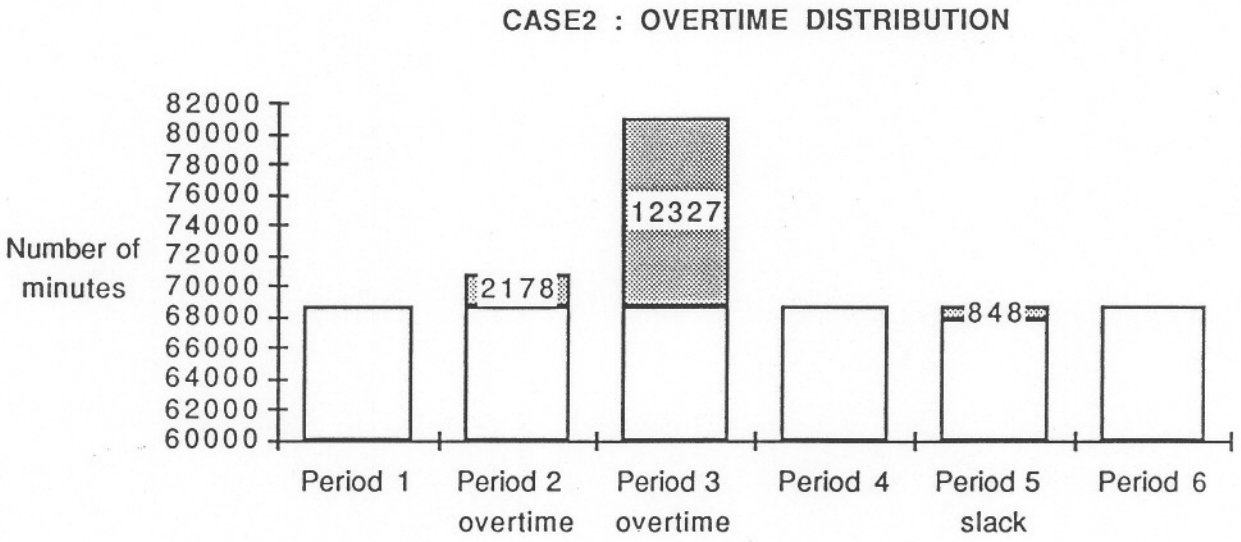

Figure 4

Sensitivity Analysis

The Right-Hand Sides of the constraints yield a higher degree of sensitivity than did case 1. The demand values for period 1 may increase only slightly to maintain the basis, and may decrease none at all. The Table 5 shows the amount of increase/decrease in period 1 that is permissible to allow the tableau to remain optimal.

Product	Demand Increase	Demand Decrease
A	0	11
B	0	8
C	0	16
D	0	11

Table 5

Examining the capacity constraint we find that the most sensitive situations occur in periods 1, 5 and 6. In period 1 a decrease of 20 minutes will cause the tableau to change basis, and in periods 5 and 6, 848 minutes will force a change. In case 1 management had to worry about time off in one period. Now three periods have are sensitive, a less desirable condition.

In looking at the RHS of the last two constraints, we find that a build plan that allows up to only 17 units of backlog at the end of period 6 will retain the basis, and the range on inventory is still zero.

Examining the range on the objective function coefficients, we see that the cost figures must be tightly controlled for all products as in case 1. The Table 6 shows the minimum cost variance for each product that would allow the basis to remain optimal. The variance values for case 2 are slightly higher than for case 1, but not to the degree that management attention to manufacturing cost could be relaxed.

Product	A	B	C	D
Cost Variance	$17.11	$22.17	$23.76	$20.82

Table 6

Examining the dual values for each constraint yields the same conclusions as in case 1 in that one additional unit of demand for any product will increase the objective function value by roughly the cost of that unit, and that the ability to carry backlog at the end of the sixth period will decrease the objective function value by the cost of the most expensive product to manufacture.

Discussion

This second formulation shows us that reducing the work force by four at the start of period 1 and maintaining that number is a plausible approach. We see that such a solution does not require excessive use of overtime, and that the overall objective function value is affected only slightly. It is also notable that the LP chooses to prescribe overtime and slack time rather than smooth the production from one period to the next and use inventory/backlog.

Armed with this knowledge and the reminder that management and the workers alike do not prefer the use of overtime, a third alternative that would force greater use of backlog and inventory is investigated in case 3.

Case 3

Model

In case 3 the LP is modified to prevent the use of overtime, and to prevent the monthly fluctuation in the work force. Our goal is still to minimize cost of production over a six period time frame. Using the same basic variable set from case 2, we formulate an LP that uses three fewer workers than in case 1, and drop O_t^+ and O_t^-. Our set of variables for the third model becomes:

P_{at} = number of product A produced during period t.
P_{bt} = number of product B produced during period t.
P_{ct} = number of product C produced during period t.
P_{dt} = number of product D produced during period t.
i_{at}^+ = inventory of product A at end of period t.
i_{bt}^+ = inventory of product B at end of period t.
i_{ct}^+ = inventory of product C at end of period t.
i_{dt}^+ = inventory of product D at end of period t.
i_{at}^- = backlog of product A at end of period t.
i_{bt}^- = backlog of product B at end of period t.
i_{ct}^- = backlog of product C at end of period t.
i_{dt}^- = backlog of product D at end of period t.

for periods t = 1,2,3,4,5,6 and products P = a,b,c,d

Our objective function is defined as: the manufacturing cost of the product + inventory cost + cost of not meeting demand. We constrain the problem in the following manner:

1. For each product,the production in any period must equal the demand for that period, minus the backlog, plus the inventory for that period.

2. Total production minutes available per period for the 12 workers is 74,880 minutes.

3. Inventories and backlogs at the end of period 6 must be zero for all products.

This formulation allows the inventory or backlog figures for periods 1 through 5 to be unconstrained, and allows the LP to show us what the difference in cost might be when carrying backlog and/or inventory .

