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Abstract: With the increased emphasis on the quality of products, consumers look to warranties as financial guarantees of a product's reliability. The nature and extent of the warranty affect the sales, market share, costs and profits of many businesses. In some cases warranties are viewed as essential to the manufacturer's competitive position. Consumers, on the other hand, must decide between products with differing warranties or between different warranty options for the same product. The problem of calculating the costs and benefits of warranty policies is an important one.

ESTIMATION OF WARRANTY RESERVES: METHODS AND MODELS

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INTRODUCTION

With the increased emphasis on the quality of products, consumers look to warranties as financial guarantees of a product's reliability. The nature and extent of the warranty affect the sales, market share, costs and profits of many businesses.

In some cases warranties are required by law. In other cases, though voluntary, warranties are viewed as essential to the manufacturer's competitive position. Consumers, on the other hand, must decide between products with differing warranties or between different warranty options for the same product. The problem of calculating the costs and benefits of warranty policies is an important one.[1]

Depending on the type of product, warranty expenses[2] may represent a major commitment of funds for a manufacturer.[3] Precise estimates of warranty costs are needed to plan the operations of the company. However, warranty expenses are affected by several stochastic parameters, including the failure rate of the product, the mean time between failures,

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[1]Mamer, John W., "Cost Analysis for Pro Rata and Free-Replacement Warranties," Naval Research Logistics Quarterly, Vol. 29, No. 2, June 1982, p. 345.

[2]Liability aspects of warranties will not be considered here; the discussion is limited to the cost of assuring that the customer be reimbursed for product failure before the end of the warranty time.

[3]In fact, if the liability is considered material, an estimate must be itemized in the firm's financial statements in accordance with the "Statement of Financial Accounting Standards No. 5: Accounting for Contingencies." Frees, Edward W., "Estimating the Cost of a Warranty," Journal of Business & Economic Statistics, Vol. 6, No. 1, Jan. 1988, p. 79 (hereinafter "Frees (1988)"). The standard requires that the after-sale costs associated with warranties be accrued by a charge to income in the period of sale when it is probable that a liability has been incurred and the amount of future costs is reasonably estimable. Amato, Henry N., Anderson, Evan E. and Harvey, David W., "A General Model of Future Period Warranty Costs," The Accounting Review, Vol. LI, No. 4, Oct. 1976, p. 854 (hereinafter "Amato, Anderson and Harvey").

and the number of failures in the warranty period. Thus the manufacturer needs the capability of predicting, with relative accuracy, the associated costs due to randomly occurring claims.[4]

There have been many proposed models of the warranty process that have tried to determine the optimal amount of warranty reserves for a given set of conditions and warranty provisions. These models will be discussed in the following sections.

PURPOSES OF WARRANTIES

Generally, a warranty is a contractual assurance from a seller to a buyer that guarantees that the product sold will perform satisfactorily during the warranty period. Further, the seller promises to replace or repair the product in case it fails to give satisfactory performance during the warranty period.[5]

So widespread is the offering of warranties that most sellers would be placed at a competitive disadvantage if they did not offer a warranty. New competitors entering a market, for example, often must provide their products with warranty protection at least equal to those of established competitors.[6]

Warranties are important to buyers and sellers alike. Buyers want a warranty as an assurance that the product will perform satisfactorily. Buyers rely on warranties to ease

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[4]Thomas, Marlin U., "Optimum Warranty Policies for Nonreparable Items," IEEE Transactions on Reliability, Vol. R-32, No. 3, Aug. 1983, p. 282 (hereinafter "Thomas (1983)").

[5]Mitra, Amitava and Patankar, Jayprakash G., "Warranty Cost Estimation: A Goal Programming Approach," Decision Sciences, Vol. 19, Spring 1988, pp. 409-423.

[6]McGuire, E. Patrick, Industrial Product Warranties: Policies and Practices, New York: The Conference Board, 1980, p. 2.

doubts they might have about imperfections in product design, since warranties can compensate for any hidden quality problems.

