ETM

ENGINEERING & TECHNOLOGY MANAGEMENT

Title: A Linear Programming Model and Analysis of Incoming Quality Control at Fujitsu Computer Products of America, Hillsboro, Oregon

Course:

Year: 1992

Author(s): D. Lee, M. Lazur and J. Teeter

Report No: P92016

Abstract: At Fujitsu Computer Products of America (FCPA), the quality of all production material is assured by Incoming Quality Control (IQC) before it is used in manufacturing. Material inspection is a critical part of this quality control process. The inspectors who conduct these inspections are typically cross trained to work with more than one commodity. The objective of this project is to optimize the use of the inspectors' time such that the weekly production demand will be met with the minimal weekly labor costs.

A LINEAR PROGRAMMING MODEL AND ANALYSIS OF INCOMING QUALITY CONTROL AT FUJITSU COMPUTER PRODUCTS OF AMERICA, HILLSBORO, OREGON

Douglas Lee, Michael Lazur and James R. Teeter

EMP-P9216

A Linear Programming Model and Analysis of Incoming Quality Control at Fujitsu Computer Products of America, Hillsboro, Oregon

Prepared by

Douglas Lee
Michael Lazur
James R. Teeter

for

Dr. Richard Deckro
Professor of Engineering Management 540
Department of Engineering Management
Portland State University

June 1, 1992

ABSTRACT

At Fujitsu Computer Products of America (FCPA), the quality of all production material is assured by Incoming Quality Control (IQC) before it is used in manufacturing. Material inspection is a critical part of this. The inspectors who conduct the inspections are typically cross trained to work with more than one commodity. The objective of this project is to optimize the use of the inspectors' time such that the weekly production demand will be met with the minimal weekly labor costs.

INTRODUCTION

Fujitsu Computer Products of America (FCPA) in Hillsboro, Oregon manufactures computer peripheral products for mini and mainframe computers. The product mix includes disk drives, tape drives, and drive controller systems. These products are designed to have high performance and high reliability. To satisfy these goals, the parts and materials used in these products need to meet stringent specifications.

Incoming Quality Control (IQC) verifies that all materials comply with their respective specifications. Typically, this is done through inspection of the parts. An inspection plan is created for each new part. This plan details the important characteristics of the part and how it will be tested. To inspect 100% of all incoming parts would be an overwhelming task, therefore parts are inspected on a sample basis. The appropriate sample size for the week is determined by taking a percentage of the total weekly forecast for the part. The percentage is generally based on Military Standard 105D[1], but is usually modified due to empirical data gained over the history of the part.

Currently, there are eleven inspectors in IQC. Each is trained to inspect one or more commodity. Each inspector earns a different salary and some inspect the same commodities at different rates.

The objective of this project is to minimize inspection labor costs for a given weekly forecast. This is subject to the constraints of allowing no overtime for the inspectors and meeting the weekly commodity demand as derived from the forecast. Linear programming is employed as the means of achieving this objective. A model is developed and tested using LINDO[2], a linear programming application software. The results are presented, analyzed, and discussed. Other approaches for solutions are investigated and their results presented. Finally, conclusions are summarized and future activities are identified.

MODEL

The linear programming model for this problem is patterned after a typical minimization problem. A theoretical framework is presented first so that the reader can understand the relationships and restrictions involved. The objective function is defined to minimize the total weekly labor costs due to inspection. This is dependent on the value of the coefficients and variables used in the model. Definitions for these are shown in Table 1. Weekly inspection demand is a requirement that must be met. The sample size is decided and assignment of inspectors to commodities is determined. Table 2 summarizes each inspector's rate of inspection for each commodity as well as the weekly inspection demand constraints. Inspectors salaries, hours spent per commodity, and labor constraints are shown in Table 3.

MODEL VARIABLE DEFINITION

NAME	TYPE	AMOUNT	DEFINITION
C_j	Objective Function Coefficient	11	Inspector I_j salary in dollars/hour
X_n	Decision Variable	30	Inspector I_j hours per Commodity M_i
a_{ij}	Technological Coefficient	30	# of Commodity M_i inspected by Inspector I_j per hour
T_i	Related Right Hand Side Constraint Variable	12	Total Commodity parts to be inspected per week
P_i	Related Right Hand Side Constraint Variable	12	Sampling Percentage for Commodity M_i
B_r	Right Hand Side Constraint Variable	12	Inspection sample size B_r for Commodity M_i per week
B_c	Right Hand Side Constraint Variable	11	Labor hours available for Inspector I_j per week
M_i	Row Index	12	Commodity identifier
I_j	Column Index	11	Inspector identifier

TABLE 1

INSPECTION RATES and FORECAST CONSTRAINTS

COMMODITY		INSPECTION RATE FOR INSPECTOR I_j, IN PARTS PER HOUR											WEEKLY FORECAST		
M_i	**DESCRIPTION**	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	**TOTAL PARTS T_i**	**SAMPLE % P_i**	**SAMPLE SIZE B_r**
M_1	ACTIVE/ICs	–	–	–	–	–	$a_{1,6}$	$a_{1,7}$	–	–	–	–	T_1	P_1	B_1
M_2	CABLES	–	$a_{2,2}$	–	–	–	–	–	$a_{2,8}$	–	–	–	T_2	P_2	B_2
M_3	ELECTRO/MECH	–	–	–	–	–	–	–	–	$a_{3,9}$	–	–	T_3	P_3	B_3
M_4	LOCKERS	–	$a_{4,2}$	$a_{4,3}$	–	$a_{4,5}$	–	–	–	–	–	–	T_4	P_4	B_4
M_5	PASSIVE COMP	–	–	–	$a_{5,4}$	–	–	–	–	–	$a_{5,10}$	–	T_5	P_5	B_5
M_6	PLASTICS	–	–	–	–	$a_{6,5}$	–	–	–	–	–	$a_{6,11}$	T_6	P_6	B_6
M_7	POWER SUPPLIES	–	–	–	–	–	–	–	–	$a_{7,9}$	$a_{7,10}$	–	T_7	P_7	B_7
M_8	PRINTED CKT BD	$a_{8,1}$	–	–	–	–	–	–	–	–	–	–	T_8	P_8	B_8
M_9	SHEET METAL	–	–	$a_{9,3}$	–	$a_{9,5}$	–	–	–	–	–	–	T_9	P_9	B_9
M_{10}	PCB ASSEMBLY	$a_{10,1}$	–	–	–	–	$a_{10,6}$	$a_{10,7}$	–	–	–	–	T_{10}	P_{10}	B_{10}
M_{11}	FASTENERS	–	$a_{11,2}$	$a_{11,3}$	–	$a_{11,5}$	–	–	$a_{11,8}$	–	–	$a_{11,11}$	T_{11}	P_{11}	B_{11}
M_{12}	LABELS	–	$a_{12,2}$	$a_{12,3}$	–	$a_{12,5}$	–	–	$a_{12,8}$	–	–	$a_{12,11}$	T_{12}	P_{12}	B_{12}

TABLE 2

LABOR COST and CONSTRAINTS

COMMODITY		INSPECTOR HOURS X_n SPENT PER COMMODITY										
M_i	**DESCRIPTION**	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}
M_1	ACTIVE COMPONENTS/ICs	-	-	-	-	-	X_{17}	X_{19}	-	-	-	-
M_2	CABLES	-	X_3	-	-	-	-	-	X_{21}	-	-	-
M_3	ELECTRO/MECHANICAL	-	-	-	-	-	-	-	-	X_{24}	-	-
M_4	LOCKERS	-	X_4	X_7	-	X_{12}	-	-	-	-	-	-
M_5	PASSIVE COMPONENTS	-	-	-	X_{11}	-	-	-	-	-	X_{26}	-
M_6	PLASTICS	-	-	-	-	X_{13}	-	-	-	-	-	X_{28}
M_7	POWER SUPPLIES	-	-	-	-	-	-	-	-	X_{25}	X_{27}	-
M_8	PRINTED CIRCUIT BOARD	X_1	-	-	-	-	-	-	-	-	-	-
M_9	SHEET METAL	-	-	X_8	-	X_{14}	-	-	-	-	-	-
M_{10}	PRINTED CIRCUIT BOARD ASSEMBLY	X_2	-	-	-	-	X_{18}	X_{20}	-	-	-	-
M_{11}	FASTENERS	-	X_5	X_9	-	X_{15}	-	-	X_{22}	-	-	X_{29}
M_{12}	LABELS	-	X_6	X_{10}	-	X_{16}	-	-	X_{23}	-	-	X_{30}
AVAILABLE LABOR HOURS B_c		B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}	B_{21}	B_{22}	B_{23}
INSPECTOR SALARY C_j IN $/HOUR		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}

TABLE 3

The general relationships of the model are shown in the following equations.