The formulation as input to Lindo is as follows:

```
MIN     2619 PA1 + 2619 PA2 + 2619 PA3 + 2619 PA4 + 2619 PA5 + 2619 PA6
     + 59 IA1PLUS + 59 IA2PLUS + 59 IA3PLUS + 59 IA4PLUS + 59 IA5PLUS
     + 59 IA6PLUS + 75 IA1MIN + 75 IA2MIN + 75 IA3MIN + 75 IA4MIN + 75 IA5MIN
     + 75 IA6MIN + 4394 PB1 + 4394 PB2 + 4394 PB3 + 4394 PB4 + 4394 PB5
     + 4394 PB6 + 137 IB1PLUS + 137 IB2PLUS + 137 IB3PLUS + 137 IB4PLUS
     + 137 IB5PLUS + 137 IB6PLUS + 75 IB1MIN + 75 IB2MIN + 75 IB3MIN
     + 75 IB4MIN + 75 IB5MIN + 75 IB6MIN + 3587 PC1 + 3587 PC2 + 3587 PC3
     + 3587 PC4 + 3587 PC5 + 3587 PC6 + 63 IC1PLUS + 63 IC2PLUS + 63 IC3PLUS
     + 63 IC4PLUS + 63 IC5PLUS + 63 IC6PLUS + 75 IC1MIN + 75 IC2MIN
     + 75 IC3MIN + 75 IC4MIN + 75 IC5MIN + 75 IC6MIN + 4145 PD1 + 4145 PD2
     + 4145 PD3 + 4145 PD4 + 4145 PD5 + 4145 PD6 + 99 ID1PLUS + 99 ID2PLUS
     + 99 ID3PLUS + 99 ID4PLUS + 99 ID5PLUS + 99 ID6PLUS + 75 ID1MIN
     + 75 ID2MIN + 75 ID3MIN + 75 ID4MIN + 75 ID5MIN + 75 ID6MIN
SUBJECT TO
! Production, inventory and backlog balance constraints
     2)  PA1 - IA1PLUS + IA1MIN =    157
     3)  PA2 + IA1PLUS - IA2PLUS - IA1MIN + IA2MIN =    157
     4)  PA3 + IA2PLUS - IA3PLUS - IA2MIN + IA3MIN =    196
     5)  PA4 + IA3PLUS - IA4PLUS - IA3MIN + IA4MIN =    157
     6)  PA5 + IA4PLUS - IA5PLUS - IA4MIN + IA5MIN =    156
     7)  PA6 + IA5PLUS - IA6PLUS - IA5MIN + IA6MIN =    196
     8)  PB1 - IB1PLUS + IB1MIN =    48
     9)  PB2 + IB1PLUS - IB2PLUS - IB1MIN + IB2MIN =    59
    10)  PB3 + IB2PLUS - IB3PLUS - IB2MIN + IB3MIN =    49
    11)  PB4 + IB3PLUS - IB4PLUS - IB3MIN + IB4MIN =    47
    12)  PB5 + IB4PLUS - IB5PLUS - IB4MIN + IB5MIN =    60
    13)  PB6 + IB5PLUS - IB6PLUS - IB5MIN + IB6MIN =    47
    14)  PC1 - IC1PLUS + IC1MIN =    45
    15)  PC2 + IC1PLUS - IC2PLUS - IC1MIN + IC2MIN =    40
```

```
  16)  PC3 + IC2PLUS - IC3PLUS - IC2MIN + IC3MIN =    102
  17)  PC4 + IC3PLUS - IC4PLUS - IC3MIN + IC4MIN =    28
  18)  PC5 + IC4PLUS - IC5PLUS - IC4MIN + IC5MIN =    31
  19)  PC6 + IC5PLUS - IC6PLUS - IC5MIN + IC6MIN =    42
  20)  PD1 - ID1PLUS + ID1MIN =    95
  21)  PD2 + ID1PLUS - ID2PLUS - ID1MIN + ID2MIN =    95
  22)  PD3 + ID2PLUS - ID3PLUS - ID2MIN + ID3MIN =    107
  23)  PD4 + ID3PLUS - ID4PLUS - ID3MIN + ID4MIN =    78
  24)  PD5 + ID4PLUS - ID5PLUS - ID4MIN + ID5MIN =    62
  25)  PD6 + ID5PLUS - ID6PLUS - ID5MIN + ID6MIN =    80
!  Production capacity constraints
  26)  203 PA1 + 263 PB1 + 135 PC1 + 190 PD1 <=    74880
  27)  203 PA2 + 263 PB2 + 135 PC2 + 190 PD2 <=    74880
  28)  203 PA3 + 263 PB3 + 135 PC3 + 190 PD3 <=    74880
  29)  203 PA4 + 263 PB4 + 135 PC4 + 190 PD4 <=    74880
  30)  203 PA5 + 263 PB5 + 135 PC5 + 190 PD5 <=    74880
  31)  203 PA6 + 263 PB6 + 135 PC6 + 190 PD6 <=    74880
! Ending inventory and backlog constraints
  38)  IA6MIN + IB6MIN + IC6MIN + ID6MIN =    0
  39)  IA6PLUS + IB6PLUS + IC6PLUS + ID6PLUS =    0
```

Solution

This LP yields an objective function value of $7,210,314 versus $7,216,094 from case 2 and $7,212,209 from case 1. Figure 5 shows that the formulation makes use of backlog only in period 3. Figure 6 shows that full capacity is used only in period 3, with all other periods having excess capacity. The increase in total demand during period 3 exceeds the available capacity, so the LP chooses to create a backlog of the most costly product, product B.

This solution is regarded as meeting all applicable goals outlined in the problem description. Overtime is avoided, the work force is held constant, demand is met no more than one period after the order is placed, and the objective value is at a minimum.

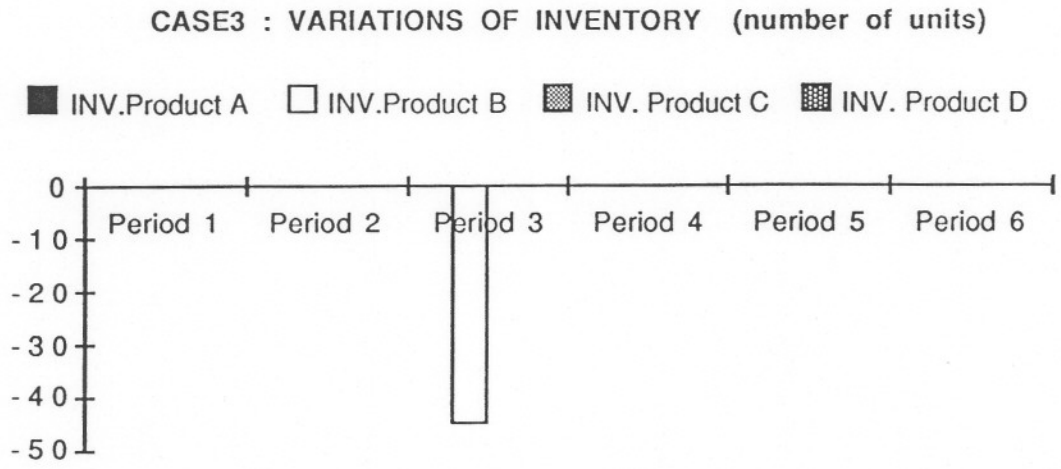

Figure 5.

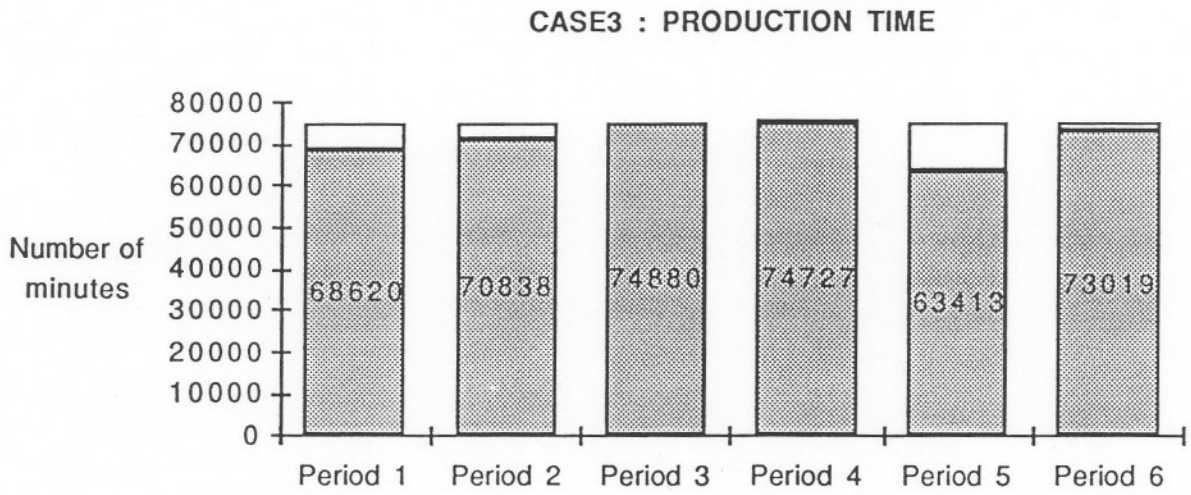

Figure 6.

Sensitivity Analysis

The Right-Hand Sides of the constraints in this case yield lower sensitivity two variation. Table 7 shows that increases in demand will affect the optimum tableau as in cases 1 and 2, but case 3 is more tolerant of decreases in demand.

Product	Demand Increase	Demand Decrease

A	0	58
B	0	45
C	1	28
D	1	62

Table 7.

Examining the capacity constraint we find that the most sensitive situations occur in periods 3 and 4. In both periods a decrease of 153 minutes will cause the tableau to change basis.

In looking at the RHS of the last two constraints, we find the range on inventory at the end of the sixth period is zero but that 47 units of backlog could be carried into period 7 without a change in the basis.

Examining the range on the objective function coefficients, we see that the cost figures must be more tightly controlled for this case than in cases 1 or 2. The table below shows the minimum cost variance for each product that would allow the basis to remain optimal.

Product	A	B	C	D
Cost Variance	$1.11	$1.43	$24.50	$20.82

Table 8.

Examining the dual values for each constraint yields the same conclusions as in cases 1 and 2 in that one additional unit of demand for any product will increase the objective function value by roughly the cost of that unit, and that the ability to carry backlog at the end of the sixth period will decrease the objective function value by the cost of the most expensive product to manufacture.

Discussion

This third and final formulation shows us that reducing the work force by three at the start of period 1 and maintaining that number remains plausible approach. In five of six periods there is unused capacity, which makes for easier contingency planning by management. The highest unused capacity is 1.8 people, occuring In period 5. We also find that the cost of manufacturing the products causes the LP to avoid inventory to a great degree.