Warranties are equally important to sellers, who use warranties for a variety of promotional and protection purposes. Warranties often communicate information about the reliability of the product. Including a warranty statement can reduce the buyer's risk and thus spur sales.[7] The degree and nature of warranty protection are often seen as significant product attributes in that they indicate the amount of after-sales service that will be provided.[8]

Sellers also use warranties to protect themselves against unreasonable buyer claims by explicitly defining their responsibility after the sale. The seller accepts the responsibility to see that the buyer gets a certain performance from the product but does so on its own terms to protect against excessive demands.[9] Warranty claims can also furnish valuable feedback to the seller about product quality and production problems.

IMPORTANCE OF PRECISE ESTIMATES

The optimum allocation of funds for covering the risk of warranty claims is a major problem for manufacturers. The planning is complicated due to uncertainties associated with product failures, quality levels, and economic conditions.

Precise estimates are needed for two distinct phases of the planning process. First, the manufacturer must determine the warranty provisions to which it will be bound. This determination must cover both the conditions of compensation

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[7]Mitra and Patankar, p. 410.

[8]McGuire, p. 2.

[9]Mitra and Patankar, p. 409.

and period of coverage. The extent of warranty coverage granted generally depends upon the nature of the product, the kinds of use it will receive, the practicality of repair, and the kinds of repair and service skills needed.[10] The models discussed below have examined relationships between warranty period and expected costs.[11]

Second, the company must determine the amount of money that must be allocated to cover future expenses for failures during a specified warranty period on current production. Underestimating the magnitude of future outlays may result in a false impression of the profit position of the firm and result in costs that must be covered in a future profit plan.[12] At the same time, overestimating the reserves needed can result in lost opportunity due to committed funds that could presumably be invested for greater returns elsewhere.[13]

In spite of efforts to estimate warranty reserves accurately, most warranty policies are established solely based on judgment, or by matching the policies of the competitors. The difficulty in analytically determining a policy seems to be in establishing an acceptable criterion for examining cost-benefit tradeoffs.[14]

DIFFICULTIES IN GETTING PRECISE ESTIMATES

Planning for warranty reserves is complicated due to the stochastic nature of product failures, quality levels, sales,

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[10] McGuire, p. 4.

[11] Thomas (1983), pp. 282-288.

[12] Thomas (1983), p. 282.

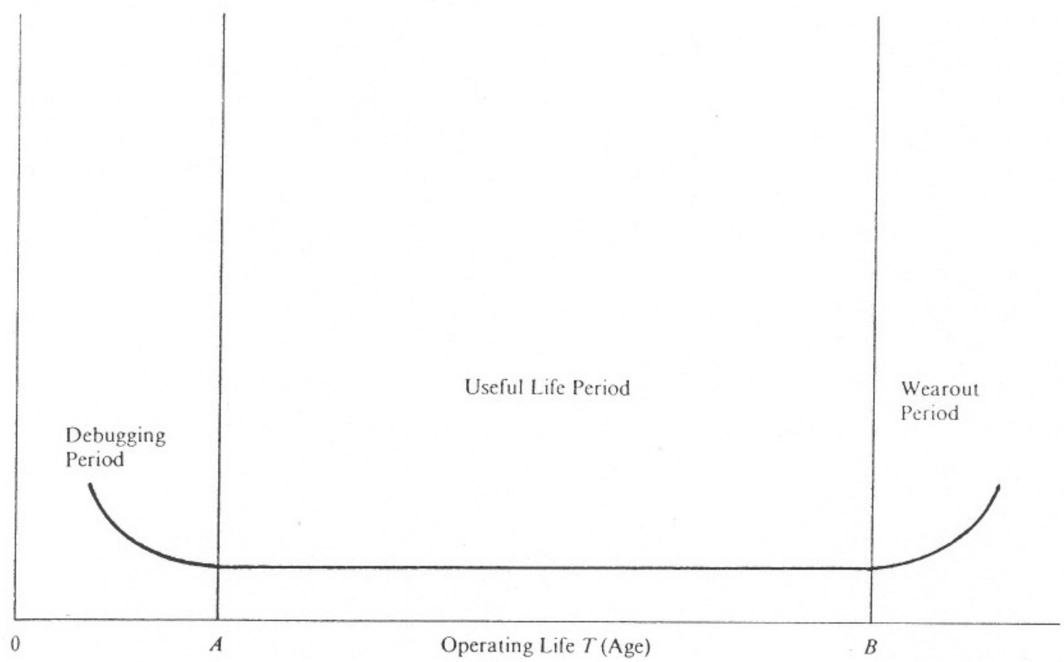
[13] Amato, H.N. and Anderson, E.E., "Determination of Warranty Reserves: An Extension," Management Science, Vol. 22, No. 12, August 1976, p. 1392 (hereinafter "Amato and Anderson").

