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Abstract: In this project we determined the minimum first-year cost of an active soil ventilation system. The results indicate the duct diameter and the fan brand that achieve the minimum cost. Some aspects of the problem were controlled in order to maintain a proper scope for the project. Initial results obtained from LINDO agreed with hand calculations. This indicates that appropriate models were constructed, and that these models were properly constrained. The sensitivity analysis indicated that fine variations can affect the basis.

MINIMIZATION OF THE FIRST-YEAR COST OF AN ACTIVE SOIL VENTILATION SYSTEM USING INTEGER PROGRAMMING TECHNIQUES

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EXECUTIVE SUMMARY

MINIMIZATION OF THE FIRST-YEAR COST OF AN ACTIVE SOIL VENTILATION SYSTEM USING INTEGER PROGRAMMING TECHNIQUES

The development of a prototype hybrid knowledge-based advisory software, RnX, which assists radon mitigators in the selection and design of indoor radon mitigation systems is one of the research projects in the Mechanical Engineering Department at Portland State University. Some mitigation methods require the installation of a simple ducting system and a fan to perform active soil ventilation. Of particular interest is the first-year cost, which includes among other things, purchase costs for the fan and ducting, and the yearly energy cost for constant operation of the fan. The fan selection module of RnX can analyze ducting systems with one, two, or three branches, and supports duct diameters of 3, 4, and 6 inches. The software contains a small database of four different brands of fans, and each brand has approximately five different size models. The purchase costs and the power consumptions of the different brands and models vary, as does the cost per foot for ducting. In a single branch system, for a given set of diagnostics measurements (the flow rate and pressure drop required at the suction points of the ventilation system), a suitable design may be achieved with either 3, 4, or 6 inch ducting, but the fan sizes will vary. One solution is to pay less for 3 inch ducting but more for the associated fan. Another is to pay a more for 6 inch ducting and get a smaller fan that doesn't need as much energy. At

present, these decisions are left to the user.

The purpose of this project is to apply mathematical programming to a real problem. We propose to create a model to determine the minimum first-year cost of an active soil ventilation system. More specifically, the result should indicate the duct diameter and the fan brand that will achieve the minimum cost. This project is interesting in that the recommendation from the fan selection module will be used in conjunction with LINDO to find the least cost using integer programming.

ABSTRACT

This project involves the determination of the minimum first-year cost of an active soil ventilation system. The results indicate the duct diameter and the fan brand that achieve the minimum cost.

Some aspects of the problem were controlled in order to maintain a proper scope for the project. Initial results obtained from LINDO agreed with hand calculations. This indicates that appropriate models were constructed, and that these models were properly constrained. The sensitivity analysis indicated that fine variations will affect the basis.

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BACKGROUND

The development of a prototype hybrid knowledge-based advisory software, RnX, which assists radon mitigators in the selection and design of indoor radon mitigation systems is one of the research projects in the Mechanical Engineering Department at Portland State University [1]. RnX is currently in the review stage.

The software was written in a modular fashion. It is composed of hierarchically activated modules that perform mitigation method selection, fan selection, and cost estimation. Some mitigation methods require the installation of a simple ducting system and a fan to perform active soil ventilation. Of particular interest is the first-year cost, which includes among other things, purchase costs for the fan and ducting, and the yearly energy cost for constant operation of the fan.

If an active soil ventilation method is recommended by the mitigation method selection module, the execution passes to the fan selection module. The user inputs the information related to the ducting configuration (duct diameter, length, and the number of fittings) for each branch of the system. The user also inputs the diagnostic measurements (these pertain to the flow rate and pressure drop required at the suction points). The software calculates the total flow rate and the total system friction loss (tsfl) for the given diagnostics and ducting information. The user is shown these values and the maximum velocity achieved in the duct. A selection of particular fan models that are capable of meeting the flow rate and tsfl requirements is also shown. The user may then select a brand for use in the cost estimation.

The fan selection module of RnX can analyze ducting systems with one, two, or three branches, and supports duct diameters of 3, 4, and 6 inches. The software contains a small database of four different brands of fans, and each brand has approximately five different size models. The purchase costs and the power consumptions of the different brands and models vary, and the cost per foot of 6 inch ducting is naturally higher than that of 3 inch ducting. Please refer to the appendix for these costs.