What is concerning about all three cases is the degree of sensitivity to manufacturing cost and demand. Neither figure for any product will be accurate to the degree necessary to insure we retain optimality, so it is likely that the LP will change basis at some point in the six-month time frame. If demand increases significantly, it is possible that the case 3 LP will become infeasible.

Extensions

This model does not take into account the cost of procuring materials and what holding costs would be associated with inventory of raw materials when a backlog is chosen. Rather, this model assumes that the material is ready and waiting at a constant cost whenever management decides to build. A Just-in-Time operation working smoothly might approximate this, but in this line the more critical, expensive components are stockpiled to some degree to insure flexibility. The cost of not building when you have the material to do so could be included in the model.

Conclusions/Recommendations

By the data made available to this team, the production manager should consider operating with fewer people than the current 15. No data was available pertaining to the reliability of the demand forecast, and it may be prudent to keep more than 12 workers if the forecasts tend to be low.

A close watch on manufacturing costs should be maintained. The costing figures were obtained via activity-based costing systems and update each period. It would be advisable to modify the final LP every period using updated figures to see if it remains feasible. The expected variation in objective function cost will remain neglibible unless the manufacturing costs change appreciably.

Finally, it would be wise to educate the employees on the line as to what has the largest leverage on the cost figures. Perhaps using some of the excess capacity that may be present on the line to track down methods of reducing cost would be a wise investment. Certainly watching for ways to control the costs to their current level is necessary.

Appendices

References Cited

1. Winston, W.L. *Introduction to Mathematical Programming Applications and Algorithms* Boston, Massachusetts: PWS-Kent Publishing Company, 1991, pp. 170-173.

2. Anderson, D.R., Sweeney, D.J., Williams, T.A. *An Introduction to Management Science - Quantitative Approaches to Decision Making* St. Paul, Minnesota: West Publishing Company, 1988, pp. 124-132.

Computer Output

Case 1

```
MIN    2619 PA1 + 2619 PA2 + 2619 PA3 + 2619 PA4 + 2619 PA5 + 2619 PA6
   + 59 IA1PLUS + 59 IA2PLUS + 59 IA3PLUS + 59 IA4PLUS + 59 IA5PLUS
   + 59 IA6PLUS + 75 IA1MIN + 75 IA2MIN + 75 IA3MIN + 75 IA4MIN + 75 IA5MIN
   + 75 IA6MIN + 4394 PB1 + 4394 PB2 + 4394 PB3 + 4394 PB4 + 4394 PB5
   + 4394 PB6 + 137 IB1PLUS + 137 IB2PLUS + 137 IB3PLUS + 137 IB4PLUS
   + 137 IB5PLUS + 137 IB6PLUS + 75 IB1MIN + 75 IB2MIN + 75 IB3MIN
   + 75 IB4MIN + 75 IB5MIN + 75 IB6MIN + 3587 PC1 + 3587 PC2 + 3587 PC3
   + 3587 PC4 + 3587 PC5 + 3587 PC6 + 63 IC1PLUS + 63 IC2PLUS + 63 IC3PLUS
   + 63 IC4PLUS + 63 IC5PLUS + 63 IC6PLUS + 75 IC1MIN + 75 IC2MIN
   + 75 IC3MIN + 75 IC4MIN + 75 IC5MIN + 75 IC6MIN + 4145 PD1 + 4145 PD2
   + 4145 PD3 + 4145 PD4 + 4145 PD5 + 4145 PD6 + 99 ID1PLUS + 99 ID2PLUS
   + 99 ID3PLUS + 99 ID4PLUS + 99 ID5PLUS + 99 ID6PLUS + 75 ID1MIN
   + 75 ID2MIN + 75 ID3MIN + 75 ID4MIN + 75 ID5MIN + 75 ID6MIN
   + 1000 W1PLUS + 1000 W2PLUS + 1000 W3PLUS + 1000 W4PLUS + 1000 W5PLUS
   + 1000 W6PLUS + 100 W1MIN + 100 W2MIN + 100 W3MIN + 100 W4MIN
   + 100 W5MIN + 100 W6MIN
SUBJECT TO
    2)  PA1 - IA1PLUS + IA1MIN =  157
    3)  PA2 + IA1PLUS - IA2PLUS - IA1MIN + IA2MIN =  157
    4)  PA3 + IA2PLUS - IA3PLUS - IA2MIN + IA3MIN =  196
    5)  PA4 + IA3PLUS - IA4PLUS - IA3MIN + IA4MIN =  157
    6)  PA5 + IA4PLUS - IA5PLUS - IA4MIN + IA5MIN =  156
    7)  PA6 + IA5PLUS - IA6PLUS - IA5MIN + IA6MIN =  196
    8)  PB1 - IB1PLUS + IB1MIN =  48
    9)  PB2 + IB1PLUS - IB2PLUS - IB1MIN + IB2MIN =  59
   10)  PB3 + IB2PLUS - IB3PLUS - IB2MIN + IB3MIN =  49
   11)  PB4 + IB3PLUS - IB4PLUS - IB3MIN + IB4MIN =  47
   12)  PB5 + IB4PLUS - IB5PLUS - IB4MIN + IB5MIN =  60
   13)  PB6 + IB5PLUS - IB6PLUS - IB5MIN + IB6MIN =  47
   14)  PC1 - IC1PLUS + IC1MIN =  45
   15)  PC2 + IC1PLUS - IC2PLUS - IC1MIN + IC2MIN =  40
   16)  PC3 + IC2PLUS - IC3PLUS - IC2MIN + IC3MIN =  102
   17)  PC4 + IC3PLUS - IC4PLUS - IC3MIN + IC4MIN =  28
   18)  PC5 + IC4PLUS - IC5PLUS - IC4MIN + IC5MIN =  31
   19)  PC6 + IC5PLUS - IC6PLUS - IC5MIN + IC6MIN =  42
   20)  PD1 - ID1PLUS + ID1MIN =  95
   21)  PD2 + ID1PLUS - ID2PLUS - ID1MIN + ID2MIN =  95
   22)  PD3 + ID2PLUS - ID3PLUS - ID2MIN + ID3MIN =  107
   23)  PD4 + ID3PLUS - ID4PLUS - ID3MIN + ID4MIN =  78
   24)  PD5 + ID4PLUS - ID5PLUS - ID4MIN + ID5MIN =  62
   25)  PD6 + ID5PLUS - ID6PLUS - ID5MIN + ID6MIN =  80
   26)  203 PA1 + 263 PB1 + 135 PC1 + 190 PD1 - 6240 W1PLUS + 6240 W1MIN =  93600
   27)  203 PA2 + 263 PB2 + 135 PC2 + 190 PD2 - 6240 W1PLUS - 6240 W2PLUS
  + 6240 W1MIN + 6240 W2MIN =  93600
```

```
    28)  203 PA3 + 263 PB3 + 135 PC3 + 190 PD3 - 6240 W1PLUS - 6240 W2PLUS
   - 6240 W3PLUS + 6240 W1MIN + 6240 W2MIN + 6240 W3MIN =    93600
    29)  203 PA4 + 263 PB4 + 135 PC4 + 190 PD4 - 6240 W1PLUS - 6240 W2PLUS
   - 6240 W3PLUS - 6240 W4PLUS + 6240 W1MIN + 6240 W2MIN + 6240 W3MIN  + 6240 W4MIN =    93600
    30)  203 PA5 + 263 PB5 + 135 PC5 + 190 PD5 - 6240 W1PLUS - 6240 W2PLUS
   - 6240 W3PLUS - 6240 W4PLUS - 6240 W5PLUS + 6240 W1MIN + 6240 W2MIN
   + 6240 W3MIN + 6240 W4MIN + 6240 W5MIN =    93600
    31)  203 PA6 + 263 PB6 + 135 PC6 + 190 PD6 - 6240 W1PLUS - 6240 W2PLUS
   - 6240 W3PLUS - 6240 W4PLUS - 6240 W5PLUS - 6240 W6PLUS + 6240 W1MIN
   + 6240 W2MIN + 6240 W3MIN + 6240 W4MIN + 6240 W5MIN + 6240 W6MIN =    93600
    32)  IA6MIN + IB6MIN + IC6MIN + ID6MIN =    0
    33)  IA6PLUS + IB6PLUS + IC6PLUS + ID6PLUS =    0
END

LP OPTIMUM FOUND AT STEP    46

     OBJECTIVE FUNCTION VALUE

     1)  7212209.00

 VARIABLE        VALUE          REDUCED COST
    PA1       157.000000          .000000
    PA2       157.000000          .000000
    PA3       196.000000          .000000
    PA4       157.000000          .000000
    PA5       156.000000          .000000
    PA6       196.000000          .000000
  IA1PLUS        .000000        23.214843
  IA2PLUS        .000000        23.214843
  IA3PLUS        .000000       116.889640
  IA4PLUS        .000000        50.576420
  IA5PLUS        .000000        12.787110
  IA6PLUS        .000000      2710.532000
   IA1MIN        .000000       110.785200
   IA2MIN        .000000       110.785200
   IA3MIN        .000000        17.110351
   IA4MIN        .000000        83.423583
   IA5MIN        .000000       121.212900
   IA6MIN        .000000      1784.615400
    PB1       48.000000          .000000
    PB2       59.000000          .000000
    PB3       46.790874          .000000
    PB4       49.209130          .000000
    PB5       60.000000          .000000
    PB6       47.000000          .000000
  IB1PLUS        .000000        90.637700
  IB2PLUS        .000000        90.637700
  IB3PLUS        .000000       212.000000
  IB4PLUS        .000000       126.086910
  IB5PLUS        .000000        77.127930
  IB6PLUS        .000000      4573.147400
   IB1MIN        .000000       121.362300
   IB2MIN        .000000       121.362300
   IB3MIN       2.209126          .000000
   IB4MIN        .000000        85.913090
   IB5MIN        .000000       134.872100
   IB6MIN        .000000          .000000
    PC1       45.000000          .000000
    PC2       40.000000          .000000
    PC3      102.000000          .000000
    PC4       28.000000          .000000
    PC5       31.000000          .000000
    PC6       42.000000          .000000
  IC1PLUS        .000000        39.201904
```