[14] Thomas (1983), p. 282.

and general economic conditions. The effectiveness of an calculated warranty policy depends how well these factors can be determined or predicted.

The typical life cycle of a product is shown in FIGURE 1. The cycle may be broken into three phases depending on the failure rate: (1) the debugging period; (2) the useful life period; and (3) the wearout period. This life cycle model is often called a "bathtub curve" due to its shape.

FIGURE 1: TYPICAL PRODUCT FAILURE PHENOMENA[15]



1. <u>The Debugging Period.</u> The beginning of a product life is often characterized by "early failures." These failures may be the result of poor product design, poor workmanship and/or

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[15]Balachandran, K.R., Maschmeyer, Richard A. and Livingstone, J. Leslie, "Product Warranty Period: A Markovian Approach to Estimation and Analysis of Repair and Replacement Costs," The Accounting Review, Vol. LVI, No. 1, Jan. 1981, p. 117.

Because a product's warranty is normally in effect during the useful life period, the failure rate is assumed to be constant during the warranty period. For mathematical simplicity, the assumption of constant failure rate is often made when computing warranty reserves.[21]

A major factor in any manufacturing decision is the indirect costs. Difficulties arise in predicting these costs for a particular warranted item. Not only is it necessary to know the failure characteristics of the item, but one must also be able to estimate the cost of failure over time, including service costs, repair costs, and replacement costs. Obviously, the more warranty service a manufacturer provides to the consumer, the higher the unit warranty costs will be.[22]

Since the expenses incurred by the manufacturer during warranty periods represent deductions to profits that lag the production, appreciable uncertainty is introduced into the profit plan.[23] If the length of the item's life cycle is sufficiently long, it may be necessary to discount future payments at a fixed rate α to the present time.[24] It is, however, usually hard to forecast the proper discount rate to use, since it is dependent on inflation/deflation, investment rates and the like.

Companies must not only know the warranty costs of existing products, but must also have a forecast of what the

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[21]Mitra and Patankar, p. 412. Note that this assumption is invalid if the debugging process is not completed prior to the sale of the product.

[22]Thomas (1983), p. 285.

[23]Thomas (1983), p. 282.

[24]Blischke, Wallace R. and Scheuer, Ernest M., "Calculation of the Cost of Warranty Policies as a Function of Estimated Life Distributions," Naval Research Logistics Quarterly, Vol. 22, No. 4, 1975, p. 684.

likely warranty costs will be for new products -- to price them at a level that can absorb such costs.[25] Typically warranty costs are treated as manufacturing costs and are included in the final price of a product to the extent the product pricing structure will allow.[26]

New product warranties are particularly difficult to evaluate due to the limited product failure information that is available when warranty commitments are made. Thomas[27] demonstrated that the calculated warranty reserves are highly sensitive to the assumed failure probability distributions.

TYPES OF WARRANTY PROVISIONS

There are two common types of warranty policies: the free replacement warranty policy and the pro rata warranty policy. Due to differences in their provisions, the amount of necessary warranty reserves will also differ.

Free Replacement

Under a free replacement warranty, the manufacturer pays the entire cost of repair or replacement if the product fails before the warranty expires and supplies as many repairs or replacements as needed during the warranty period.[28] If the product fails after the warranty period, the consumer pays the full price for the replacement. Any replacement made during

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[25]McGuire, p. 17.