In a single branch system, for a given set of diagnostics, a suitable design may be achieved with 3, 4, or 6 inch ducting, but the fan sizes will vary. One solution is to pay less for 3 inch ducting but more for the associated fan, since it will be larger and require more energy. Another is to pay a more for 6 inch ducting and get a smaller fan that doesn't need as much energy. At present, these decisions are left to the user. The user is also required to guard against the possibility of a system being unsuitable. Unsuitable systems are those in which a duct has such a high velocity in it that a noise problem erupts, or if the main trunk of a two or three branch system has a diameter that is less than that of the minor branches.

RnX is one of the first of its kind. Other radon mitigation advisory software has been attempted, but did not reach the testing and revision phase. Also, no form of fan selection or cost estimation was present [2,3].

LITERATURE SEARCH

It has been established that RnX in itself is a prototype, but the minimization of cost is a frequent assignment of operations research, and numerous applications may be found. The literature search for this project focuses first on radon mitigation, and second on the emergence of knowledge-based systems.

Radon Mitigation

The issue of radon mitigation for residential housing has surfaced during the past decade. The problem is relatively new and the mitigation technology is equally new. Radon alone poses very little danger. It is a noble element, and does not react chemically. It can be breathed in and out, and the chance of lung tissue irradiation is small due to the length of the half-life (3.8 days). Radon decay products, which are not inert, are created and decay in less than an hour. These may react with dust particles which may become lodged in lung tissue. There have been incidences of lung cancer linked to radon by-products, but the data available are still limited and it is difficult to assess risk due to elevated levels in the home.

The initial efforts to combat indoor radon problems began with a publication from the Environmental Protection Agency in 1986 [4]. This publication was focused on public education about the risks of radon exposure. The publications that followed explained and illustrated many radon mitigation strategies [5-9]. By far the most successful strategies are termed active soil ventilation methods. Using these methods, the soil beneath the house is depressurized to counteract the movement of soil gas into the house.

Radon reduction is a relatively new field, and since the need for it varies regionally, not all contractors have sufficient expertise in dealing with a radon problem. One solution is to provide the mitigator with a knowledge-based advisory system capable of disseminating the knowledge in the present literature and assisting them in various aspects of radon mitigation work.

Knowledge-Based Systems [10]

The term and use of knowledge-based systems is becoming more familiar to the industry. Some problem-types that are particularly suited to the application of knowledge-based systems are diagnosis, design, data interpretation, planning/selection, configuration, and computer-aided learning. Knowledge-based systems are an aspect of artificial intelligence. There is much debate among experts as to what artificial intelligence means. In general, an artificial intelligence should be able to:

Use rules and heuristics to solve problems with a defined area
Cope with uncertainty and incomplete information
Receive input and communicate output in a natural language
Explain how conclusions have been reached
Grade the accuracy of conclusions

Knowledge-based systems are becoming more popular mostly to reduce the cost of expertise. Knowledge-based systems can be more attractive if:

People with expertise are in short supply

There are many factors involved in a decision

A poor decision will make a significant difference

No individual knows all about the subject

Competitors are consistently performing better

The knowledge base is fairly narrow

There is no need for background or common sense

It takes a significant amount of time to do manually

Outcomes can be evaluated

There are experts available in the subject

The knowledge base is fairly static

Although the above list is neither mutually exclusive nor exhaustive, some of the items are applicable to RnX.

PROBLEM DEFINITION

The purpose of this project is to apply mathematical programming to a real problem. We propose to create a model to determine the minimum first-year cost of an active soil ventilation system. More specifically, the result should indicate the duct diameter and the fan brand that will achieve the minimum cost. This project is interesting in that the recommendation from the fan selection module will be used in conjunction with LINDO to find the least cost using integer programming.

MODEL FORMULATION

As discussed earlier, RnX will support the design of ducting systems with one, two, or three branches. Multi-branch systems are permitted to have varying branch diameters as long as the configuration is acceptable (An unacceptable configuration would occur if the main branch was specified as having a smaller diameter than the lesser branches.). To maintain this project at an manageable scope, the multi-branch system models were restricted to have constant diameter. If the models were not constrained in this manner, there would be 56 combinations of possible solutions for a two-branch system, and 216 for a three-branch. The number of constraints would exponentiate. Although the two and three branch models will approach that of the single branch model, three separate models for the cost minimization were constructed. They are quite similar, the two and three-branch models simply having more variables and constraints, but it will be much easier to expand on the two- and three-branch models in a future project.