1)	TOTAL WEEKLY LABOR COST	WEEKLY LABOR COST DUE TO
	$= C_1 * X_1 + C_1 * X_2 +$	INSPECTOR 1
	$C_2 * X_3 + C_2 * X_4 + C_2 * X_5 + C_2 * X_6 +$	INSPECTOR 2
	$C_3 * X_7 + C_3 * X_8 + C_3 * X_9 + C_3 * X_{10} +$	INSPECTOR 3
	$C_4 * X_{11} +$	INSPECTOR 4
	$C_5 * X_{12} + C_5 * X_{13} + C_5 * X_{14} + C_5 * X_{15} +$	
	$C_5 * X_{16} +$	INSPECTOR 5
	$C_6 * X_{17} + C_6 * X_{18} +$	INSPECTOR 6
	$C_7 * X_{19} + C_7 * X_{20} +$	INSPECTOR 7
	$C_8 * X_{21} + C_8 * X_{22} + C_8 * X_{23} +$	INSPECTOR 8
	$C_9 * X_{24} + C_9 * X_{25} +$	INSPECTOR 9
	$C_{10} * X_{26} + C_{10} * X_{27} +$	INSPECTOR 10
	$C_{11} * X_{28} + C_{11} * X_{29} + C_{11} * X_{30}$	INSPECTOR 11
	SUBJECT TO	WEEKLY FORECAST CONSTRAINTS FOR
2)	$a_{1,6} * X_{17} + a_{1,7} * X_{19} \geq B_1$	ACTIVE COMP/IC
3)	$a_{2,2} * X_3 + a_{2,1} * X_{21} \geq B_2$	CABLES
4)	$a_{3,9} * X_{24} \geq B_3$	ELECTRO/MECH
5)	$a_{4,2} * X_4 + a_{4,3} * X_7 + a_{4,5} * X_{12} \geq B_4$	LOCKERS
6)	$a_{5,4} * X_{11} + a_{5,10} * X_{26} \geq B_5$	PASSIVE DEVICES
7)	$a_{6,5} * X_{13} + a_{6,11} * X_{28} \geq B_6$	PLASTICS
8)	$a_{7,9} * X_{25} + a_{7,10} * X_{27} \geq B_7$	POWER SUPPLIES
9)	$a_{8,1} * X_1 \geq B_8$	PCBS
10)	$a_{9,3} * X_8 + a_{9,5} * X_{14} \geq B_9$	SHEET METAL
11)	$a_{10,1} * X_2 + a_{10,6} * X_{18} + a_{10,7} * X_{20} \geq B_{10}$	PCBAS
12)	$a_{11,2} * X_5 + a_{11,3} * X_9 + a_{11,5} * X_{15} +$	
	$a_{11,8} * X_{22} + a_{11,11} * X_{29} \geq B_{11}$	FASTENERS
13)	$a_{12,2} * X_6 + a_{12,3} * X_{10} + a_{12,5} * X_{16} +$	
	$a_{12,8} * X_{23} + a_{12,11} * X_{30} \geq B_{12}$	LABELS
	AND SUBJECT TO	WEEKLY LABOR CONSTRAINTS FOR
14)	$X_1 + X_2 \leq B_{13}$	INSPECTOR 1
15)	$X_3 + X_4 + X_5 + X_6 \leq B_{14}$	INSPECTOR 2
16)	$X_7 + X_8 + X_9 + X_{10} \leq B_{15}$	INSPECTOR 3
17)	$X_{11} \leq B_{16}$	INSPECTOR 4
18)	$X_{12} + X_{13} + X_{14} + X_{15} + X_{16} \leq B_{17}$	INSPECTOR 5
19)	$X_{17} + X_{18} \leq B_{18}$	INSPECTOR 6
20)	$X_{19} + X_{20} \leq B_{19}$	INSPECTOR 7
21)	$X_{21} + X_{22} + X_{23} \leq B_{20}$	INSPECTOR 8
22)	$X_{24} + X_{25} \leq B_{21}$	INSPECTOR 9
23)	$X_{26} + X_{27} \leq B_{22}$	INSPECTOR 10
24)	$X_{28} + X_{29} + X_{30} \leq B_{23}$	INSPECTOR 11
25)	X_n (for n = 1 to 30) ≥ 0	NON-NEGATIVITY CONSTRAINT

SOLUTION

Initial Linear Program

Right Hand Side and coefficient values needed to be chosen in order to solve the model's equations. The inspection rates used in the model have been determined empirically and are average rates based on the past six months. In addition to the actual inspection time, these rates include setup and documentation times as well. The weekly demand forecast is obtained from the IQC data base and is a weekly average based on the past three months. The sample size is generally based on Military Standard 105D, but is modified using a sampling percentage based on a running average of the previous six month sample size for each commodity. The sample size is calculated by multiplying a commodity's weekly total by the corresponding sample percentage. The initial values used are shown in Table 4. The inspector's salaries used are actuals. All inspectors, except Inspectors 1 and 9 have 40 hours per week available for inspection. Inspectors 1 and 9 have other duties besides inspection therefore, they have only 20 and 30 hours respectively available for inspection. The salaries and initial values used for available hours are shown in Table 5. The Linear Program was run using LINDO. Appendix 1 contains the LINDO printout of the LP and it's results. Since the Locker constraint, as shown in Row 5 of the LINDO printout, cannot be met, the results are that the LP is infeasible.

INSPECTION RATES and FORECAST CONSTRAINTS

COMMODITY		INSPECTION RATE FOR INSPECTOR I_j, IN PARTS PER HOUR											WEEKLY FORECAST		
M_i	DESCRIPTION	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	TOTAL PARTS T_i	SAMPLE % P_i	SAMPLE SIZE B_r
M_1	ACTIVE/ICs	-	-	-	-	-	169	169	-	-	-	-	47632	5.5	2619
M_2	CABLES	-	13	-	-	-	-	-	13	-	-	-	9005	10.5	945
M_3	ELECTRO/MECH	-	-	-	-	-	-	-	-	12	-	-	1859	10	185
M_4	LOCKERS	-	0.5	1	-	1	-	-	-	-	-	-	54	100	54
M_5	PASSIVE COMP	-	-	-	82	-	-	-	-	-	82	-	320217	1.5	4803
M_6	PLASTICS	-	-	-	-	8	-	-	-	-	-	13	6585	4	263
M_7	POWER SUPPLIES	-	-	-	-	-	-	-	-	6	3	-	510	25	127
M_8	PRINTED CKT BD	20	-	-	-	-	-	-	-	-	-	-	905	8	72
M_9	SHEET METAL	-	-	17	-	17	-	-	-	-	-	-	19755	3.3	651
M_{10}	PCB ASSEMBLY	3	-	-	-	-	3	3	-	-	-	-	388	22	85
M_{11}	FASTENERS	-	176	176	-	176	-	-	176	-	-	176	563232	0.5	2816
M_{12}	LABELS	-	13	13	-	13	-	-	13	-	-	13	3524	0.2	7