IC2PLUS	.000000	39.201904
IC3PLUS	.000000	101.498040
IC4PLUS	.000000	57.398193
IC5PLUS	.000000	32.267333
IC6PLUS	.000000	3671.635000
IC1MIN	.000000	98.798100
IC2MIN	.000000	98.798100
IC3MIN	.000000	36.501953
IC4MIN	.000000	80.601810
IC5MIN	.000000	105.732700
IC6MIN	.000000	827.512930
PD1	95.000000	.000000
PD2	95.000000	.000000
PD3	107.000000	.000000
PD4	78.000000	.000000
PD5	62.000000	.000000
PD6	80.000000	.000000
ID1PLUS	.000000	65.506350
ID2PLUS	.000000	65.506350
ID3PLUS	.000000	153.182610
ID4PLUS	.000000	91.115722
ID5PLUS	.000000	55.746582
ID6PLUS	.000000	4274.449000
ID1MIN	.000000	108.493700
ID2MIN	.000000	108.493700
ID3MIN	.000000	20.817382
ID4MIN	.000000	82.884280
ID5MIN	.000000	118.253410
ID6MIN	.000000	260.698730
W1PLUS	.000000	1100.000000
W2PLUS	.355449	.000000
W3PLUS	2.460897	.000000
W4PLUS	.000000	1100.000000
W5PLUS	.000000	420.535330
W6PLUS	1.539423	.000000
W1MIN	4.003205	.000000
W2MIN	.000000	1100.000000
W3MIN	.000000	1100.000000
W4MIN	3.650801	.000000
W5MIN	.000000	679.464700
W6MIN	.000000	1100.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-2583.215000
3)	.000000	-2619.000000
4)	.000000	-2654.785100
5)	.000000	-2596.896000
6)	.000000	-2605.319000
7)	.000000	-2651.532000
8)	.000000	-4347.638000
9)	.000000	-4394.000000
10)	.000000	-4440.362300
11)	.000000	-4365.362300
12)	.000000	-4376.275300
13)	.000000	-4436.147400
14)	.000000	-3563.202000
15)	.000000	-3587.000000
16)	.000000	-3610.798000
17)	.000000	-3572.300000
18)	.000000	-3577.902000
19)	.000000	-3608.635000
20)	.000000	-4111.506300
21)	.000000	-4145.000000
22)	.000000	-4178.494000
23)	.000000	-4124.311000

24)	.000000	-4132.195300
25)	.000000	-4175.449000
26)	.000000	-.176282
27)	.000000	0.000000
28)	.000000	.176282
29)	.000000	-.108889
30)	.000000	-.067393
31)	.000000	.160256
32)	.000000	4361.147400
33)	.000000	.000000

NO. ITERATIONS= 46

RANGES IN WHICH THE BASIS IS UNCHANGED:

	OBJ COEFFICIENT RANGES		
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
PA1	2619.000000	110.785200	23.214843
PA2	2619.000000	23.214843	23.214843
PA3	2619.000000	17.110351	110.785200
PA4	2619.000000	83.423583	17.110351
PA5	2619.000000	50.576420	12.787110
PA6	2619.000000	12.787110	121.212900
IA1PLUS	59.000000	INFINITY	23.214843
IA2PLUS	59.000000	INFINITY	23.214843
IA3PLUS	59.000000	INFINITY	116.889640
IA4PLUS	59.000000	INFINITY	50.576420
IA5PLUS	59.000000	INFINITY	12.787110
IA6PLUS	59.000000	INFINITY	2710.532000
IA1MIN	75.000000	INFINITY	110.785200
IA2MIN	75.000000	INFINITY	110.785200
IA3MIN	75.000000	INFINITY	17.110351
IA4MIN	75.000000	INFINITY	83.423583
IA5MIN	75.000000	INFINITY	121.212900
IA6MIN	75.000000	INFINITY	1784.615400
PB1	4394.000000	121.362300	90.637700
PB2	4394.000000	90.637700	90.637700
PB3	4394.000000	16.566551	17.724490
PB4	4394.000000	17.724490	16.566551
PB5	4394.000000	126.086910	77.127930
PB6	4394.000000	77.127930	134.872100
IB1PLUS	137.000000	INFINITY	90.637700
IB2PLUS	137.000000	INFINITY	90.637700
IB3PLUS	137.000000	INFINITY	212.000000
IB4PLUS	137.000000	INFINITY	126.086910
IB5PLUS	137.000000	INFINITY	77.127930
IB6PLUS	137.000000	INFINITY	4573.147400
IB1MIN	75.000000	INFINITY	121.362300
IB2MIN	75.000000	INFINITY	121.362300
IB3MIN	75.000000	17.724490	16.566551
IB4MIN	75.000000	INFINITY	85.913090
IB5MIN	75.000000	INFINITY	134.872100
IB6MIN	75.000000	260.698730	INFINITY
PC1	3587.000000	98.798100	39.201904
PC2	3587.000000	39.201904	39.201904
PC3	3587.000000	36.501953	98.798100
PC4	3587.000000	80.601810	36.501953
PC5	3587.000000	57.398193	32.267333
PC6	3587.000000	32.267333	105.732700
IC1PLUS	63.000000	INFINITY	39.201904
IC2PLUS	63.000000	INFINITY	39.201904
IC3PLUS	63.000000	INFINITY	101.498040
IC4PLUS	63.000000	INFINITY	57.398193