[26]Menke, Warren W., "Determination of Warranty Reserves," Management Science, Vol. 15, No. 10, June 1969, pp. B-542-49 (hereinafter "Menke (1969)").

[27]Thomas, Marlin U., "A Prediction Model for Manufacturer Warranty Reserves," Management Science, Vol. 35, No. 1, Dec. 1989, pp. 1515-19 (hereinafter "Thomas (1989)").

[28]Nguyen D.G. and Murthy, D.N.P., "Cost Analysis of Warranty Policies," Naval Research Logisitics Quarterly, Vol. 31, 1984, p. 525.

the warranty period is warranted only until the end of the period, thus the warranty is not renewed after each replacement. This means that the consumer will receive as many free replacements as needed during the warranty period, but must buy the replacements at full cost after that time.

For a long warranty period, the warranty costs can be very large, depending upon the number of failures and the cost of correcting the failure. Furthermore, an increase in warranty period will reduce the number of replacements the consumer needs to purchase over the product life cycle, which in turn reduces the total profits. The free replacement warranty generally favors the consumer at the expense of the manufacturer.[29]

Pro Rata

Under a pro rata warranty policy, if the product fails during the warranty period, both the manufacturer and the consumer pay a cost that depends on the age of the product.[30] The consumer pays an amount proportional to the remaining useful life of the item for a replacement that is covered by an identical warranty. This makes this type of warranty more appealing to the manufacturer but less attractive to the consumer since she may have to purchase a new item, at a cost, should the earlier item have a very short useful life. Thus, a pro rata warranty favors the manufacturer at the expense of the consumer.[31]

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[29]Nguyen and Murthy, p. 526.

[30]Frees, Edward W, and Nam, Seong-Hyun, "Approximating Expected Warranty Costs," Management Science, Vol. 34, No. 12, Dec. 1988, p. 1442.

[31]Nguyen and Murthy, p. 526.

In reality, under a pro rata policy there is a point beyond which the customer will forego the warranty claim opportunity to avoid incurring any personal costs. It has been proposed that in terms of warranty reserves, the accumulation of unclaimed warranty monies offsets the assumed expenses that would be incurred by warranties on the replaced units. This "renewable warranty" situation has also been explored.[32]

METHODS OF CALCULATING WARRANTY RESERVES

There have been a number of approaches taken to formulate models to approximate the warranty process and calculate the required amount of warranty reserves. These approaches include: (1) general probabilistic methods; (2) renewal theory methods; (3) goal programming methods; and (4) Markovian methods. Each of these methods are discussed below.

GENERAL PROBABILISTIC METHODS

One of the first articles dealing with the determination of necessary warranty reserves was by Lowerre.[33] His main objective was to determine a selling price that would provide a contingency fund against which warranty costs could be charged.[34]

Lowerre assumed that each unit produced has the same independent probability p of being defective[35], and calculated the expected cost of the warranty using a binomial distribution. Thus the expected cost of the warranty ($) is

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[32]Thomas (1983), pp. 282-288.

[33]Lowerre, James M., "On Warranties," Journal of Industrial Engineering, Vol. 19, No. 7, July 1968, pp. 359-60.

[34]Lowerre, p. 359.

[35]If field data is not available, the value of p will have to be established by decision methods. Schlaifer's standard gamble is suggested as being appropriate. Lowerre, p. 359.

failures are stochastic events whose probability of occurrence at any time can be described by a single number. Further, the probability of failure is an exponential function described by:

$$\text{Probability of failure} = P = 1 - e^{-t/m}$$

where t = time to failure and m = MTTF. Fortunately, this is true for most products.[40] Since this approach to warranty calculations is statistical in nature, its validity is highest when large quantities of products are considered.

Knowing the MTTF, the probability of failure as a function of time can be determined. From this, the number of products that will probably fail in any small time interval, dt, is calculated. Multiplying the probable failures by the cost of replacement or the cost of repair at time t produces the increment of warranty reserves that must be set aside for failures during the interval. Finally, adding such warranty cost increments for all dt from t = 0 to the end of the warranty period results in the total cost of the warranty reserves.[41] Menke calculated the total cost of warranty reserves for the free replacement and the pro rata warranty and discussed the applicability of both types of warranties.