TABLE 4

LABOR COST and CONSTRAINTS

COMMODITY		INSPECTOR HOURS X_n SPENT PER COMMODITY										
M_i	**DESCRIPTION**	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}
M_1	ACTIVE COMPONENTS/ICs	–	–	–	–	–	X_{17}	X_{19}	–	–	–	–
M_2	CABLES	–	X_3	–	–	–	–	–	X_{21}	–	–	–
M_3	ELECTRO/MECHANICAL	–	–	–	–	–	–	–	–	X_{24}	–	–
M_4	LOCKERS	–	X_4	X_7	–	X_{12}	–	–	–	–	–	–
M_5	PASSIVE COMPONENTS	–	–	–	X_{11}	–	–	–	–	–	X_{26}	–
M_6	PLASTICS	–	–	–	–	X_{13}	–	–	–	–	–	X_{28}
M_7	POWER SUPPLIES	–	–	–	–	–	–	–	–	X_{25}	X_{27}	–
M_8	PRINTED CIRCUIT BOARD	X_1	–	–	–	–	–	–	–	–	–	–
M_9	SHEET METAL	–	–	X_8	–	X_{14}	–	–	–	–	–	–
M_{10}	PRINTED CIRCUIT ASSEMBLY	X_2	–	–	–	–	X_{18}	X_{20}	–	–	–	–
M_{11}	FASTENERS	–	X_5	X_9	–	X_{15}	–	–	X_{22}	–	–	X_{29}
M_{12}	LABELS	–	X_6	X_{10}	–	X_{16}	–	–	X_{23}	–	–	X_{30}
AVAILABLE LABOR HOURS B_c		20	40	40	40	40	40	40	40	30	40	40
INSP SALARY C_j IN $/HOUR		11.15	7.97	7.79	7.73	10.28	8.72	8.85	10.77	8.56	8.35	9.50

TABLE 5

Final Linear Program

Since the initial LP is infeasible, the model needs to be modified. The weekly forecast is a hard constraint and cannot be changed. The initial LP shows that the Locker inspection constraint cannot be met. Because of this, the labor constraints are adjusted for two of the locker inspectors. Inspector 2's hours (B_{14}) are increased to 50 per week and Inspector 3's hours (B_{15}) are increased to 45 per week. Allowing overtime for any inspector contradicts the original premise but is reasonable and necessary to get the job done. The new Linear Program is run using LINDO and results in a feasible solution. The objective function and decision variable values are shown in Table 6. Appendix 2 contains the LINDO printout of the final LP.

FINAL LINEAR PROGRAM SOLUTION

DECISION VARIABLE	HOURS	DECISION VARIABLE	HOURS	DECISION VARIABLE	HOURS
X_1	3.60	X_{11}	40.00	X_{21}	37.28
X_2	0.00	X_{12}	40.00	X_{22}	0.00
X_3	35.41	X_{13}	0.00	X_{23}	0.00
X_4	14.59	X_{14}	0.00	X_{24}	15.42
X_5	0.00	X_{15}	0.00	X_{25}	14.58
X_6	0.00	X_{16}	0.00	X_{26}	18.57
X_7	6.71	X_{17}	15.50	X_{27}	13.17
X_8	38.29	X_{18}	24.50	X_{28}	20.23
X_9	0.00	X_{19}	0.00	X_{29}	16.00
X_{10}	0.00	X_{20}	3.83	X_{30}	0.54
OBJECTIVE FUNCTION VALUE = $3164.94					

TABLE 6

SENSITIVITY ANALYSIS

Sensitivity analysis was run, using LINDO, on the final version of the Linear Program. The LINDO printout of the results is shown in Appendix 2. LINDO calculates the Slack or Surplus and the associated Shadow (Dual) Prices for each of the constraints. LINDO also calculates the ranges for the objective coefficients, C_j, and the right hand side variables, B_r and B_c. Unfortunately, Lindo does not calculate the ranges of the technological variables, a_{ij}. These ranges were manually calculated by applying the methods learned in Engineering Management 540.

Slack

The commodity inspection constraints, B_r, are modelled as "greater than or equal to" equations. Therefore, they are unbounded and are all met with zero slack. The labor constraints, B_c, present the real bounds in this model. Significant results for the labor constraints include slacks of 16.4, 36.2, 2.7, 8.2, and 3.2 hours for Inspectors 1, 7, 8, 10, and 11, respectively. This indicates that Inspectors 1, 7, and 10 are greatly under utilized.

Shadow Prices

The shadow prices for each of the commodities are shown in rows 2 through 13 of the Lindo printout in Appendix 2. Each price indicates the additional cost of inspecting one more part of the given commodity. For these inspection constraints, B_r, one particular shadow price appears significant. The Locker inspection constraint (row 5 of LINDO printout) shows a shadow price of \$21.54. This indicates that the addition of 1 locker to the weekly requirement will result in an overall labor cost increase of \$21.54. However, the range of the RHS for the locker constraint shows that the requirement can increase only 1.36 extra lockers (effectively 1) inspected and still maintain the same basic solution.

For the labor constraints, B_c, the shadow price indicates the reduction in total cost if one more inspection hour is available. Row 16 of the Lindo printout shows that an additional hour of Inspector 3's time would reduce the total cost by $13.75. Rows 18 and 22 also show significant results for Inspectors 5 and 9. Their shadow prices are $11.26 and $8.14 respectively.

Objective Function Coefficients

The value of the objective function coefficient, C_j, is particularly significant since it is the salary of each inspector and contributes directly to the total inspection labor cost. The allowable increases and decreases to the current coefficient are shown in the Lindo printout in Appendix 2. These increases/decreases form the range over which the value of C_j can vary without changing the basic solution. An increase in C_j would typically occur by an inspector receiving a raise. Salaries are seldom lowered but decreases can occur if one of the current inspectors is replaced by a lower salaried inspector.

Most of the salary ranges are so large that any reasonable change would not affect the basic solution. However, there are two cases where salary changes could occur and change the basis. Reference Table 3 for Inspector, decision variable, and cost relationships. In the first case, the allowable increase for decision variable X_{11} is $0.62. Thus, if Inspector 4 receives a $0.63 pay increase, the basis would change. Inspector 4 would spend 8.26 less hours inspecting Passive Components, whereas Inspector 10 would increase his time inspecting Passive Components by 8.26 hours. The new total cost would be $3190.05. Conversely, the allowable decrease for Inspector 10's salary was $0.62. If it decreased $0.63 the hours would change as previously stated for Inspector 4's increase, but the new total cost would be obviously lower at $3153.15. Similarly, and even more likely, the allowable increase for X_{18} is $0.13. If Inspector 6 receives a pay increase of $0.14, his hours spent inspecting Printed Circuit Board Assemblies would go from 24.5 to 0. Inspector 7's hours for inspecting PCBAs would correspondingly increase from 3.83 to 28.33. The new total cost would be $3168.12, an increase of $3.18. Conversely, if Inspector 7's pay decreased $0.14, the hours would change as previously stated and the new total cost would be $3164.15, a decrease of $0.78.

<u>Right Hand Side Values</u>

The data that would most realistically change is the forecast. It is important to know how much the forecast can change before the basis changes or before the constraints have to be modified to avoid infeasibility. The Lindo printout shown in Appendix 2 presents the allowable increases and decreases for the right hand side.

Most of the ranges for the forecast constraints, B_r, are large enough that any reasonable change would not affect the basic solution. However, the Locker constraint (Row 5 of LINDO printout) only allows an increase of 1.36 lockers before the basis changes. If the Locker requirement increases by 2 with no other changes, the resulting LP becomes infeasible. Overtime would have to be increased for Inspectors 2, 3, or 5 in order to have enough available hours to handle the additional lockers. For example, increasing Inspector 3's hours to 46 makes the LP feasible again.

Technological Coefficients

Skill levels and efficiency varies for each inspector. If an inspector improves his capability, the corresponding technological coefficient, $a_{i,j}$, will increase. If he becomes less efficient, the corresponding $a_{i,j}$ will decrease. These coefficients affect inspection time which in turn determines the total inspection labor cost. The range of the technological coefficient for the forecast constraints was calculated to determine the LP's sensitivity to variations of the $a_{i,j}$'s,. The ranges are presented in Table 7. It is important to note that the $a_{i,j}$'s associated with the basic variables always resides in constraints in which the slack variable is binding, that is, where the slack is equal to zero. For non-basic variables, the constraints of concern were always " ≥ " constraints. The analysis of the $a_{i,j}$'s associated with basic decision variables shows that improvement of inspector performance will decrease the overall cost of the labor. In addition, $a_{i,j}$'s associated with non-basic variables appear to be important. If a non-basic $a_{i,j}$ exceeds it's upper limit, the basis is changed and the total cost is reduced. This results in redistributing the workload among the inspectors.