It is interesting to note that integer programming techniques greatly reduce the number of constraints. An attempt was made to construct a model without taking advantage of integer programming. The model worked, but there were 84 constraints. This was due to the constraints that were needed to fool LINDO into assigning the decision variables a value of zero or one. A subsequent attempt at model formulation took advantage of integer programming, and there were only 9 constraints.

The variables for a general model and the nature of the constraints are defined and shown below.

L = total length of ducting used (feet)

F = total number of fittings used

Xnn = total cost (purchase and operation) for fan nn

The above variables represent given information. They are not decision variables, but are factors of the objective function coefficients. The variables that follow are the decision variables. All of these are to be integer variables, that is, they can assume values of zero or one. Zero would indicate a condition of "false" and one would indicate a condition of "true" for the associated definition.

L1 = 3" diameter ducting to be used

L2 = 4" diameter ducting to be used

L3 = 6" diameter ducting to be used

F1 = 3" fittings used

F2 = 4" fittings used

F3 = 6" fittings used

FAN11 = fan brand 1 for 3" system used

FAN12 = fan brand 2 for 3" system used

FAN13 = fan brand 3 for 3" system used

FAN14 = fan brand 4 for 3" system used

FAN21 = fan brand 1 for 4" system used

FAN22 = fan brand 2 for 4" system used

FAN23 = fan brand 3 for 4" system used

FAN24 = fan brand 4 for 4" system used

FAN31 = fan brand 1 for 6" system used

FAN32 = fan brand 2 for 6" system used

FAN33 = fan brand 3 for 6" system used

FAN34 = fan brand 4 for 6" system used

The constraints that follow restrict the diameter and configuration of the duct and fittings, and assure that the correct fan size is specified for the selected duct diameter.

L1, L2, and L3 can be equal to zero or one, and only one of the three may equal one. This is because the diameter of the duct must be constant. This is also true for F1, F2, and F3.

$$L1 + L2 + L3 = 1$$

$$F1 + F2 + F3 = 1$$

Also, only one fan may be selected.

FAN11 + FAN12 + FAN13 + FAN14

+ FAN21 + FAN22 + FAN23 + FAN24

+ FAN31 + FAN32 + FAN33 + FAN34 = 1

Since 3" diameter ducting cannot be used with 4" diameter fittings or with a fan that is specified for a 6" diameter system, the following constraints also exist:

L1 - F1 = 0

L2 - F2 = 0

L3 - F3 = 0

L1 - FAN11 - FAN12 - FAN13 - FAN14 = 0

L2 - FAN21 - FAN22 - FAN23 - FAN24 = 0

L3 - FAN31 - FAN32 - FAN33 - FAN34 = 0

The decision variables are integer variables. The value of the decision variables may be zero or one. It effectively denotes the presence or absence of a particular ducting or fitting diameter, and the corresponding fan. The coefficients of the decision variables represent the costs. The fan cost coefficients take into account both the purchase price and the first year of energy cost. The following generalized objective function may be formulated:

Minimize:

$$L\,[(0.5)L1 + (1.0)L2 + (1.5)L3] + F\,[(3.0)F1 + (3.5)F2 + (4.0)F3]$$
$$+ X11\ FAN11 + X12\ FAN12 + X13\ FAN13 + X14\ FAN14$$
$$+ X21\ FAN21 + X22\ FAN22 + X23\ FAN23 + X24\ FAN24$$
$$+ X31\ FAN31 + X32\ FAN32 + X33\ FAN33 + X34\ FAN34$$

The coefficients of L1, L2,and L3 indicate the cost per foot (in dollars) for the corresponding diameter of ducting, and the coefficients of F1, F2, and F3 are the cost per fitting for each size of fitting.

The general model shown on the previous pages was used to construct the models for one-, two-, and three-branch systems. Please refer to the appendix of this paper to see these models and their solutions for given inputs.