Blischke and Scheuer[42] next extended Menke's analysis by considering more general failure distributions. They also introduced the idea of a product's life cycle (roughly, its economic usefulness) which, in turn, uses tools arising in

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[40]Menke (1969), p. B-543.

[41]Menke (1969), p. B-543. This assumes that all failures within the warranty period are claimed.

[42]Blischke, Wallace R. and Scheuer, Ernest M., "Calculation of the Cost of Warranty Policies as a Function of Estimated Life Distributions," Naval Research Logistics Quarterly, 22(4), 1975, pp. 681-696.

renewal theory.[43] Blischke and Scheuer derived the expected costs for pro rata and free replacement warranties as expressions involving renewal functions.

The key random variable in the analysis is N, the number of replacements required. This variable is a function of the type and length of the warranty, and of the relationship of the warranty length to the life distribution of the item. Thus a statistical approach to the problem begins with an analysis of life-test data.[44]

In the free replacement warranty situation, the seller's revenue per warranty period is fixed. Costs are a random variable since the number of free replacements that must be made is random. The time between payments is also a random variable.[45] In the pro rata warranty, it is the revenue per transaction that is a random variable.[46] The expected value of

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[43]A renewal process is a counting process for which the time between successive events are independent and identically distributed with an arbitrary distribution. Thus, the time until the first event occursi has some distribution F, the time between the first and second events has, independently of the first event, the same distribution F, and so on. When an event occurs, we say that a renewal has taken place. Ross, Sheldon M., Introduction to Probability Models, 4th ed., Boston: Academic Press, 1989, p. 297.

[44]Blischke and Scheuer, p. 682.

[45]Blischke and Scheuer, p. 687. The authors acknowledged that their analysis assumed knowledge of the time-to-failure cumulative distribution function F(•) of the item in question, as well as its mean μ, and the probability distribution of the associated renewal process, N(•) , which in turn require successive convolutions with itself. In most practical situations, F(•) will not be known. In some situations the form of F(•) may be known (e.g., normal, exponential, Weibull, or the like) but not the specific parameters. In other situations, not even the form of F(•) is known. In the former situation, the parameters must be estimated; in the latter, one may either follow a nonparametric approach or first try to elicit the form of F(•) and then estimate the parameters within that form. How these estimation tasks are performed depends on the kind of data available.

[46]Blischke and Scheuer, p. 686.

the revenue per transaction is a function of the product life and the number of breakdowns. In the free replacement warranty case, the time between payments is also a random variable.[47]

An important extension of this analysis is to the case in which there are multiple competing modes of failure. If the causes of the failure act independently of one another, the extension of the analysis is conceptually straightforward. If they do not, additional complications are encountered. Blischke and Scheuer limited their consideration to the the case of independent failure modes, and assume that each leads to a normal distribution of lifetimes.[48]

Amato and Anderson[49] were concerned with an explicit warranty rebate program on repairable products. They present, in a general form, mathematical functions capable of estimating the future cost of a warranty program. They extend Menke's model for nonreparable products by discounting future warranty costs to their present value, and by adjusting for expected changes in the general price level. By ignoring these two factors, the original model overstated the required warranty reserve and per unit product price derived therefrom.[50]

The expected money value of consumer claims within the warranty period occur in the future after the product is sold. Therefore their present value is reduced by the opportunity to invest and earn profits from the per unit price premium (r) which the firm receives to cover the eventual cost of warranty

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[47]Blischke and Scheuer, p. 687.

[48]Blischke and Scheuer, p. 690.

[49]Amato, H.N. and Anderson, E.E., "Determination of Warranty Reserves: An Extension," Management Science, Vol. 22, No. 12, Aug. 1976, pp. 1391-1394.