The upper limits of each $a_{i,j}$ must be examined as to whether it is reasonable to expect an inspector to achieve these rates. In many cases it may not be reasonable to expect a two or three-fold increase in inspection rates. For example, Inspector 3 may not be able to increase his inspection rate, $a_{11,3}$, from 176 Fasteners/hour to 400 Fasteners/hour, an increase of 127%. Therefore, the a_{ij}'s of interest are those with upper limits that increase only a small amount above their current value.

This approach leads to a important observation for Inspector 5. If this inspector increases his Sheet Metal inspection rate, $a_{9,5}$, from 17 parts/hour to 18 parts/hour, the minimal operation cost would decrease from $3164.94 to $3119.11. This is a savings of $45.83 per week. Similarly, if Inspector 2 increases his PCBA inspection rate, $a_{10,1}$, from 3 to 4 PCBAs per hour, the overall cost is reduced by $8.32. This could be an unrealistic expectation since this means an efficiency increase of 33% above the original rate.

Table 8 presents the cost savings associated with all of the non-basic a_{ij}'s once the inspector reaches the crossover value. The crossover value is the next integer rate that is greater than the upper limit for the $a_{i,j}$.

Reduced Cost

The "reduced cost" concept appears to have little application to the analysis of this linear program. By definition, the reduced cost is the amount an objective function coefficient of a non-basic variable, X_k, must change in order to cause a change to the LP's optimal solution[3]. This causes the previously non-basic variable X_k to become part of the basis. In the LP described in this paper, the reduced cost value represents the decrease of the hourly salary of each inspector, C_j. Theoretically, salaries can decrease, but this rarely happens in practice. The coefficient could be reduced if a different inspector with lower pay replaced a current inspector. The analysis of this situation is currently not important and therefore we choose not to address it in this paper.

RANGE VALUES FOR TECHNOLOGICAL COEFFICIENTS

DESCRIPTION	COEFFICIENT VALUES									
TECHNOLOGICAL COEFFICIENT, $a_{i,j}$	$a_{8,1}$	$a_{10,1}$	$a_{2,2}$	$a_{4,2}$	$a_{11,2}$	$a_{12,2}$	$a_{4,3}$	$a_{9,3}$	$a_{11,3}$	$a_{12,3}$
LOWER LIMIT	20	$-\infty$	13	.5	$-\infty$	$-\infty$	1	17	$-\infty$	$-\infty$
CURRENT VALUE	20	3	13	.5	176	13	1	17	176	13
UPPER LIMIT	20	3.8	13	.5	199.6	14.7	1	17	399.1	29.5
TECHNOLOGICAL COEFFICIENT, $a_{i,j}$	$a_{5,4}$	$a_{4,5}$	$a_{6,5}$	$a_{9,5}$	$a_{11,5}$	$a_{12,5}$	$a_{1,6}$	$a_{10,6}$	$a_{1,7}$	$a_{10,7}$
LOWER LIMIT	82	1	$-\infty$	$-\infty$	$-\infty$	$-\infty$	169	3	$-\infty$	3
CURRENT VALUE	82	1	8	17	176	13	169	3	169	3
UPPER LIMIT	82	1	29.5	17	399.1	29.5	169	3	169	3
TECHNOLOGICAL COEFFICIENT, $a_{i,j}$	$a_{2,8}$	$a_{11,8}$	$a_{12,8}$	$a_{3,9}$	$a_{7,9}$	$a_{5,10}$	$a_{7,10}$	$a_{6,11}$	$a_{11,11}$	$a_{12,11}$
LOWER LIMIT	13	$-\infty$	$-\infty$	12	6	82	3	13	176	13
CURRENT VALUE	13	176	13	12	6	82	3	13	176	13
UPPER LIMIT	13	199.5	14.7	12	6	82	3	13	176	13

TABLE 7

COST SENSITIVITY FOR NON-BASIC TECHNOLOGICAL COEFFICIENT

COMMODITY M_i	DESCRIPTION	TECHNOLOGICAL COEFFICIENT $a_{i,j}$	CURRENT VALUE (PARTS/HOUR)	CROSSOVER VALUE (+1 > UPPER LIMIT)	COST REDUCTION ($)
M_1	ACTIVE COMPONENTS/ICs	$a_{1,7}$	169	170	0.20
M_6	PLASTICS	$a_{6,5}$	8	30	0.52
M_9	SHEET METAL	$a_{9,5}$	17	18	45.83
M_{10}	PRINTED CIRCUIT BOARD ASSEMBLY	$a_{10,1}$	3	4	8.32
M_{11}	FASTENERS	$a_{11,2}$	176	200	0.07
		$a_{11,3}$	176	400	0.07
		$a_{11,5}$	176	400	0.07
		$a_{11,8}$	176	200	0.07
M_{12}	LABELS	$a_{12,2}$	13	15	0.09
		$a_{12,3}$	13	30	0.09
		$a_{12,5}$	13	30	0.09
		$a_{12,8}$	13	15	0.09

TABLE 8

In formulating the original LP, we looked at many variables including inspection equipment times and capabilities and personnel availability and efficiencies. We decided to group all of this into an aggregate inspection rate for each inspector. The original objective was to minimize overtime for inspectors, but this quickly evolved into minimizing total inspection labor cost.

To make it as realistic as possible, the model was defined using parameters that were derived from the IQC data base. Realism was demonstrated when the first LP turned out to be infeasible. No overtime was allowed in the first LP, when in reality certain inspectors work overtime every week. The final LP included adjustments for overtime for these inspectors and the new LP proved to be feasible. One assumption that made the problem easier was that labor costs due to overtime were the same as regular time, that is, no time-and-a-half pay was included.

After solving the LP, we expected to have the hours that each inspector must work for each commodity and at the minimum cost. This means that the inspectors with the highest inspection rates and the lowest salaries would get the majority of the work. This is basically what occurred as the results in Table 6 show.

Observations of the LP results include:

- Active components/ICs inspection is satisfied by Inspector 6.
- Inspectors 2 and 8 perform all required cable inspection.
- Inspector 11 can satisfy Plastics requirements using only 20 hours per week.
- There are too many inspectors trained in fasteners and labels compared to the demand. Requirement is met by Inspector 11 and there is a 3 hour surplus.
- Only 3.6 hours per week of Inspector 1's time is needed to inspect Printed Circuit Boards.
- Inspectors 1, 7, 10 are under utilized with slack times of 16.4, 36.2, and 8.2 hours respectively.
- Inspector 5 spends all his time on lockers and is not used on any other commodity.

Several of these observations are significant.
Inspector 1 is the highest paid inspector. This is due to the fact that he functions primarily as a Quality Technician. Since only 3.6 of his hours are needed to inspect Printed Circuit Boards, it may be cost effective to train a lower paid inspector to do this inspection.

Inspectors 1, 7, and 10 are under utilized whereas all of Inspector 5's time is needed for Locker inspection. If practical, it may be cost effective to train Inspectors 1, 7, or 10 to do Locker inspection.

The Locker inspection requirement appears to be the most critical of the commodity inspection constraints listed in the LP. The LP solution shows that the range of the Right Hand Side for Lockers allows for the least amount of variation before the basis changes. This is further demonstrated by the relatively high shadow price of $21.54 for locker inspection. In addition, it is recognized that locker inspection is the most time consuming and the most difficult inspection task as far as obtaining high quality units. This is indicated by the 100% "sampling" used in inspection.

These results show that it is necessary to concentrate on ways to improve Locker inspection. Some combination of training additional inspectors, reducing the sample percentage without reducing quality, and increasing inspection rates needs to be pursued by Fujitsu Quality Management.

Minimizing overtime is an important issue. The final LP allowed overtime for both Inspectors 2 and 3. This was necessary in order to keep the LP feasible. Achieving the results given by the LP solution, even though it includes overtime, would be a vast improvement over the current situation at Fujitsu.