SOLUTION

Before actually finding a solution to a given problem, the objective function coefficients needed to be determined using the fan selection module of RnX. Three RnX runs were performed, each one specifying a 3", 4", or 6" diameter for the duct. The ducting configuration for each run was defined as composed of 40 feet of ducting with two 90 degree elbow fittings. Each run was given the same diagnostic measurement input. At the end of each RnX run, four brands of appropriately sized fans are presented along with their prices and wattages. Please refer to the appendix for a complete list of the prices and wattages and total costs. With the information obtained from RnX, the following objective function for a one-branch system is determined:

MIN 20 L1 + 40 L2 + 60 L3 + 6 F1 + 7 F2 + 8 F3 + 222 FAN11 +
182.25 FAN12 + 195 FAN13 + 172.5 FAN14 + 137.5 FAN21 +
126.5 FAN22 + 123 FAN23 + 139.75 FAN24 + 137.5 FAN31 +
106.5 FAN32 + 103.3 FAN33 + 139.75 FAN34

This function was entered into LINDO with the following constraints:

SUBJECT TO

2) L1 + L2 + L3 = 1

3) F1 + F2 + F3 = 1

4) FAN11 + FAN12 + FAN13 + FAN14 + FAN21 + FAN22 + FAN23 + FAN24 + FAN31 + FAN32 + FAN33 + FAN34 = 1

5) L1 - F1 = 0

6) L2 - F2 = 0

7) L3 - F3 = 0

8) L1 - FAN11 - FAN12 - FAN13 - FAN14 = 0

9) L2 - FAN21 - FAN22 - FAN23 - FAN24 = 0

10) L3 - FAN31 - FAN32 - FAN33 - FAN34 = 0

INTE 18

The INTE 18 command indicates that the first 18 variables in the objective function are to be 0/1 variables. Initially, the command INTEGER N was typed in, where N represented the 0/1 variable, but LINDO consolidated all of these into the INTE 18 command [11].

The solution to this particular problem determined that variables L2, F2, and FAN23 were equal to one. All other variables were equal to zero. These values are interpreted as meaning to use 4" diameter ducting and fittings (variable L2 and F2), and fan brand 3 (variable FAN23). The minimum value of the objective function was determined to be 170.00. This means that the minimum cost is $170.00. The LINDO output for this problem is shown on the following page, as it is not very lengthy. The inputs, objective functions, and solutions for two and three branch models are included in the appendix.

LP OPTIMUM FOUND AT STEP 4
OBJECTIVE VALUE = 170.000000
ENUMERATION COMPLETE. BRANCHES = 0 PIVOTS = 4

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

1) 170.00000

VARIABLE	VALUE	REDUCED COST
L1	.000000	20.000000
L2	1.000000	40.000000
L3	.000000	60.000000
F1	.000000	6.000000
F2	1.000000	7.000000
F3	.000000	8.000000
FAN11	.000000	222.000000
FAN12	.000000	182.250000
FAN13	.000000	195.000000
FAN14	.000000	172.500000
FAN21	.000000	137.500000
FAN22	.000000	126.500000
FAN23	1.000000	123.000000
FAN24	.000000	139.750000
FAN31	.000000	137.500000
FAN32	.000000	106.500000
FAN33	.000000	103.300000
FAN34	.000000	139.750000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000

NO. ITERATIONS = 4

BRANCHES = 0 DETERM. = 1.000E 0

SENSITIVITY ANALYSIS

The typical approach to sensitivity analysis in linear programming is to determine the ranges on the right hand sides and the objective function coefficients for which the current basis will remain optimal [12]. Instead, incremental changes in costs were introduced, and the effect upon the solution was determined. There are several approaches to the cost variation. The purchase cost of the ducting, fittings, and fan may be altered, as there are real differences due to regional and quality variations. Also the cost of energy may be changed. Changes in cost per kiloWatt hour are more subtle than the changes in purchase costs. A change in energy cost would affect all of the coefficients that represent fan costs. This aspect of the sensitivity analysis will be featured.

The initial computer solutions were determined with a cost of $0.04 per kiloWatt hour. The single branch model was used to perform the sensitivity analysis. At first, the energy cost was simply doubled to $0.08 per kiloWatt hour. This immediately affected the basis. The solution changed to recommend that 6" diameter ducting be used instead of 4". However, the fan brand stayed the same. Please refer to the appendix for the complete output of the sensitivity analysis.

Next, it was determined at what point the basis had actually changed. This involved changing the energy cost by increments of $0.01 and determining the new basis. At a cost of $0.05, the basis remained the same, but at a cost of $0.06, it had changed.

RESULTS

The results obtained from LINDO for each of the models created agreed with hand calculations. The results have shown that it is not always less costly to have a large diameter duct and a smaller fan. However, the results are for the first-year cost only. A different time period would probably result in a different basis.