IC5PLUS	63.000000	INFINITY	32.267333
IC6PLUS	63.000000	INFINITY	3671.635000
IC1MIN	75.000000	INFINITY	98.798100
IC2MIN	75.000000	INFINITY	98.798100
IC3MIN	75.000000	INFINITY	36.501953
IC4MIN	75.000000	INFINITY	80.601810
IC5MIN	75.000000	INFINITY	105.732700
IC6MIN	75.000000	INFINITY	827.512930
PD1	4145.000000	108.493700	65.506350
PD2	4145.000000	65.506350	65.506350
PD3	4145.000000	20.817382	108.493700
PD4	4145.000000	82.884280	20.817382
PD5	4145.000000	91.115722	55.746582
PD6	4145.000000	55.746582	118.253410
ID1PLUS	99.000000	INFINITY	65.506350
ID2PLUS	99.000000	INFINITY	65.506350
ID3PLUS	99.000000	INFINITY	153.182610
ID4PLUS	99.000000	INFINITY	91.115722
ID5PLUS	99.000000	INFINITY	55.746582
ID6PLUS	99.000000	INFINITY	4274.449000
ID1MIN	75.000000	INFINITY	108.493700
ID2MIN	75.000000	INFINITY	108.493700
ID3MIN	75.000000	INFINITY	20.817382
ID4MIN	75.000000	INFINITY	82.884280
ID5MIN	75.000000	INFINITY	118.253410
ID6MIN	75.000000	INFINITY	260.698730
W1PLUS	1000.000000	INFINITY	1100.000000
W2PLUS	1000.000000	356.799600	713.599120
W3PLUS	1000.000000	356.799600	420.535400
W4PLUS	1000.000000	INFINITY	1100.000000
W5PLUS	1000.000000	INFINITY	420.535330
W6PLUS	1000.000000	196.530940	1100.000000
W1MIN	100.000000	713.599120	1100.000000
W2MIN	100.000000	INFINITY	1100.000000
W3MIN	100.000000	INFINITY	1100.000000
W4MIN	100.000000	196.530940	210.267700
W5MIN	100.000000	INFINITY	679.464700
W6MIN	100.000000	INFINITY	1100.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	157.000000	10.926110	157.000000
3	157.000000	75.645320	10.926110
4	196.000000	INFINITY	75.645320
5	157.000000	2.862069	60.620690
6	156.000000	47.320194	2.862069
7	196.000000	INFINITY	47.320194
8	48.000000	8.433460	48.000000
9	59.000000	58.387832	8.433460
10	49.000000	INFINITY	46.790874
11	47.000000	2.209126	46.790874
12	60.000000	36.524711	2.209126
13	47.000000	INFINITY	36.524711
14	45.000000	16.429630	45.000000
15	40.000000	113.748140	16.429630
16	102.000000	INFINITY	102.000000
17	28.000000	4.303704	28.000000
18	31.000000	71.155550	4.303704
19	42.000000	INFINITY	42.000000
20	95.000000	11.673684	95.000000
21	95.000000	80.821052	11.673684
22	107.000000	INFINITY	80.821052
23	78.000000	3.057895	64.768420

24	62.000000	50.557891	3.057895
25	80.000000	INFINITY	50.557891
26	93600.000000	INFINITY	2218.000000
27	93600.000000	2218.000000	15356.000000
28	93600.000000	15356.000000	INFINITY
29	93600.000000	12306.000000	581.000000
30	93600.000000	581.000000	9606.000000
31	93600.000000	9606.000000	INFINITY
32	.000000	36.524711	.000000
33	.000000	.000000	.000000

Case 2

```
MIN    2619 PA1 + 2619 PA2 + 2619 PA3 + 2619 PA4 + 2619 PA5 + 2619 PA6
   + 59 IA1PLUS + 59 IA2PLUS + 59 IA3PLUS + 59 IA4PLUS + 59 IA5PLUS
   + 59 IA6PLUS + 75 IA1MIN + 75 IA2MIN + 75 IA3MIN + 75 IA4MIN + 75 IA5MIN
   + 75 IA6MIN + 4394 PB1 + 4394 PB2 + 4394 PB3 + 4394 PB4 + 4394 PB5
   + 4394 PB6 + 137 IB1PLUS + 137 IB2PLUS + 137 IB3PLUS + 137 IB4PLUS
   + 137 IB5PLUS + 137 IB6PLUS + 75 IB1MIN + 75 IB2MIN + 75 IB3MIN
   + 75 IB4MIN + 75 IB5MIN + 75 IB6MIN + 3587 PC1 + 3587 PC2 + 3587 PC3
   + 3587 PC4 + 3587 PC5 + 3587 PC6 + 63 IC1PLUS + 63 IC2PLUS + 63 IC3PLUS
   + 63 IC4PLUS + 63 IC5PLUS + 63 IC6PLUS + 75 IC1MIN + 75 IC2MIN
   + 75 IC3MIN + 75 IC4MIN + 75 IC5MIN + 75 IC6MIN + 4145 PD1 + 4145 PD2
   + 4145 PD3 + 4145 PD4 + 4145 PD5 + 4145 PD6 + 99 ID1PLUS + 99 ID2PLUS
   + 99 ID3PLUS + 99 ID4PLUS + 99 ID5PLUS + 99 ID6PLUS + 75 ID1MIN
   + 75 ID2MIN + 75 ID3MIN + 75 ID4MIN + 75 ID5MIN + 75 ID6MIN
   + 0.43 O1PLUS + 0.43 O2PLUS + 0.43 O3PLUS + 0.43 O4PLUS + 0.43 O5PLUS
   + 0.43 O6PLUS
SUBJECT TO
    2)  PA1 - IA1PLUS + IA1MIN =  157
    3)  PA2 + IA1PLUS - IA2PLUS - IA1MIN + IA2MIN =  157
    4)  PA3 + IA2PLUS - IA3PLUS - IA2MIN + IA3MIN =  196
    5)  PA4 + IA3PLUS - IA4PLUS - IA3MIN + IA4MIN =  157
    6)  PA5 + IA4PLUS - IA5PLUS - IA4MIN + IA5MIN =  156
    7)  PA6 + IA5PLUS - IA6PLUS - IA5MIN + IA6MIN =  196
    8)  PB1 - IB1PLUS + IB1MIN =  48
    9)  PB2 + IB1PLUS - IB2PLUS - IB1MIN + IB2MIN =  59
   10)  PB3 + IB2PLUS - IB3PLUS - IB2MIN + IB3MIN =  49
   11)  PB4 + IB3PLUS - IB4PLUS - IB3MIN + IB4MIN =  47
   12)  PB5 + IB4PLUS - IB5PLUS - IB4MIN + IB5MIN =  60
   13)  PB6 + IB5PLUS - IB6PLUS - IB5MIN + IB6MIN =  47
   14)  PC1 - IC1PLUS + IC1MIN =  45
   15)  PC2 + IC1PLUS - IC2PLUS - IC1MIN + IC2MIN =  40
   16)  PC3 + IC2PLUS - IC3PLUS - IC2MIN + IC3MIN =  102
   17)  PC4 + IC3PLUS - IC4PLUS - IC3MIN + IC4MIN =  28
   18)  PC5 + IC4PLUS - IC5PLUS - IC4MIN + IC5MIN =  31
   19)  PC6 + IC5PLUS - IC6PLUS - IC5MIN + IC6MIN =  42
   20)  PD1 - ID1PLUS + ID1MIN =  95
   21)  PD2 + ID1PLUS - ID2PLUS - ID1MIN + ID2MIN =  95
   22)  PD3 + ID2PLUS - ID3PLUS - ID2MIN + ID3MIN =  107
   23)  PD4 + ID3PLUS - ID4PLUS - ID3MIN + ID4MIN =  78
   24)  PD5 + ID4PLUS - ID5PLUS - ID4MIN + ID5MIN =  62
   25)  PD6 + ID5PLUS - ID6PLUS - ID5MIN + ID6MIN =  80
   26)  203 PA1 + 263 PB1 + 135 PC1 + 190 PD1 - O1PLUS + O1MIN =  68640
   27)  203 PA2 + 263 PB2 + 135 PC2 + 190 PD2 - O2PLUS + O2MIN =  68640
   28)  203 PA3 + 263 PB3 + 135 PC3 + 190 PD3 - O3PLUS + O3MIN =  68640
   29)  203 PA4 + 263 PB4 + 135 PC4 + 190 PD4 - O4PLUS + O4MIN =  68640
   30)  203 PA5 + 263 PB5 + 135 PC5 + 190 PD5 - O5PLUS + O5MIN =  68640
   31)  203 PA6 + 263 PB6 + 135 PC6 + 190 PD6 - O6PLUS + O6MIN =  68640
   32)  O1PLUS <=  25920
   33)  O2PLUS <=  25920
   34)  O3PLUS <=  25920
   35)  O4PLUS <=  25920
   36)  O5PLUS <=  25920
   37)  O6PLUS <=  25920
   38)  IA6MIN + IB6MIN + IC6MIN + ID6MIN =  0
   39)  IA6PLUS + IB6PLUS + IC6PLUS + ID6PLUS =  0
END

LP OPTIMUM FOUND AT STEP   33

     OBJECTIVE FUNCTION VALUE

     1)  7216094.00
```