Additional opportunities for cost reduction can be found from reviewing Table 8. The "COST REDUCTION" column shows savings associated with inspection rate improvements for $a_{i,j}$'s associated with the non-basic decision variables. The table shows that attempts to improve efficiency in most of these areas would not directly reduce labor costs significantly, except for Sheet Metal. The table makes it easy to estimate the cost savings due to an improvement in inspection rate. The difficult part is in achieving these improvements. Unfortunately, these questions are beyond the scope of this paper.

Several potential cost saving areas have been identified in this paper. Management needs to assess the practicality of changing the inspection work load and cross training additional inspectors such that current and future demands are met while achieving the projected labor cost savings.

EXTENSION

In our first LP model, we found that if the inspection requirements are too large, that the inspectors cannot satisfy the demand without overtime and that the LP becomes infeasible. After reviewing the LP to determine the cause of the infeasibility, we found that we needed to increase the hours worked by Inspectors 2 and 3 in order to allow them to work the necessary overtime. In real life the forecasts are constantly changing, therefore it would be difficult to continue using this model. Also, if we are to minimize labor costs we need to know how much cost is due to overtime. Therefore, we decided that a better model was needed. The new and improved model should tell us if any inspectors need to work overtime and how many overtime hours are necessary. The new model should also tell us whether or not we will meet or exceed our budget and by how much. The material forecast is a hard constraint that we must always meet. The amount by which we exceed the budget must be minimized. Concurrently, we also want to minimize overtime costs. Several methods were investigated in order to develop a better model. The allocation model[4] was reviewed but did not seem to be applicable. Assignment modeling[5] was evaluated but rejected since the problem was not limited to assigning only one inspector to one inspection task. The Matching concept[6] was investigated but applicability could not be determined, therefore it was also rejected. Finally, Pre-emptive Goal Programming (PGP)[7] was reviewed. Its' concept of combining hard and soft constraints while minimizing goal variables well meets our needs.

The new PGP model is based on the original LP and is shown in the LINDO printout in Appendix 3. The original objective function was added as a budget constraint with the RHS equal to $3500 per week. In order to account for overtime and slack time, two decision variables, D_{1P} and D_{1M} respectively, were added to each inspector labor constraint. The solution for these decision variables tells us how many overtime or slack hours any inspector has based on a given weekly forecast. The coefficients for the D_{1P}s are the corresponding hourly overtime salaries. Two additional decision variables, D_{12P} and D_{12M}, were added to tell us by how much we were over or under budget. Our new objective is to meet our forecast, minimize the amount over budget and minimize overtime. Since we have to meet the forecast, we left the forecast constraints as "hard" constraints. The new objective function is:

MIN Z =

$$P1*(D_{12P}) +$$

$$P2*(D_{1P} + D_{2P} + D_{3P} + D_{4P} + D_{5P} + D_{6P} + D_{7P} + D_{8P} + D_{9P} + D_{10P} + D_{11P})$$

with P1 as the first priority coefficient and P2 as the second priority coefficient.

In order to solve this LP with LINDO, two iterations have to be performed. Since minimizing the amount over budget is the first priority, we try to make the amount over budget, D_{12P}, as close to zero as possible in the first iteration. Therefore, the objective function becomes, MIN $Z = D_{12P}$. The LP is feasible and the objective function value equals zero. These results are shown in the LINDO printout in Appendix 3.

The new constraint of $D_{12P} = 0$ is added for the second iteration. The objective function now becomes,

$$\text{MIN } Z = D_{1P} + D_{2P} + D_{3P} + D_{4P} + D_{5P} + D_{6P} + D_{7P} + D_{8P} + D_{9P} + D_{10P} + D_{11P}$$

The LP is feasible and the objective function equals 8.94. Appendix 4 shows the second iteration equations and solutions. The solution means that 8.64 hours of overtime are required. By examining the decision variables we know that Inspector 3 must work this overtime since $D_{3P} = 8.64$. We also know that we are \$217.83 under budget, since $D_{12M} = 217.83$, .

By using this new Pre-emptive Goal Programming model, we can much more easily solve the LP and access the information that we really need. As forecasts change, the RHS of the corresponding constraint can be changed and the LP rerun. The solution will let us know which inspectors will need to work overtime and how our budget will be affected.

CONCLUSION

There are different ways to approach this problem. Our first idea was to minimize inspector time and therefore, cost. Using Linear Programming we found these solutions. Instead, we could have determined maximum IQC throughput and found the bottlenecks in the IQC process.

By using goal programming, we immediately found where overtime was needed and how our budget was affected. The LPs gave us solutions, but realistically it may not be the best real world solution to the problem. For example, any changes should include concerns about resource leveling, that is, distributing the work load more evenly, employee moral, and personal development goals for employees.

Based on the original Linear Program, the immediate solution to meeting an increasing forecast would be to extend work hours and allow overtime. The long term solution would include additional cross training of the current inspectors so that assignments could more easily be optimized. Improving inspection rates, either through training or more efficient inspection equipment, will reduce costs. Developing partnerships with our vendors and pushing the cost of inspection back to them would require some investment, but may eventually result in a cost savings. Whether these are practical or worth the necessary investment is left as an future exercise.

REFERENCES

[1] MIL-STD-105D, "Sampling Procedures and Tables for Inspectors by Attributes", Department of Defense, April 1963.

[2] Schrage, Linus. "User's Manual for Linear, Integer, and Quadratic Programming with Lindo, Release 5.0", San Francisco: The Scientific Press, 1991.

[3] Winston, Wayne L. "Introduction to Mathematical Programming Applications and Algorithms", Boston: PWS-Kent Publishing Company, 1991.

[4] Loomba, Nareada "Linear Programming: A Managerial Perspective", San Francisco: McMillan Publishing Company, 1976.

[5] Hillier, Frederick S. and Lieberman Gerald J. "Operations Research", San Francisco, Holden-Day, Inc., 1974.

[6] Cook, W. and Pulleyblank, W. R. "Linear Systems for Constrained Matching Problems", Mathematics of Operations Research, vol. 12, no. 1, pp. 97-120, February 1987.

[7] Ignizio, James P. "Introduction to Linear Goal Programming", Sage University Paper series on Quantitative Applications in the Social Sciences, series no. 07-056. Beverly Hills: Sage Publications, 1985.

APPENDIX 1

Initial LP with labor constraints fixed for regular time only.

```
MIN      11.15 X1 + 11.15 X2 + 7.97 X3 + 7.97 X4 + 7.97 X5 + 7.97 X6
     + 7.79 X7 + 7.79 X8 + 7.79 X9 + 7.79 X10 + 7.73 X11 + 10.28 X12
     + 10.28 X13 + 10.28 X14 + 10.28 X15 + 10.28 X16 + 8.72 X17 + 8.72 X18
     + 8.85 X19 + 8.85 X20 + 10.77 X21 + 10.77 X22 + 10.77 X23 + 8.56 X24
     + 8.56 X25 + 8.35 X26 + 8.35 X27 + 9.5 X28 + 9.5 X29 + 9.5 X30

SUBJECT TO
       2)   169 X17 + 169 X19 >=   2619
       3)   13 X3 + 13 X21 >=   945
       4)   12 X24 >=   185
       5)   0.5 X4 + X7 + X12 >=   54
       6)   82 X11 + 82 X26 >=   4803
       7)   8 X13 + 13 X28 >=   263
       8)   6 X25 + 3 X27 >=   127
       9)   20 X1 >=   72
      10)   17 X8 + 17 X14 >=   651
      11)   3 X2 + 3 X18 + 3 X20 >=   85
      12)   176 X5 + 176 X9 + 176 X15 + 176 X22 + 176 X29 >=   2816
      13)   13 X6 + 13 X10 + 13 X16 + 13 X23 + 13 X30 >=   7
      14)   X1 + X2 <=   20
      15)   X3 + X4 + X5 + X6 <=   40
      16)   X7 + X8 + X9 + X10 <=   40
      17)   X11 <=   40
      18)   X12 + X13 + X14 + X15 + X16 <=   40
      19)   X17 + X18 <=   40
      20)   X19 + X20 <=   40
      21)   X21 + X22 + X23 <=   40
      22)   X24 + X25 <=   30
      23)   X26 + X27 <=   40
      24)   X28 + X29 + X30 <=   40
END


NO FEASIBLE SOLUTION AT STEP    20
SUM OF INFEASIBILITIES=  8.64027

VIOLATED ROWS HAVE NEGATIVE SLACK,
OR (EQUALITY ROWS) NONZERO SLACKS.
ROWS CONTRIBUTING TO INFEASIBILITY
HAVE NONZERO DUAL PRICE.
```