The results for the two- and three-branch models were correct given that this problem was constrained to a constant diameter. Multi-branch systems rarely have constant diameter unless the flow rates are very low. This is because the combined flow in the main trunk causes additional friction.

The results of the sensitivity analysis, while limited, show that even subtle changes may affect the outcome of a problem. In this region of the country, the cost of electricity is fairly reasonable. However, on the east coast, where radon mitigation is a much bigger business, the cost of energy is somewhat higher. This was why the energy cost was selected for sensitivity analysis. Also, even though there are regional and quality variations for material costs, mitigation contractors are in a position to obtain items at wholesale.

FUTURE EXTENSIONS

In the future, it would be interesting to try a non-linear programming technique for this problem. The duct loss calculations are non-linear, and are based on the diagnostic measurements. That is why RnX was used for the generation of the objective function coefficients. If a non-linear technique is employed, the diagnostic measurements would become factors of the objective function coefficients, and RnX would not be required to generate them.

The two-and three-branch models may be expanded on to remove the constraint of constant diameter. Although this will necessitate the inclusion of many more constraints to eliminate infeasible configurations, it is much more realistic.

Other possible future work would involve somehow having RnX access LINDO through some sort of batch file, and feeding it the necessary inputs for a given problem. The hard part would be getting the information back from LINDO in a form that RnX would understand and could present to the user. Another difficulty with this potential work is that RnX was developed as part of a federal grant, and is considered to be public domain, but LINDO is not.

CONCLUSION

This project has effectively demonstrated the application of integer programming to a cost minimization problem. The results indicate that appropriate models were constructed, and that these models were suitably constrained. It was necessary to approach the sensitivity analysis in a restricted manner due to the nature of integer programming, but even those results revealed that it was quite relevant to the problem, and it simulated the least controllable (and genuinely potential) occurrence.

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APPENDIX

Two-branch model and solution

MIN 5 L11 + 10 L12 + 15 L13 + 5 L21 + 10 L22 + 15 L23 + 10 L31 + 20 L32 + 30 L33 + 3 F11 + 3.5 F12 + 4 F13 + 3 F21 + 3.5 F22 + 4 F23 + 3 F31 + 3.5 F32 + 4 F33 + 253.8 FAN11 + 182.3 FAN12 + 135.04 FAN13 + 202.56 FAN14 + 137.52 FAN21 + 126.54 FAN22 + 123.03 FAN23 + 139.78 FAN24 + 137.52 FAN31 + 106.52 FAN32 + 103.32 FAN33 + 139.78 FAN34

SUBJECT TO

2) L11 + L12 + L13 = 1

3) L21 + L22 + L23 = 1

4) L31 + L32 + L33 = 1

5) F11 + F12 + F13 = 1

6) F21 + F22 + F23 = 1

7) F31 + F32 + F33 = 1

8) FAN11 + FAN12 + FAN13 + FAN14 + FAN21 + FAN22 + FAN23 + FAN24 + FAN31 + FAN32 + FAN33 + FAN34 = 1

9) L11 - FAN11 - FAN12 - FAN13 - FAN14 = 0

10) L12 - FAN21 - FAN22 - FAN23 - FAN24 = 0

11) L13 - FAN31 - FAN32 - FAN33 - FAN34 = 0

12) L11 - L21 = 0

13) L11 - L31 = 0

14) L12 - L22 = 0

15) L12 - L32 = 0

```
16)  L13 - L23 = 0

17)  L13 - L33 = 0

18)  L11 - F11 = 0

19)  L11 - F21 = 0

20)  L11 - F31 = 0

21)  L12 - F12 = 0

22)  L12 - F22 = 0

23)  L12 - F32 = 0

24)  L13 - F13 = 0

25)  L13 - F23 = 0

26)  L13 - F33 = 0

END

INTE    30

LP OPTIMUM FOUND AT STEP 1
OBJECTIVE VALUE = 164.040000
ENUMERATION COMPLETE.  BRANCHES = 0   PIVOTS = 1

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      164.04000

  VARIABLE        VALUE          REDUCED COST

       L11         1.000000          5.000000
       L12          .000000         10.000000
       L13          .000000         15.000000
       L21         1.000000          5.000000
       L22          .000000         10.000000
       L23          .000000         15.000000
       L31         1.000000         10.000000
       L32          .000000         20.000000
       L33          .000000         30.000000
       F11         1.000000          3.000000
       F12          .000000          3.500000
       F13          .000000          4.000000
```