VARIABLE	VALUE	REDUCED COST
PA1	157.098510	.000000
PA2	156.901500	.000000
PA3	196.000000	.000000
PA4	157.000000	.000000
PA5	177.571420	.000000
PA6	174.428600	.000000
IA1PLUS	.098522	.000000
IA2PLUS	.000000	59.000000
IA3PLUS	.000000	116.889900
IA4PLUS	.000000	88.400150
IA5PLUS	21.571430	.000000
IA6PLUS	.000000	2737.000000
IA1MIN	.000000	134.000000
IA2MIN	.000000	75.000000
IA3MIN	.000000	17.110110
IA4MIN	.000000	45.599853
IA5MIN	.000000	134.000000
IA6MIN	.000000	1792.438400
PB1	48.000000	.000000
PB2	59.000000	.000000
PB3	26.916350	.000000
PB4	69.083650	.000000
PB5	60.000000	.000000
PB6	47.000000	.000000
IB1PLUS	.000000	60.561523
IB2PLUS	.000000	137.000000
IB3PLUS	.000000	212.000000
IB4PLUS	.000000	175.089840
IB5PLUS	.000000	60.561523
IB6PLUS	.000000	4607.438400
IB1MIN	.000000	151.438500
IB2MIN	.000000	75.000000
IB3MIN	22.083650	.000000
IB4MIN	.000000	36.910160
IB5MIN	.000000	151.438500
IB6MIN	.000000	.000000
PC1	45.000000	.000000
PC2	40.000000	.000000
PC3	102.000000	.000000
PC4	28.000000	.000000
PC5	31.000000	.000000
PC6	42.000000	.000000
IC1PLUS	.000000	23.763430
IC2PLUS	.000000	63.000000
IC3PLUS	.000000	101.498300
IC4PLUS	.000000	82.551760
IC5PLUS	.000000	23.763671
IC6PLUS	.000000	3689.236300
IC1MIN	.000000	114.236600
IC2MIN	.000000	75.000000
IC3MIN	.000000	36.501710
IC4MIN	.000000	55.448242
IC5MIN	.000000	114.236320
IC6MIN	.000000	844.202140
PD1	95.000000	.000000
PD2	95.000000	.000000
PD3	107.000000	.000000
PD4	78.000000	.000000
PD5	62.000000	.000000
PD6	80.000000	.000000
ID1PLUS	.000000	43.778320
ID2PLUS	.000000	99.000000

ID3PLUS	.000000	153.182610
ID4PLUS	.000000	126.517600
ID5PLUS	.000000	43.778320
ID6PLUS	.000000	4299.222000
ID1MIN	.000000	130.221700
ID2MIN	.000000	75.000000
ID3MIN	.000000	20.817382
ID4MIN	.000000	47.482421
ID5MIN	.000000	130.221700
ID6MIN	.000000	270.216800
O1PLUS	.000000	.290640
O2PLUS	2178.000000	.000000
O3PLUS	12327.000000	.000000
O4PLUS	.000000	.285172
O5PLUS	.000000	.430000
O6PLUS	.000000	.139360
O1MIN	.000000	.139360
O2MIN	.000000	.430000
O3MIN	.000000	.430000
O4MIN	.000000	.144828
O5MIN	848.000000	.000000
O6MIN	.000000	.290640

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-2647.290000
3)	.000000	-2706.290000
4)	.000000	-2706.290000
5)	.000000	-2648.400100
6)	.000000	-2619.000000
7)	.000000	-2678.000000
8)	.000000	-4430.651300
9)	.000000	-4507.090000
10)	.000000	-4507.090000
11)	.000000	-4432.090000
12)	.000000	-4394.000000
13)	.000000	-4470.438400
14)	.000000	-3605.813400
15)	.000000	-3645.050000
16)	.000000	-3645.050000
17)	.000000	-3606.552000
18)	.000000	-3587.000000
19)	.000000	-3626.236300
20)	.000000	-4171.479000
21)	.000000	-4226.700100
22)	.000000	-4226.700100
23)	.000000	-4172.518000
24)	.000000	-4145.000000
25)	.000000	-4200.222000
26)	.000000	.139360
27)	.000000	.430000
28)	.000000	.430000
29)	.000000	.144828
30)	.000000	.000000
31)	.000000	.290640
32)	25920.000000	.000000
33)	23742.000000	.000000
34)	13593.000000	.000000
35)	25920.000000	.000000
36)	25920.000000	.000000
37)	25920.000000	.000000
38)	.000000	4395.438400
39)	.000000	.000000

NO. ITERATIONS= 33

RANGES IN WHICH THE BASIS IS UNCHANGED:

	OBJ COEFFICIENT RANGES		
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
PA1	2619.000000	28.290040	58.999961
PA2	2619.000000	58.999961	28.290040
PA3	2619.000000	17.110110	75.000000
PA4	2619.000000	45.599853	17.110110
PA5	2619.000000	28.290004	45.599853
PA6	2619.000000	59.000000	28.290004
IA1PLUS	59.000000	28.290040	58.999961
IA2PLUS	59.000000	INFINITY	59.000000
IA3PLUS	59.000000	INFINITY	116.889900
IA4PLUS	59.000000	INFINITY	88.400150
IA5PLUS	59.000000	28.290004	59.000000
IA6PLUS	59.000000	INFINITY	2737.000000
IA1MIN	75.000000	INFINITY	134.000000
IA2MIN	75.000000	INFINITY	75.000000
IA3MIN	75.000000	INFINITY	17.110110
IA4MIN	75.000000	INFINITY	45.599853
IA5MIN	75.000000	INFINITY	134.000000
IA6MIN	75.000000	INFINITY	1792.438400
PB1	4394.000000	151.438500	60.561523
PB2	4394.000000	60.561523	137.000000
PB3	4394.000000	36.910160	22.167282
PB4	4394.000000	22.167282	59.077644
PB5	4394.000000	151.438500	36.910160
PB6	4394.000000	60.561523	151.438500
IB1PLUS	137.000000	INFINITY	60.561523
IB2PLUS	137.000000	INFINITY	137.000000
IB3PLUS	137.000000	INFINITY	212.000000
IB4PLUS	137.000000	INFINITY	175.089840
IB5PLUS	137.000000	INFINITY	60.561523
IB6PLUS	137.000000	INFINITY	4607.438400
IB1MIN	75.000000	INFINITY	151.438500
IB2MIN	75.000000	INFINITY	75.000000
IB3MIN	75.000000	22.167282	36.910160
IB4MIN	75.000000	INFINITY	36.910160
IB5MIN	75.000000	INFINITY	151.438500
IB6MIN	75.000000	270.216800	INFINITY
PC1	3587.000000	114.236600	23.763430
PC2	3587.000000	23.763430	63.000000
PC3	3587.000000	36.501710	75.000000
PC4	3587.000000	55.448242	36.501710
PC5	3587.000000	82.551760	23.763671
PC6	3587.000000	23.763671	114.236320
IC1PLUS	63.000000	INFINITY	23.763430
IC2PLUS	63.000000	INFINITY	63.000000
IC3PLUS	63.000000	INFINITY	101.498300
IC4PLUS	63.000000	INFINITY	82.551760
IC5PLUS	63.000000	INFINITY	23.763671
IC6PLUS	63.000000	INFINITY	3689.236300
IC1MIN	75.000000	INFINITY	114.236600
IC2MIN	75.000000	INFINITY	75.000000
IC3MIN	75.000000	INFINITY	36.501710
IC4MIN	75.000000	INFINITY	55.448242
IC5MIN	75.000000	INFINITY	114.236320
IC6MIN	75.000000	INFINITY	844.202140
PD1	4145.000000	130.221700	43.778320
PD2	4145.000000	43.778320	99.000000
PD3	4145.000000	20.817382	75.000000

PD4	4145.000000	47.482421	20.817382
PD5	4145.000000	126.517600	43.778320
PD6	4145.000000	43.778320	130.221700
ID1PLUS	99.000000	INFINITY	43.778320
ID2PLUS	99.000000	INFINITY	99.000000
ID3PLUS	99.000000	INFINITY	153.182610
ID4PLUS	99.000000	INFINITY	126.517600
ID5PLUS	99.000000	INFINITY	43.778320
ID6PLUS	99.000000	INFINITY	4299.222000
ID1MIN	75.000000	INFINITY	130.221700
ID2MIN	75.000000	INFINITY	75.000000
ID3MIN	75.000000	INFINITY	20.817382
ID4MIN	75.000000	INFINITY	47.482421
ID5MIN	75.000000	INFINITY	130.221700
ID6MIN	75.000000	INFINITY	270.216800
O1PLUS	.430000	INFINITY	.290640
O2PLUS	.430000	.285171	.139360
O3PLUS	.430000	.140343	.144828
O4PLUS	.430000	INFINITY	.285172
O5PLUS	.430000	INFINITY	.430000
O6PLUS	.430000	INFINITY	.139360
O1MIN	.000000	INFINITY	.139360
O2MIN	.000000	INFINITY	.430000
O3MIN	.000000	INFINITY	.430000
O4MIN	.000000	INFINITY	.144828
O5MIN	.000000	.140343	.139360
O6MIN	.000000	INFINITY	.290640