OBJECTIVE FUNCTION VALUE

1) 3075.5740

VARIABLE	VALUE	REDUCED COST
X1	3.600000	.000000
X2	.000000	.000000
X3	32.692310	.000000
X4	7.307693	.000000
X5	.000000	.500000
X6	.000000	.500000
X7	1.705882	.000000
X8	38.294120	.000000
X9	.000000	1.000000
X10	.000000	1.000000
X11	40.000000	.000000
X12	40.000000	.000000
X13	.000000	1.000000
X14	.000000	.000000
X15	.000000	1.000000
X16	.000000	1.000000
X17	15.497040	.000000
X18	24.502960	.000000
X19	.000000	.000000
X20	3.830375	.000000
X21	40.000000	.000000
X22	.000000	.500000
X23	.000000	.500000
X24	15.416670	.000000
X25	14.583330	.000000
X26	18.573170	.000000
X27	13.166670	.000000
X28	20.230770	.000000
X29	16.000000	.000000
X30	.538462	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	-.038462
4)	.000000	.000000
5)	-8.640271	-1.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	-.058824
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	16.400000	.000000
15)	.000000	.500000
16)	.000000	1.000000
17)	.000000	.000000
18)	.000000	1.000000
19)	.000000	.000000
20)	36.169620	.000000
21)	.000000	.500000
22)	.000000	.000000
23)	8.260162	.000000
24)	3.230769	.000000

NO. ITERATIONS= 20

APPENDIX 2

Final LP with overtime allowed for labor constraints.

```
MIN      11.15 X1 + 11.15 X2 + 7.97 X3 + 7.97 X4 + 7.97 X5 + 7.97 X6
     + 7.79 X7 + 7.79 X8 + 7.79 X9 + 7.79 X10 + 7.73 X11 + 10.28 X12
     + 10.28 X13 + 10.28 X14 + 10.28 X15 + 10.28 X16 + 8.72 X17
     + 8.72 X18 + 8.85 X19 + 8.85 X20 + 10.77 X21 + 10.77 X22
     + 10.77 X23 + 8.56 X24 + 8.56 X25 + 8.35 X26 + 8.35 X27 + 9.5 X28
     + 9.5 X29 + 9.5 X30

SUBJECT TO
       2)   169 X17 + 169 X19 >=   2619
       3)   13 X3 + 13 X21 >=   945
       4)   12 X24 >=   185
       5)   0.5 X4 + X7 + X12 >=   54
       6)   82 X11 + 82 X26 >=   4803
       7)   8 X13 + 13 X28 >=   263
       8)   6 X25 + 3 X27 >=   127
       9)   20 X1 >=   72
      10)   17 X8 + 17 X14 >=   651
      11)   3 X2 + 3 X18 + 3 X20 >=   85
      12)   176 X5 + 176 X9 + 176 X15 + 176 X22 + 176 X29 >=   2816
      13)   13 X6 + 13 X10 + 13 X16 + 13 X23 + 13 X30 >=   7
      14)   X1 + X2 <=   20
      15)   X3 + X4 + X5 + X6 <=   50
      16)   X7 + X8 + X9 + X10 <=   45
      17)   X11 <=   40
      18)   X12 + X13 + X14 + X15 + X16 <=   40
      19)   X17 + X18 <=   40
      20)   X19 + X20 <=   40
      21)   X21 + X22 + X23 <=   40
      22)   X24 + X25 <=   30
      23)   X26 + X27 <=   40
      24)   X28 + X29 + X30 <=   40
END

LP OPTIMUM FOUND AT STEP      22
```

OBJECTIVE FUNCTION VALUE

1) 3164.9360

VARIABLE	VALUE	REDUCED COST
X1	3.600000	.000000
X2	.000000	2.299999
X3	35.411770	.000000
X4	14.588230	.000000
X5	.000000	1.270000
X6	.000000	1.270000
X7	6.705883	.000000
X8	38.294120	.000000
X9	.000000	12.040000
X10	.000000	12.040000
X11	40.000000	.000000
X12	40.000000	.000000
X13	.000000	15.693850
X14	.000000	.000000
X15	.000000	12.040000
X16	.000000	12.040000
X17	15.497040	.000000
X18	24.502960	.000000
X19	.000000	.000000
X20	3.830375	.000000
X21	37.280540	.000000
X22	.000000	1.270000
X23	.000000	1.270000
X24	15.416670	.000000
X25	14.583330	.000000
X26	18.573170	.000000
X27	13.166670	.000000
X28	20.230770	.000000
X29	16.000000	.000000
X30	.538462	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-.052367
3)	.000000	-.828462
4)	.000000	-1.391667
5)	.000000	-21.540000
6)	.000000	-.101829
7)	.000000	-.730769
8)	.000000	-2.783334
9)	.000000	-.557500
10)	.000000	-1.267059
11)	.000000	-2.950000
12)	.000000	-.053977
13)	.000000	-.730769
14)	16.400000	.000000
15)	.000000	2.800001
16)	.000000	13.750000
17)	.000000	.620000
18)	.000000	11.260000
19)	.000000	.130000
20)	36.169620	.000000
21)	2.719457	.000000
22)	.000000	8.140000
23)	8.260162	.000000
24)	3.230769	.000000

NO. ITERATIONS= 22

RANGES IN WHICH THE BASIS IS UNCHANGED:

		OBJ COEFFICIENT RANGES	
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	11.150000	INFINITY	11.150000
X2	11.150000	INFINITY	2.299999
X3	7.970000	1.270000	INFINITY
X4	7.970000	INFINITY	5.630001
X5	7.970000	INFINITY	1.270000
X6	7.970000	INFINITY	1.270000
X7	7.790000	12.040000	.000000
X8	7.790000	.000000	21.540000
X9	7.790000	INFINITY	12.040000
X10	7.790000	INFINITY	12.040000
X11	7.730000	.620000	INFINITY
X12	10.280000	.000000	INFINITY
X13	10.280000	INFINITY	15.693850
X14	10.280000	INFINITY	.000000
X15	10.280000	INFINITY	12.040000
X16	10.280000	INFINITY	12.040000
X17	8.720000	.000000	8.850000
X18	8.720000	.130000	.000000
X19	8.850000	INFINITY	.000000
X20	8.850000	.000000	.130000
X21	10.770000	INFINITY	1.270000
X22	10.770000	INFINITY	1.270000
X23	10.770000	INFINITY	1.270000
X24	8.560000	INFINITY	16.700000
X25	8.560000	8.140000	INFINITY
X26	8.350000	INFINITY	.620000
X27	8.350000	INFINITY	4.070000
X28	9.500000	25.502500	9.499999
X29	9.500000	1.270000	9.500000
X30	9.500000	1.270000	9.499999

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	2619.000000	4141.000000	647.333300
3	945.000000	35.352940	484.647100
4	185.000000	49.560970	79.000000
5	54.000000	1.359728	7.294117
6	4803.000000	677.333300	1523.000000
7	263.000000	42.000000	263.000000
8	127.000000	24.780490	39.500000
9	72.000000	328.000000	72.000000
10	651.000000	23.115380	124.000000
11	85.000000	108.508900	11.491120
12	2816.000000	568.615400	2816.000000
13	7.000000	42.000000	7.000000
14	20.000000	INFINITY	16.400000
15	50.000000	37.280540	2.719457
16	45.000000	7.294117	1.359728
17	40.000000	18.573170	8.260162
18	40.000000	7.294117	1.359728
19	40.000000	3.830375	24.502960
20	40.000000	INFINITY	36.169620
21	40.000000	INFINITY	2.719457
22	30.000000	6.583333	4.130081
23	40.000000	INFINITY	8.260162
24	40.000000	INFINITY	3.230769