F21	1.000000	3.000000
F22	.000000	3.500000
F23	.000000	4.000000
F31	1.000000	3.000000
F32	.000000	3.500000
F33	.000000	4.000000
FAN11	.000000	253.800000
FAN12	.000000	182.300000
FAN13	1.000000	135.040000
FAN14	.000000	202.560000
FAN21	.000000	137.520000
FAN22	.000000	126.540000
FAN23	.000000	123.030000
FAN24	.000000	139.780000
FAN31	.000000	137.520000
FAN32	.000000	106.520000
FAN33	.000000	103.320000
FAN34	.000000	139.780000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000

NO. ITERATIONS = 1

BRANCHES = 0 DETERM. = 1.000E 0

Three-branch model and solution

MIN 5 L11 + 10 L12 + 15 L13 + 5 L21 + 10 L22 + 15 L23 + 5 L31 + 10 L32 + 15 L33 + 2.5 L41 + 5 L42 + 7.5 L43 + 10 L51 + 20 L52 + 30 L53 + 3 F11 + 3.5 F12 + 4 F13 + 3 F21 + 3.5 F22 + 4 F23 + 3 F31 + 3.5 F32 + 4 F33 + 3 F41 + 3.5 F42 + 4 F43 + 3 F51 + 3.5 F52 + 4 F53 + 253.8 FAN11 + 182.3 FAN12 + 296.1 FAN13 + 200.1 FAN14 + 137.52 FAN21 + 126.54 FAN22 + 123.03 FAN23 + 139.78 FAN24 + 137.52 FAN31 + 106.52 FAN32 + 103.32 FAN33 + 139.78 FAN34

SUBJECT TO

2) L11 + L12 + L13 = 1

3) L21 + L22 + L23 = 1

4) L31 + L32 + L33 = 1

5) L41 + L42 + L43 = 1

6) L51 + L52 + L53 = 1

7) F11 + F12 + F13 = 1

8) F21 + F22 + F23 = 1

9) F31 + F32 + F33 = 1

10) F41 + F42 + F43 = 1

11) F51 + F52 + F53 = 1

12) FAN11 + FAN12 + FAN13 + FAN14 + FAN21 + FAN22 + FAN23 + FAN24 + FAN31 + FAN32 + FAN33 + FAN34 = 1

13) L11 - FAN11 - FAN12 - FAN13 - FAN14 = 0

14) L12 - FAN21 - FAN22 - FAN23 - FAN24 = 0

15) L13 - FAN31 - FAN32 - FAN33 - FAN34 = 0

16) L11 - L21 = 0

17) L11 - L31 = 0

18) L11 - L41 = 0

19) L11 - L51 = 0

20) L12 - L22 = 0

21) L12 - L32 = 0

22) L12 - L42 = 0

23) L12 - L52 = 0

24) L13 - L23 = 0

25) L13 - L33 = 0

26) L13 - L43 = 0

27) L13 - L53 = 0

28) L11 - F11 = 0

29) L11 - F21 = 0

30) L11 - F31 = 0

31) L11 - F41 = 0

32) L11 - F51 = 0

33) L12 - F12 = 0

34) L12 - F22 = 0

35) L12 - F32 = 0

36) L12 - F42 = 0

37) L12 - F52 = 0

38) L13 - F13 = 0

39) L13 - F23 = 0

40) L13 - F33 = 0

41) L13 - F43 = 0

42) L13 - F53 = 0

END

INTE 42

LP OPTIMUM FOUND AT STEP 36
OBJECTIVE VALUE = 195.530000
ENUMERATION COMPLETE. BRANCHES = 0 PIVOTS = 36

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

1) 195.53000

VARIABLE	VALUE	REDUCED COST
L11	.000000	5.000000
L12	1.000000	10.000000
L13	.000000	15.000000
L21	.000000	5.000000
L22	1.000000	10.000000
L23	.000000	15.000000
L31	.000000	5.000000
L32	1.000000	10.000000
L33	.000000	15.000000
L41	.000000	2.500000
L42	1.000000	5.000000
L43	.000000	7.500000
L51	.000000	10.000000
L52	1.000000	20.000000
L53	.000000	30.000000
F11	.000000	3.000000
F12	1.000000	3.500000
F13	.000000	4.000000
F21	.000000	3.000000
F22	1.000000	3.500000
F23	.000000	4.000000
F31	.000000	3.000000
F32	1.000000	3.500000
F33	.000000	4.000000
F41	.000000	3.000000
F42	1.000000	3.500000
F43	.000000	4.000000