[50]Amato and Anderson, p. 1391.

claims as part of the selling price of the product.[51]

In addition to the time value of r, the presence of changes in the general price level may cause the real value of rebate costs to be greater or less than their nominal values in future periods. If the general price level is tending to increase, this inflationary process will erode the purchasing power of r, and thereby reduce the real warranty burden of the firm.[52] Since the product's unit price without a warranty is not affected by warranty costs, the discounting of the costs associated with future warranty claims would reduce the product's price by reducing the component associated with the warranty reserves.[53] If there are increases in the general price level between the time of sales and the time of warranty claims, the purchasing power of the price premium r is eroded. Thus, the effect of inflation is to reduce the real present value of future warranty costs.[54]

In commenting on the Amato and Anderson extension to his model, Menke[55] cautions that the investment opportunities presented by the present value approach should also be examined in light of the number of failures expected during the warranty period. The cash flow requirements of the company must be

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[51]Amato and Anderson, p. 1392.

[52]Amato and Anderson, p. 1392.

[53]This result may have important implications for the firm depending on the price elasticity of demand. If the demand for the firm's product, for example, is price elastic, by reducing is unit price, the firm would expand its sales and revenues. Furthermore, in terms of the aggregate economy, the expectation of increases in the general price level in the future will encourage the firm to reduce the current prices. Amato and Anderson, p. 1392.

[54]Amato and Anderson, p. 1394.

[55]Menke, W.W., "Comments on Amato and Anderson's Paper, 'Determination of Warranty Reserves: An Extension,'" Management Science, Vol. 22, No. 12, August 1976, pp. 1395-96 (hereinafter "Menke (1976)").

analyzed carefully. The stochastic nature of the failures must not be overlooked; the investor of reserve funds may be walking a thin line in trying to balance sage investment durations against liquidity requirements to satisfy claims for failures. If he falls off the line he may be forced to borrow money to replace reserve funds that are locked into an investment. It therefore appears that short term investments are viable, but that the larger the ratio of warranty period to MTTF, the shorter the term of the investment.[56]

Glickman and Berger[57] formulated the expression for unit profit as an expectation so their model is applicable to products requiring random, possibly multiple, repairs under warranty. They also provided an expression for the expected sales volume to show the dependence of the demand on price and warranty period. An economic sensitivity analysis provided some insight into the economic effects of the optimal profit, price and warranty length by considering the sensitivity of the optimal solution to variations in these parameters.[58]

By assuming the failure rate to be monotonic, Mamer[59] developed an alternative approximation to the expected warranty cost. He obtained the exact total costs over the product life cycle (short-run costs) and the expected cost per unit of time for an infinite life cycle (long-run costs). He noted that the average cost and profit to both the manufacturer and the consumer depend on both the cumulative hazard function and the

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[56]Menke (1976), p. 1395.

[57]Glickman, Theodore S. and Berger, Paul D., "Optimal Price and Protection Period Decisions for a Product Under Warranty," Management Science, Vol. 22, No. 12, Aug. 1976, pp. 1381-90.

[58]Glickman and Berger, pp. 1385.

[59]Mamer, John W., "Cost Analysis for Pro Rata and Free-Replacement Warranties," Naval Research Logistics Quarterly, Vol. 29, No. 2, June 1982, pp. 345-356.

mean of the distribution of product lifetime.[60]

To calculate the short-run total costs for the case of a pro rata warranty, Mamer assumes that the product has only two possible states (working or failed) and that each failed product is replaced by a working unit. Thus, there is no distinction between repair and replacement of the product. He assumes that after each failure the customer replaces the product immediately with an identical unit carrying an identical warranty. Under these conditions the sequence of breakdowns is described by a renewal process. An expression for the total expected replacement costs up to time T, namely E[R(T)], is given.[61]

The short-run total cost analysis of the free-replacement warranty is more complex than that of the pro rata warranty. Based on the provisions of a free-replacement warranty, the consumer pays every W + e(W) units of time, where e(W) is the excess life of the process X_1, X_2, ... at time W. Thus the equation for the short-run total cost under this type of warranty is a function that can be solved by conditioning on the time to the first failure.[62]

In many situations the long-run averages of cost and profit are better criteria for comparing alternative policies. This is particularly true in situations in which the product life-cycle is expected to be quite long. The exact formulas for the long-run average cost and profit for a product under three possible conditions, a no warranty, a pro rata warranty and a free-replacement warranty are calculated.[63]

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[60]Mamer, p. 351.