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	157.000000	.098522	10.729063
3	157.000000	116.955700	10.729063
4	196.000000	66.960590	60.724140
5	157.000000	28.610840	34.871921
6	156.000000	4.177340	177.571420
7	196.000000	4.177340	21.571430
8	48.000000	.076046	8.281368
9	59.000000	90.273760	8.281368
10	49.000000	51.684410	26.916350
11	47.000000	22.083650	26.916350
12	60.000000	3.224334	60.000000
13	47.000000	3.224334	16.650190
14	45.000000	.148148	16.133333
15	40.000000	175.866700	16.133333
16	102.000000	100.688900	91.311111
17	28.000000	43.022224	28.000000
18	31.000000	6.281481	31.000000
19	42.000000	6.281481	32.437034
20	95.000000	.105263	11.463160
21	95.000000	124.957900	11.463160
22	107.000000	71.542110	64.878944
23	78.000000	30.568422	37.257900
24	62.000000	4.463158	62.000000
25	80.000000	4.463158	23.047370
26	68640.000000	2178.000000	20.000000
27	68640.000000	2178.000000	23742.000000
28	68640.000000	12327.000000	13593.000000
29	68640.000000	7079.000000	5808.000000
30	68640.000000	INFINITY	848.000000
31	68640.000000	4379.000000	848.000000
32	25920.000000	INFINITY	25920.000000
33	25920.000000	INFINITY	23742.000000
34	25920.000000	INFINITY	13593.000000
35	25920.000000	INFINITY	25920.000000
36	25920.000000	INFINITY	25920.000000
37	25920.000000	INFINITY	25920.000000
38	.000000	16.650190	.000000
39	.000000	.000000	.000000

Case 3

MIN 2619 PA1 + 2619 PA2 + 2619 PA3 + 2619 PA4 + 2619 PA5 + 2619 PA6
+ 59 IA1PLUS + 59 IA2PLUS + 59 IA3PLUS + 59 IA4PLUS + 59 IA5PLUS
+ 59 IA6PLUS + 75 IA1MIN + 75 IA2MIN + 75 IA3MIN + 75 IA4MIN + 75 IA5MIN
+ 75 IA6MIN + 4394 PB1 + 4394 PB2 + 4394 PB3 + 4394 PB4 + 4394 PB5
+ 4394 PB6 + 137 IB1PLUS + 137 IB2PLUS + 137 IB3PLUS + 137 IB4PLUS
+ 137 IB5PLUS + 137 IB6PLUS + 75 IB1MIN + 75 IB2MIN + 75 IB3MIN
+ 75 IB4MIN + 75 IB5MIN + 75 IB6MIN + 3587 PC1 + 3587 PC2 + 3587 PC3
+ 3587 PC4 + 3587 PC5 + 3587 PC6 + 63 IC1PLUS + 63 IC2PLUS + 63 IC3PLUS
+ 63 IC4PLUS + 63 IC5PLUS + 63 IC6PLUS + 75 IC1MIN + 75 IC2MIN
+ 75 IC3MIN + 75 IC4MIN + 75 IC5MIN + 75 IC6MIN + 4145 PD1 + 4145 PD2
+ 4145 PD3 + 4145 PD4 + 4145 PD5 + 4145 PD6 + 99 ID1PLUS + 99 ID2PLUS
+ 99 ID3PLUS + 99 ID4PLUS + 99 ID5PLUS + 99 ID6PLUS + 75 ID1MIN

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    + 75 ID2MIN + 75 ID3MIN + 75 ID4MIN + 75 ID5MIN + 75 ID6MIN
SUBJECT TO
      2)   PA1 - IA1PLUS + IA1MIN =    157
      3)   PA2 + IA1PLUS - IA2PLUS - IA1MIN + IA2MIN =    157
      4)   PA3 + IA2PLUS - IA3PLUS - IA2MIN + IA3MIN =    196
      5)   PA4 + IA3PLUS - IA4PLUS - IA3MIN + IA4MIN =    157
      6)   PA5 + IA4PLUS - IA5PLUS - IA4MIN + IA5MIN =    156
      7)   PA6 + IA5PLUS - IA6PLUS - IA5MIN + IA6MIN =    196
      8)   PB1 - IB1PLUS + IB1MIN =    48
      9)   PB2 + IB1PLUS - IB2PLUS - IB1MIN + IB2MIN =    59
     10)   PB3 + IB2PLUS - IB3PLUS - IB2MIN + IB3MIN =    49
     11)   PB4 + IB3PLUS - IB4PLUS - IB3MIN + IB4MIN =    47
     12)   PB5 + IB4PLUS - IB5PLUS - IB4MIN + IB5MIN =    60
     13)   PB6 + IB5PLUS - IB6PLUS - IB5MIN + IB6MIN =    47
     14)   PC1 - IC1PLUS + IC1MIN =    45
     15)   PC2 + IC1PLUS - IC2PLUS - IC1MIN + IC2MIN =    40
     16)   PC3 + IC2PLUS - IC3PLUS - IC2MIN + IC3MIN =    102
     17)   PC4 + IC3PLUS - IC4PLUS - IC3MIN + IC4MIN =    28
     18)   PC5 + IC4PLUS - IC5PLUS - IC4MIN + IC5MIN =    31
     19)   PC6 + IC5PLUS - IC6PLUS - IC5MIN + IC6MIN =    42
     20)   PD1 - ID1PLUS + ID1MIN =    95
     21)   PD2 + ID1PLUS - ID2PLUS - ID1MIN + ID2MIN =    95
     22)   PD3 + ID2PLUS - ID3PLUS - ID2MIN + ID3MIN =    107
     23)   PD4 + ID3PLUS - ID4PLUS - ID3MIN + ID4MIN =    78
     24)   PD5 + ID4PLUS - ID5PLUS - ID4MIN + ID5MIN =    62
     25)   PD6 + ID5PLUS - ID6PLUS - ID5MIN + ID6MIN =    80
     26)    203 PA1 + 263 PB1 + 135 PC1 + 190 PD1 <=    74880
     27)    203 PA2 + 263 PB2 + 135 PC2 + 190 PD2 <=    74880
     28)    203 PA3 + 263 PB3 + 135 PC3 + 190 PD3 <=    74880
     29)    203 PA4 + 263 PB4 + 135 PC4 + 190 PD4 <=    74880
     30)    203 PA5 + 263 PB5 + 135 PC5 + 190 PD5 <=    74880
     31)    203 PA6 + 263 PB6 + 135 PC6 + 190 PD6 <=    74880
     32)   IA6MIN + IB6MIN + IC6MIN + ID6MIN =    0
     33)   IA6PLUS + IB6PLUS + IC6PLUS + ID6PLUS =    0
END