APPENDIX 3

ITERATION 1: Minimize the highest priority goal of limiting the over budget deviation variable, that is, try to make $D_{12P} = 0$.

```
 MIN      D12P

 SUBJECT TO
        2) - D12P + 16.725 D1P + 11.955 D2P + 11.685 D3P + 11.595 D4P
      + 15.42 D5P + 13.08 D6P + 13.275 D7P + 16.155 D8P + 12.84 D9P
      + 12.525 D10P + 14.25 D11P + 11.15 X1 + 11.15 X2 + 7.97 X3 + 7.97 X4
      + 7.97 X5 + 7.97 X6 + 7.79 X7 + 7.79 X8 + 7.79 X9 + 7.79 X10
      + 7.73 X11 + 10.28 X12 + 10.28 X13 + 10.28 X14 + 10.28 X15 + 10.28 X16
      + 8.72 X17 + 8.72 X18 + 8.85 X19 + 8.85 X20 + 10.77 X21 + 10.77 X22
      + 10.77 X23 + 8.56 X24 + 8.56 X25 + 8.35 X26 + 8.35 X27 + 9.5 X28
      + 9.5 X29 + 9.5 X30 + D12M =     3500
        3)    169 X17 + 169 X19 >=    2619
        4)    13 X3 + 13 X21 >=   945
        5)    12 X24 >=    185
        6)    0.5 X4 + X7 + X12 >=    54
        7)    82 X11 + 82 X26 >=    4803
        8)    8 X13 + 13 X28 >=   263
        9)    6 X25 + 3 X27 >=   127
       10)    20 X1 >=    72
       11)    17 X8 + 17 X14 >=    651
       12)    3 X2 + 3 X18 + 3 X20 >=    85
       13)    176 X5 + 176 X9 + 176 X15 + 176 X22 + 176 X29 >=    2816
       14)    13 X6 + 13 X10 + 13 X16 + 13 X23 + 13 X30 >=    7
       15) - D1P + X1 + X2 + D1M =     20
       16) - D2P + X3 + X4 + X5 + X6 + D2M =     40
       17) - D3P + X7 + X8 + X9 + X10 + D3M =     40
       18) - D4P + X11 + D4M =     40
       19) - D5P + X12 + X13 + X14 + X15 + X16 + D5M =     40
       20) - D6P + X17 + X18 + D6M =     40
       21) - D7P + X19 + X20 + D7M =     40
       22) - D8P + X21 + X22 + X23 + D8M =     40
       23) - D9P + X24 + X25 + D9M =     30
       24) - D10P + X26 + X27 + D10M =     40
       25) - D11P + X28 + X29 + X30 + D11M =     40
 END

 LP OPTIMUM FOUND AT STEP      21
```

OBJECTIVE FUNCTION VALUE

1) .00000000

VARIABLE	VALUE	REDUCED COST
D12P	.000000	1.000000
D1P	.000000	.000000
D2P	8.692307	.000000
D3P	12.294120	.000000
D4P	13.648640	.000000
D5P	.000000	.000000
D6P	.000000	.000000
D7P	.000000	.000000
D8P	.000000	.000000
D9P	.000000	.000000
D10P	.000000	.000000
D11P	.000000	.000000
X1	3.600000	.000000
X2	3.830375	.000000
X3	32.692310	.000000
X4	.000000	.000000
X5	16.000000	.000000
X6	.000000	.000000
X7	52.294120	.000000
X8	.000000	.000000
X9	.000000	.000000
X10	.000000	.000000
X11	53.648640	.000000
X12	1.705882	.000000
X13	.000000	.000000
X14	38.294120	.000000
X15	.000000	.000000
X16	.000000	.000000
X17	.000000	.000000
X18	.000000	.000000
X19	15.497040	.000000
X20	24.502960	.000000
X21	40.000000	.000000
X22	.000000	.000000
X23	.000000	.000000
X24	15.416670	.000000
X25	14.583330	.000000
X26	4.924535	.000000
X27	13.166670	.000000
X28	20.230770	.000000
X29	.000000	.000000
X30	.538462	.000000

VARIABLE	VALUE	REDUCED COST
D12M	.000000	.000000
D1M	12.569620	.000000
D2M	.000000	.000000
D3M	.000000	.000000
D4M	.000000	.000000
D5M	.000000	.000000
D6M	40.000000	.000000
D7M	.000000	.000000
D8M	.000000	.000000
D9M	.000000	.000000
D10M	21.908800	.000000
D11M	19.230770	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.000000	.000000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000

NO. ITERATIONS= 21

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES ALLOWABLE INCREASE	ALLOWABLE DECREASE
D12P	1.000000	INFINITY	1.000000
D1P	.000000	INFINITY	.000000
D2P	.000000	.000000	.000000
D3P	.000000	.000000	.000000
D4P	.000000	.000000	.000000
D5P	.000000	INFINITY	.000000
D6P	.000000	INFINITY	.000000
D7P	.000000	INFINITY	.000000
D8P	.000000	INFINITY	.000000
D9P	.000000	INFINITY	.000000
D10P	.000000	INFINITY	.000000
D11P	.000000	INFINITY	.000000
X1	.000000	INFINITY	.000000
X2	.000000	.000000	.000000
X3	.000000	.000000	.000000
X4	.000000	INFINITY	.000000
X5	.000000	.000000	.000000
X6	.000000	INFINITY	.000000
X7	.000000	.000000	.000000
X8	.000000	INFINITY	.000000
X9	.000000	INFINITY	.000000
X10	.000000	INFINITY	.000000
X11	.000000	.000000	.000000
X12	.000000	.000000	.000000
X13	.000000	INFINITY	.000000
X14	.000000	.000000	.000000
X15	.000000	INFINITY	.000000
X16	.000000	INFINITY	.000000
X17	.000000	INFINITY	.000000
X18	.000000	INFINITY	.000000
X19	.000000	.000000	.000000
X20	.000000	.000000	.000000
X21	.000000	.000000	.000000
X22	.000000	INFINITY	.000000
X23	.000000	INFINITY	.000000
X24	.000000	INFINITY	.000000
X25	.000000	.000000	.000000
X26	.000000	.000000	.000000
X27	.000000	.000000	.000000
X28	.000000	.000000	.000000
X29	.000000	INFINITY	.000000
X30	.000000	.000000	.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
D12M	.000000	INFINITY	.000000
D1M	.000000	.000000	.000000
D2M	.000000	INFINITY	.000000
D3M	.000000	INFINITY	.000000
D4M	.000000	INFINITY	.000000
D5M	.000000	INFINITY	.000000
D6M	.000000	.000000	.000000
D7M	.000000	INFINITY	.000000
D8M	.000000	INFINITY	.000000
D9M	.000000	INFINITY	.000000
D10M	.000000	.000000	.000000
D11M	.000000	.000000	.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	3500.000000	54.046770	149.793800
3	2619.000000	2124.267000	647.333300
4	945.000000	97.732450	35.262630
5	185.000000	74.654300	38.836000
6	54.000000	7.691593	2.775187
7	4803.000000	1020.275000	229.331700
8	263.000000	204.980900	73.958730
9	127.000000	37.327150	19.418000
10	72.000000	251.392500	72.000000
11	651.000000	29.000000	47.178180
12	85.000000	37.708870	11.491120
13	2816.000000	1323.147000	477.401900
14	7.000000	204.980900	7.000000
15	20.000000	INFINITY	12.569620
16	40.000000	4.520851	12.529800
17	40.000000	4.625312	12.819320
18	40.000000	4.661213	20.737300
19	40.000000	5.877843	1.705882
20	40.000000	INFINITY	40.000000
21	40.000000	3.830375	12.569620
22	40.000000	5.903525	16.361970
23	30.000000	6.583333	7.990996
24	40.000000	INFINITY	21.908800
25	40.000000	INFINITY	19.230770