[61]Mamer, p. 346.

[62]Mamer, p. 348.

[63]Mamer, p. 349-52.

Nguyen and Murthy[64] derived expressions for the expected costs to the consumer and the expected profits to the manufacturer from product sales under the two general warranty policies. From these results they estimated the costs and profits for pro rata warranty and the free replacement warranty policies.

Thomas[65] formulated an extension of Menke and Amato & Anderson (1976) that looked at a general product failure distribution. He also provided results for some nonexponential distributions that are of particular interest in product reliability. Since, in general, manufactured products become more failure prone as they get older due to wear and age deterioration, their failure times have increasing rates in that the failure rate will not decrease with $t \geq 0$. The exponential distribution can be an appropriate time-to-failure model for complex systems. It has been justified empirically for representing failure times for certain electronic components. However, for other products, particularly those have mechanical elements, the failure rate will be an increasing function with age.[66]

Thomas notes that it is particularly difficult to evaluate warranty issues such as required reserves for new products, due to the uncertainty of product failures. Such analyses typically require syntheses and evaluations, using rather diverse and somewhat skimpy data. A variety of alternative

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[64]Nguyen D.G. and Murthy, D.N.P., "Cost Analysis of Warranty Policies," Naval Research Logisitics Quarterly, Vol. 31, 1984, pp. 525-541.

[65]Thomas, Marlin U., "A Prediction Model for Manufacturer Warranty Reserves," Management Science, Vol. 35, No. 1, Dec. 1989, pp. 1515-19.

[66]Thomas (1989), p. 1516. See Hahn, G.J. and Shapiro, S.S., Statistical Methods in Engineering, New York: John Wiley & Sons, 1967, ch. 3.

product failure distributions should be considered. The uniform, gamma and Weibull are candidate increasing failure rate distributions for modeling warranty reserves.[67]

RENEWAL THEORY METHODS

Warranty reserves have also been viewed from a renewal theory perspective, where the expected number of replacements (renewals) in the warranty period is calculated.[68] Frees[69] investigated the idea of viewing expected warranty costs as a summary measure of the distribution function that is to be estimated from the available data. Estimation of the expected cost of a warranty for a stochastically failing unit is closely tied to estimation of the renewal function.[70]

Frees supposes that with the purchase of a stochastically failing item, the manufacturer agrees to replace the item at failure, free of charge, for a certain length of time. It is assumed that after each failure, the owner of the product instantaneously purchases an identical replacement. The assumption of instantaneous replacement permits the sequence of product lifetimes to be described as a renewal process.[71]

The warranty cost to the manufacturer depends on the cost per replacement item, the duration W, and the lifetime

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[67]Thomas (1989), p. 1516.

[68]See fn. 43.

[69]Frees, Edward W., "Warranty Analysis and Renewal Function Estimation," Naval Research Logistics Quarterly, vol. 33, 1986, pp. 361-372 (hereinafter "Frees (1986)") and Frees, Edward W., "Estimating the Cost of a Warranty," Journal of Business & Economic Statistics, Vol. 6, No. 1, January 1988, pp. 79-86 (hereinafter "Frees (1988)").

[70]Frees (1986), p. 361.

[71]Mamer, p. 351.

distribution of the unit. The expected cost C(W) of the warranty is

$$C(W) = c * H(W)$$

where H(W) is the renewal function over the warranty period.[72] If a manufacturer would like to offer this type of warranty for a product whose lifetime distribution is not completely known, it is necessary to estimate H(W), and thus C(W), based on a finite number of observed lifetimes of the units.[73] However, the renewal function is very difficult to calculate, even when the distribution function is known. Therefore, the authors discuss various estimators of the renewal function.[74]

Due to these computational difficulties, Frees assumes that the product is a stochastically failing item whose potential lifetime is a random variable governed by some distribution function, F. From the purchaser's point of view, the value of the warranty is dependent on the realized lifetime of the product available and thus is a random variable.[75] Frees has departed from the assumptions of Blischke & Scheuer, Mamer, and Nguyen & Murthy, who all used the idea of a product life cycle.