LP OPTIMUM FOUND AT STEP    27

     OBJECTIVE FUNCTION VALUE

        1)    7210314.00

 VARIABLE        VALUE          REDUCED COST
      PA1      157.000000          .000000
      PA2      157.000000          .000000
      PA3      196.000000          .000000
      PA4      157.000000          .000000
      PA5      156.000000          .000000
      PA6      196.000000          .000000
   IA1PLUS        .000000        59.000000
   IA2PLUS        .000000         1.110352
   IA3PLUS        .000000       116.889640
   IA4PLUS        .000000        59.000000
   IA5PLUS        .000000        59.000000
```

IA6PLUS	.000000	2678.000000
IA1MIN	.000000	75.000000
IA2MIN	.000000	132.889640
IA3MIN	.000000	17.110351
IA4MIN	.000000	75.000000
IA5MIN	.000000	75.000000
IA6MIN	.000000	1775.000000
PB1	48.000000	.000000
PB2	59.000000	.000000
PB3	3.771863	.000000
PB4	92.228134	.000000
PB5	60.000000	.000000
PB6	47.000000	.000000
IB1PLUS	.000000	137.000000
IB2PLUS	.000000	62.000000
IB3PLUS	.000000	212.000000
IB4PLUS	.000000	137.000000
IB5PLUS	.000000	137.000000
IB6PLUS	.000000	4531.000000
IB1MIN	.000000	75.000000
IB2MIN	.000000	150.000000
IB3MIN	45.228134	.000000
IB4MIN	.000000	75.000000
IB5MIN	.000000	75.000000
IB6MIN	.000000	.000000
PC1	45.000000	.000000
PC2	40.000000	.000000
PC3	102.000000	.000000
PC4	28.000000	.000000
PC5	31.000000	.000000
PC6	42.000000	.000000
IC1PLUS	.000000	63.000000
IC2PLUS	.000000	24.501953
IC3PLUS	.000000	101.498040
IC4PLUS	.000000	63.000000
IC5PLUS	.000000	63.000000
IC6PLUS	.000000	3650.000000
IC1MIN	.000000	75.000000
IC2MIN	.000000	113.498040
IC3MIN	.000000	36.501953
IC4MIN	.000000	75.000000
IC5MIN	.000000	75.000000
IC6MIN	.000000	807.000000
PD1	95.000000	.000000
PD2	95.000000	.000000
PD3	107.000000	.000000
PD4	78.000000	.000000
PD5	62.000000	.000000
PD6	80.000000	.000000
ID1PLUS	.000000	99.000000
ID2PLUS	.000000	44.817382
ID3PLUS	.000000	153.182610
ID4PLUS	.000000	99.000000
ID5PLUS	.000000	99.000000

ID6PLUS	.000000	4244.000000
ID1MIN	.000000	75.000000
ID2MIN	.000000	129.182610
ID3MIN	.000000	20.817382
ID4MIN	.000000	75.000000
ID5MIN	.000000	75.000000
ID6MIN	.000000	249.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-2619.000000
3)	.000000	-2619.000000
4)	.000000	-2676.890000
5)	.000000	-2619.000000
6)	.000000	-2619.000000
7)	.000000	-2619.000000
8)	.000000	-4394.000000
9)	.000000	-4394.000000
10)	.000000	-4469.000000
11)	.000000	-4394.000000
12)	.000000	-4394.000000
13)	.000000	-4394.000000
14)	.000000	-3587.000000
15)	.000000	-3587.000000
16)	.000000	-3625.498000
17)	.000000	-3587.000000
18)	.000000	-3587.000000
19)	.000000	-3587.000000
20)	.000000	-4145.000000
21)	.000000	-4145.000000
22)	.000000	-4199.183000
23)	.000000	-4145.000000
24)	.000000	-4145.000000
25)	.000000	-4145.000000
26)	6260.000000	.000000
27)	4042.000000	.000000
28)	.000000	.285171
29)	153.000000	.000000
30)	11467.000000	.000000
31)	1861.000000	.000000
32)	.000000	4319.000000
33)	.000000	.000000

NO. ITERATIONS= 27

RANGES IN WHICH THE BASIS IS UNCHANGED:

	OBJ COEFFICIENT RANGES		
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
PA1	2619.000000	75.000000	59.000000
PA2	2619.000000	59.000000	1.110352
PA3	2619.000000	1.110352	116.889640
PA4	2619.000000	75.000000	17.110351

PA5	2619.000000	59.000000	59.000000
PA6	2619.000000	59.000000	75.000000
IA1PLUS	59.000000	INFINITY	59.000000
IA2PLUS	59.000000	INFINITY	1.110352
IA3PLUS	59.000000	INFINITY	116.889640
IA4PLUS	59.000000	INFINITY	59.000000
IA5PLUS	59.000000	INFINITY	59.000000
IA6PLUS	59.000000	INFINITY	2678.000000
IA1MIN	75.000000	INFINITY	75.000000
IA2MIN	75.000000	INFINITY	132.889640
IA3MIN	75.000000	INFINITY	17.110351
IA4MIN	75.000000	INFINITY	75.000000
IA5MIN	75.000000	INFINITY	75.000000
IA6MIN	75.000000	INFINITY	1775.000000
PB1	4394.000000	75.000000	137.000000
PB2	4394.000000	137.000000	62.000000
PB3	4394.000000	75.000000	1.438534
PB4	4394.000000	1.438534	75.000000
PB5	4394.000000	75.000000	75.000000
PB6	4394.000000	137.000000	75.000000
IB1PLUS	137.000000	INFINITY	137.000000
IB2PLUS	137.000000	INFINITY	62.000000
IB3PLUS	137.000000	INFINITY	212.000000
IB4PLUS	137.000000	INFINITY	137.000000
IB5PLUS	137.000000	INFINITY	137.000000
IB6PLUS	137.000000	INFINITY	4531.000000
IB1MIN	75.000000	INFINITY	75.000000
IB2MIN	75.000000	INFINITY	150.000000
IB3MIN	75.000000	1.438534	75.000000
IB4MIN	75.000000	INFINITY	75.000000
IB5MIN	75.000000	INFINITY	75.000000
IB6MIN	75.000000	249.000000	INFINITY
PC1	3587.000000	75.000000	63.000000
PC2	3587.000000	63.000000	24.501953
PC3	3587.000000	24.501953	101.498040
PC4	3587.000000	75.000000	36.501953
PC5	3587.000000	63.000000	63.000000
PC6	3587.000000	63.000000	75.000000
IC1PLUS	63.000000	INFINITY	63.000000
IC2PLUS	63.000000	INFINITY	24.501953
IC3PLUS	63.000000	INFINITY	101.498040
IC4PLUS	63.000000	INFINITY	63.000000
IC5PLUS	63.000000	INFINITY	63.000000
IC6PLUS	63.000000	INFINITY	3650.000000
IC1MIN	75.000000	INFINITY	75.000000
IC2MIN	75.000000	INFINITY	113.498040
IC3MIN	75.000000	INFINITY	36.501953
IC4MIN	75.000000	INFINITY	75.000000
IC5MIN	75.000000	INFINITY	75.000000
IC6MIN	75.000000	INFINITY	807.000000
PD1	4145.000000	75.000000	99.000000
PD2	4145.000000	99.000000	44.817382
PD3	4145.000000	20.817382	129.182610
PD4	4145.000000	75.000000	20.817382

PD5	4145.000000	75.000000	75.000000
PD6	4145.000000	99.000000	75.000000
ID1PLUS	99.000000	INFINITY	99.000000
ID2PLUS	99.000000	INFINITY	44.817382
ID3PLUS	99.000000	INFINITY	153.182610
ID4PLUS	99.000000	INFINITY	99.000000
ID5PLUS	99.000000	INFINITY	99.000000
ID6PLUS	99.000000	INFINITY	4244.000000
ID1MIN	75.000000	INFINITY	75.000000
ID2MIN	75.000000	INFINITY	129.182610
ID3MIN	75.000000	INFINITY	20.817382
ID4MIN	75.000000	INFINITY	75.000000
ID5MIN	75.000000	INFINITY	75.000000
ID6MIN	75.000000	INFINITY	249.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	157.000000	30.837440	157.000000
3	157.000000	19.911330	157.000000
4	196.000000	.753695	58.596054
5	157.000000	.753695	157.000000
6	156.000000	56.487682	156.000000
7	196.000000	9.167487	196.000000
8	48.000000	23.802280	48.000000
9	59.000000	15.368821	59.000000
10	49.000000	.581749	45.228134
11	47.000000	.581749	92.228134
12	60.000000	43.600760	60.000000
13	47.000000	7.076046	47.000000
14	45.000000	46.370370	45.000000
15	40.000000	29.940740	40.000000
16	102.000000	1.133333	88.111110
17	28.000000	1.133333	28.000000
18	31.000000	84.940734	31.000000
19	42.000000	13.785184	42.000000
20	95.000000	32.947370	95.000000
21	95.000000	21.273683	95.000000
22	107.000000	.805263	62.605260
23	78.000000	.805263	78.000000
24	62.000000	60.352630	62.000000
25	80.000000	9.794737	80.000000
26	74880.000000	INFINITY	6260.000000
27	74880.000000	INFINITY	4042.000000
28	74880.000000	11895.000000	153.000000
29	74880.000000	INFINITY	153.000000
30	74880.000000	INFINITY	11467.000000
31	74880.000000	INFINITY	1861.000000
32	.000000	47.000000	.000000
33	.000000	.000000	.000000