APPENDIX 4

ITERATION 2: Minimize the second priority goal of limiting the overtime allowed by any of the inspectors. Try to make as many D_{iP}s = 0 as possible.

```
 MIN      D1P + D2P + D3P + D4P + D5P + D6P + D7P + D8P + D9P + D10P
     + D11P

 SUBJECT TO
        2)    16.725 D1P + 11.955 D2P + 11.685 D3P + 11.595 D4P + 15.42 D5P
     + 13.08 D6P + 13.275 D7P + 16.155 D8P + 12.84 D9P + 12.525 D10P
     + 14.25 D11P + 11.15 X1 + 11.15 X2 + 7.97 X3 + 7.97 X4 + 7.97 X5
     + 7.97 X6 + 7.79 X7 + 7.79 X8 + 7.79 X9 + 7.79 X10 + 7.73 X11
     + 10.28 X12 + 10.28 X13 + 10.28 X14 + 10.28 X15 + 10.28 X16 + 8.72 X17
     + 8.72 X18 + 8.85 X19 + 8.85 X20 + 10.77 X21 + 10.77 X22 + 10.77 X23
     + 8.56 X24 + 8.56 X25 + 8.35 X26 + 8.35 X27 + 9.5 X28 + 9.5 X29
     + 9.5 X30 - D12P + D12M =    3500
        3)    169 X17 + 169 X19 >=   2619
        4)    13 X3 + 13 X21 >=   945
        5)    12 X24 >=   185
        6)    0.5 X4 + X7 + X12 >=   54
        7)    82 X11 + 82 X26 >=   4803
        8)    8 X13 + 13 X28 >=   263
        9)    6 X25 + 3 X27 >=   127
       10)    20 X1 >=   72
       11)    17 X8 + 17 X14 >=   651
       12)    3 X2 + 3 X18 + 3 X20 >=   85
       13)    176 X5 + 176 X9 + 176 X15 + 176 X22 + 176 X29 >=   2816
       14)    13 X6 + 13 X10 + 13 X16 + 13 X23 + 13 X30 >=   7
       15)  - D1P + X1 + X2 + D1M =    20
       16)  - D2P + X3 + X4 + X5 + X6 + D2M =    40
       17)  - D3P + X7 + X8 + X9 + X10 + D3M =    40
       18)  - D4P + X11 + D4M =    40
       19)  - D5P + X12 + X13 + X14 + X15 + X16 + D5M =    40
       20)  - D6P + X17 + X18 + D6M =    40
       21)  - D7P + X19 + X20 + D7M =    40
       22)  - D8P + X21 + X22 + X23 + D8M =    40
       23)  - D9P + X24 + X25 + D9M =    30
       24)  - D10P + X26 + X27 + D10M =    40
       25)  - D11P + X28 + X29 + X30 + D11M =    40
       26)    D12P =    0
 END

 LP OPTIMUM FOUND AT STEP      21
```

OBJECTIVE FUNCTION VALUE

1) 8.6402710

VARIABLE	VALUE	REDUCED COST
D1P	.000000	1.000000
D2P	.000000	.500000
D3P	8.640271	.000000
D4P	.000000	1.000000
D5P	.000000	.000000
D6P	.000000	1.000000
D7P	.000000	1.000000
D8P	.000000	.500000
D9P	.000000	1.000000
D10P	.000000	1.000000
D11P	.000000	1.000000
X1	3.600000	.000000
X2	.000000	.000000
X3	32.692310	.000000
X4	7.307693	.000000
X5	.000000	.500000
X6	.000000	.500000
X7	48.640270	.000000
X8	.000000	.000000
X9	.000000	1.000000
X10	.000000	1.000000
X11	40.000000	.000000
X12	1.705882	.000000
X13	.000000	1.000000
X14	38.294120	.000000
X15	.000000	1.000000
X16	.000000	1.000000
X17	.000000	.000000
X18	3.830375	.000000
X19	15.497040	.000000
X20	24.502960	.000000
X21	40.000000	.000000
X22	.000000	.500000
X23	.000000	.500000
X24	15.416670	.000000
X25	10.453250	.000000
X26	18.573170	.000000
X27	21.426830	.000000
X28	20.230770	.000000
X29	16.000000	.000000
X30	.538462	.000000

VARIABLE	VALUE	REDUCED COST
D12P	.000000	.000000
D12M	217.835600	.000000
D1M	16.400000	.000000
D2M	.000000	.500000
D3M	.000000	1.000000
D4M	.000000	.000000
D5M	.000000	1.000000
D6M	36.169620	.000000
D7M	.000000	.000000
D8M	.000000	.500000
D9M	4.130081	.000000
D10M	.000000	.000000
D11M	3.230769	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	-.038462
5)	.000000	.000000
6)	.000000	-1.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	-.058824
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.500000
17)	.000000	1.000000
18)	.000000	.000000
19)	.000000	1.000000
20)	.000000.	.000000
21)	.000000	.000000
22)	.000000	.500000
23)	.000000	.000000
24)	.000000	.000000
25)	.000000	.000000
26)	.000000	.000000

NO. ITERATIONS= 21

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES ALLOWABLE INCREASE	ALLOWABLE DECREASE
D1P	1.000000	INFINITY	1.000000
D2P	1.000000	INFINITY	.500000
D3P	1.000000	.000000	1.000000
D4P	1.000000	INFINITY	1.000000
D5P	1.000000	INFINITY	.000000
D6P	1.000000	INFINITY	1.000000
D7P	1.000000	INFINITY	1.000000
D8P	1.000000	INFINITY	.500000
D9P	1.000000	INFINITY	1.000000
D10P	1.000000	INFINITY	1.000000
D11P	1.000000	INFINITY	1.000000
X1	.000000	INFINITY	.000000
X2	.000000	INFINITY	.000000
X3	.000000	.500000	.500000
X4	.000000	.500000	.500000
X5	.000000	INFINITY	.500000
X6	.000000	INFINITY	.500000
X7	.000000	.000000	1.000000
X8	.000000	INFINITY	.000000
X9	.000000	INFINITY	1.000000
X10	.000000	INFINITY	1.000000
X11	.000000	.000000	1.000000
X12	.000000	1.000000	.000000
X13	.000000	INFINITY	1.000000
X14	.000000	.000000	1.000000
X15	.000000	INFINITY	1.000000
X16	.000000	INFINITY	1.000000
X17	.000000	INFINITY	.000000
X18	.000000	.000000	.000000
X19	.000000	.000000	.000000
X20	.000000	.000000	.000000
X21	.000000	.500000	.500000
X22	.000000	INFINITY	.500000
X23	.000000	INFINITY	.500000
X24	.000000	INFINITY	.000000
X25	.000000	2.000000	.000000
X26	.000000	1.000000	.000000
X27	.000000	.000000	1.000000
X28	.000000	1.625000	.000000
X29	.000000	.500000	.000000
X30	.000000	.500000	.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

		OBJ COEFFICIENT RANGES	
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
D12P	.000000	INFINITY	.000000
D12M	.000000	.000000	.000000
D1M	.000000	.000000	1.000000
D2M	.000000	INFINITY	.500000
D3M	.000000	INFINITY	1.000000
D4M	.000000	INFINITY	.000000
D5M	.000000	INFINITY	1.000000
D6M	.000000	.000000	.000000
D7M	.000000	INFINITY	.000000
D8M	.000000	INFINITY	.500000
D9M	.000000	.000000	1.000000
D10M	.000000	INFINITY	.000000
D11M	.000000	.000000	.500000

		RIGHTHAND SIDE RANGES	
ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	3500.000000	INFINITY	217.835600
3	2619.000000	4141.000000	647.333300
4	945.000000	95.000000	224.647000
5	185.000000	49.560970	185.000000
6	54.000000	11.185400	8.640271
7	4803.000000	677.333300	1523.000000
8	263.000000	42.000000	263.000000
9	127.000000	24.780490	62.719510
10	72.000000	328.000000	72.000000
11	651.000000	29.000000	146.884600
12	85.000000	74.943440	11.491120
13	2816.000000	568.615400	2816.000000
14	7.000000	42.000000	7.000000
15	20.000000	INFINITY	16.400000
16	40.000000	17.280540	7.307693
17	40.000000	8.640271	18.642330
18	40.000000	18.573170	8.260162
19	40.000000	8.640271	1.705882
20	40.000000	INFINITY	36.169620
21	40.000000	3.830375	24.502960
22	40.000000	17.280540	7.307693
23	30.000000	INFINITY	4.130081
24	40.000000	20.906500	8.260162
25	40.000000	INFINITY	3.230769
26	.000000	.000000	.000000