F51	.000000	3.000000
F52	1.000000	3.500000
F53	.000000	4.000000
FAN11	.000000	253.800000
FAN12	.000000	182.300000
FAN13	.000000	296.100000
FAN14	.000000	200.100000
FAN21	.000000	137.520000
FAN22	.000000	126.540000
FAN23	1.000000	123.030000
FAN24	.000000	139.780000
FAN31	.000000	137.520000
FAN32	.000000	106.520000
FAN33	.000000	103.320000
FAN34	.000000	139.780000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000
27)	.000000	.000000
28)	.000000	.000000
29)	.000000	.000000
30)	.000000	.000000
31)	.000000	.000000
32)	.000000	.000000
33)	.000000	.000000
34)	.000000	.000000

35)	.000000	.000000
36)	.000000	.000000
37)	.000000	.000000
38)	.000000	.000000
39)	.000000	.000000
40)	.000000	.000000
41)	.000000	.000000
42)	.000000	.000000

NO. ITERATIONS = 36

BRANCHES = 0   DETERM. = 1.000E 0

Sensitivity Analysis for single branch, $0.08 per kWh

```
MIN  20 L1 + 40 L2 + 60 L3 + 6 F1 + 7 F2 + 8 F3 + 243 FAN11 + 204.2
     FAN12 + 212.56 FAN13 + 198.8 FAN14 + 146.3 FAN21 + 142.3 FAN22
     + 135 FAN23 + 154.68 FAN24 + 146.3 FAN31 + 115.28 FAN32 +
     111.73 FAN33 + 154.68 FAN34

SUBJECT TO

2)   L1 + L2 + L3 = 1
3)   F1 + F2 + F3 = 1
4)   FAN11 + FAN12 + FAN13 + FAN14 + FAN21 + FAN22 + FAN23 + FAN24
     + FAN31 + FAN32 + FAN33 + FAN34 = 1
5)   L1 - F1 = 0
6)   L2 - F2 = 0
7)   L3 - F3 = 0
8)   L1 - FAN11 - FAN12 - FAN13 - FAN14 = 0
9)   L2 - FAN21 - FAN22 - FAN23 - FAN24 = 0
10)  L3 - FAN31 - FAN32 - FAN33 - FAN34 = 0
END
INTE    18

LP OPTIMUM FOUND AT STEP 11
OBJECTIVE VALUE = 179.730000
ENUMERATION COMPLETE.  BRANCHES = 0   PIVOTS = 11

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      179.73000
```

VARIABLE	VALUE	REDUCED COST
L1	.000000	20.000000
L2	.000000	40.000000
L3	1.000000	60.000000
F1	.000000	6.000000
F2	.000000	7.000000
F3	1.000000	8.000000
FAN11	.000000	243.000000
FAN12	.000000	204.200000
FAN13	.000000	212.560000
FAN14	.000000	198.800000
FAN21	.000000	146.300000
FAN22	.000000	142.300000
FAN23	.000000	135.000000
FAN24	.000000	154.680000
FAN31	.000000	146.300000
FAN32	.000000	115.280000
FAN33	1.000000	111.730000
FAN34	.000000	154.680000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000

NO. ITERATIONS = 11
BRANCHES = 0 DETERM. = 1.000E 0

Sensitivity Analysis $0.05 per kWh

```
MIN  20 L1 + 40 L2 + 60 L3 + 6 F1 + 7 F2 + 8 F3 + 227.25 FAN11 +
     187.74 FAN12 + 199.39 FAN13 + 179.08 FAN14 + 139.7 FAN21 +
     130.45 FAN22 + 126 FAN23 + 143.49 FAN24 + 139.7 FAN31 + 108.7
     FAN32 + 105.41 FAN33 + 143.49 FAN34

SUBJECT TO

2)   L1 + L2 + L3 = 1
3)   F1 + F2 + F3 = 1
4)   FAN11 + FAN12 + FAN13 + FAN14 + FAN21 + FAN22 + FAN23 + FAN24
     + FAN31 + FAN32 + FAN33 + FAN34 = 1
5)   L1 - F1 = 0
6)   L2 - F2 = 0
7)   L3 - F3 = 0
8)   L1 - FAN11 - FAN12 - FAN13 - FAN14 = 0
9)   L2 - FAN21 - FAN22 - FAN23 - FAN24 = 0
10)  L3 - FAN31 - FAN32 - FAN33 - FAN34 = 0
END
INTE   18

LP OPTIMUM FOUND AT STEP 4
OBJECTIVE VALUE = 173.000000
ENUMERATION COMPLETE.  BRANCHES = 0   PIVOTS = 4

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      173.00000
```

VARIABLE	VALUE	REDUCED COST
L1	.000000	20.000000
L2	1.000000	40.000000
L3	.000000	60.000000
F1	.000000	6.000000
F2	1.000000	7.000000
F3	.000000	8.000000
FAN11	.000000	227.250000
FAN12	.000000	187.740000
FAN13	.000000	199.390000
FAN14	.000000	179.080000
FAN21	.000000	139.700000
FAN22	.000000	130.450000
FAN23	1.000000	126.000000
FAN24	.000000	143.490000
FAN31	.000000	139.700000
FAN32	.000000	108.700000
FAN33	.000000	105.410000
FAN34	.000000	143.490000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000

NO. ITERATIONS = 4
BRANCHES = 0 DETERM. = 1.000E 0

Sensitivity Analysis $0.06 per kWh

```
MIN  20 L1 + 40 L2 + 60 L3 + 6 F1 + 7 F2 + 8 F3 + 232.5 FAN11 +
     193.23 FAN12 + 203.78 FAN13 + 185.65 FAN14 + 141.9 FAN21 +
     134.4 FAN22 + 129 FAN23 + 147.22 FAN24 + 141.9 FAN31 + 110.89
     FAN32 + 107.52 FAN33 + 147.22 FAN34

SUBJECT TO

2)   L1 + L2 + L3 = 1
3)   F1 + F2 + F3 = 1
4)   FAN11 + FAN12 + FAN13 + FAN14 + FAN21 + FAN22 + FAN23 + FAN24
     + FAN31 + FAN32 + FAN33 + FAN34 = 1
5)   L1 - F1 = 0
6)   L2 - F2 = 0
7)   L3 - F3 = 0
8)   L1 - FAN11 - FAN12 - FAN13 - FAN14 = 0
9)   L2 - FAN21 - FAN22 - FAN23 - FAN24 = 0
10)  L3 - FAN31 - FAN32 - FAN33 - FAN34 = 0
END
INTE    18

LP OPTIMUM FOUND AT STEP 4
OBJECTIVE VALUE = 175.520000
ENUMERATION COMPLETE.  BRANCHES = 0   PIVOTS = 4

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      175.52000
```

VARIABLE	VALUE	REDUCED COST
L1	.000000	20.000000
L2	.000000	40.000000
L3	1.000000	60.000000
F1	.000000	6.000000
F2	.000000	7.000000
F3	1.000000	8.000000
FAN11	.000000	232.500000
FAN12	.000000	193.230000
FAN13	.000000	203.780000
FAN14	.000000	185.650000
FAN21	.000000	141.900000
FAN22	.000000	134.400000
FAN23	.000000	129.000000
FAN24	.000000	147.220000
FAN31	.000000	141.900000
FAN32	.000000	110.890000
FAN33	1.000000	107.520000
FAN34	.000000	147.220000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000

NO. ITERATIONS = 4
BRANCHES = 0 DETERM. = 1.000E 0

Prices

3" ducting = $0.50 per foot

4" ducting = $1.00 per foot

6" ducting = $1.50 per foot

3" fitting = $3.00 each

4" fitting = $3.50 each

6" fitting = $4.00 each

Fan brand 1

model 1	$120.00	50W
model 2	$150.00	80W
model 3	$180.00	120W
model 4	$210.00	125W
model 5	$240.00	230W
model 6	$270.00	240W

Fan brand 2

model 1	$89.00	50W
model 2	$93.00	50W
model 3	$95.00	90W
model 4	$138.50	125W

Fan brand 3

model 1	$86.50	48W
model 2	$98.50	70W
model 3	$102.00	90W
model 4	$160.00	100W
model 5	$215.50	230W

Fan brand 4

model 1	$90.00	45W
model 2	$100.00	45W
model 3	$110.00	85W
model 4	$120.00	150W
model 5	$130.00	200W
model 6	$140.00	200W2