For warranty purposes, the product life cycle was the period of time over which the consumer desired to have the product operable. The advantage of considering the lifetime and the cost of the warranty as random variables is that this framework immediately suggests other measures of the warranty expense, such as the variability.[76]

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[72]Frees (1986), p. 361.

[73]Frees (1986), p. 361.

[74]Frees (1986), p. 362.

[75]Frees (1988), pp. 79-86.

[76]Frees (1988), p. 79.

While it is important for the manufacturer to have a good approximation of the expected warranty liability, it is also important for long-range planning to know how volatile costs can be. One summary measure of this volatility is the variance of the warranty cost. The variance can be estimated using the techniques used to estimate the renewal function.[77] In addition to the mean and variance of the warranty liability, Frees also discusses the incremental cost in increasing the warranty duration from W to, say, $W + \Delta W$.[78]

GOAL PROGRAMMING METHOD

In the past, research has dealt mostly with the optimization of a single objective, such as the minimization of the warranty reserve cost or the maximization of profit. However, a manufacturer often has many goals in mind when offering a warranty on product. Some of these goals may be conflicting.

Mitra and Patankar[79] proposed the use of a goal programming approach in a warranty cost estimation problem to meet the conflicting criteria of the manufacturer. If desired, these goals can often be given priorities based on the manufacturer's perception as to which ones are more important than others. They considered three goals:[80]

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[77]Frees (1988), pp. 83-84. The techniques used to estimate the expected number of renewals (the renewal function) discussed include: (1) straight-line approximation estimator; (2) parametric estimator; (3) nonparametric estimator; and (4) resampling estimator.

[78]Frees (1988), p. 85.

[79]Mitra, Amitava and Patankar, Jayprakash G., "Warranty Cost Estimation: A Goal Programming Approach," Decision Sciences, Vol. 19, Spring 1988, pp. 409-423.

[80]Mitra and Patankar, p. 411.

Goal 1: Minimize the total warranty reserve costs per unit price subject to system constraints on the price and warranty time.

Goal 2: Offer a warranty time, w, greater than or equal to a specified value that is based on an allowable proportion of failures during that time.

Goal 3: Achieve a certain level of market share of the product.

In formulating the problem, Mitra and Patankar assumed that a linear pro rata rebate plan exists.[81] This means that the rebate to the customer varies linearly, from the unit product price to zero, as a function of the failure time of the product (as long as the failure time is within the warranty period).[82]

For Goal 1, the ratio of the warranty reserve cost per unit to the unit price of the item is minimized, subject to the allowable price range for the item and the allowable warranty time range. The minimum warranty time could be selected by considering the quality of the product and the warranty time offered by competitors on similar products. The maximum warranty time would be based on cost effectiveness and product liability information. While selecting a longer warranty period is attractive from a marketing standpoint, the company must also protect its image and goodwill by ensuring that the

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[81]The model was subsequently extended to a nonlinear goal programming problem.

[82]The authors also describe the Weibull distribution, which is a more general distribution, compared to the exponential, that has been used to describe failure distributions. The Weibull distribution has been used in the field of reliability and quality control and has been suggested as time-to-failure model for electron tubes, relays and ball bearings. Mitra and Patankar, pp. 411-12.

warranty period is not so long as to cause "too many" failures within that warranty time. A long warranty time would also tend to increase the replacement and liability costs to the manufacturer.[83]

For Goal 2, the objective is to prevent the manufacturer from setting a warranty time so small that hardly any product fails within the time. In that case the benefit obtained by the customer would be negligible. The goal is therefore formulated such that the warranty time will contain a desirable proportion of failures.[84] For Goal 3, the market share is computed as a function of price elasticity, displaced warranty period elasticity, and the warranty period.[85]

A goal programming software package was used to solve a sample problem with nonlinear goal constraints. The authors found that the nonlinear constraints that arise when dealing with the minimization of the warranty-reserve-cost-per-unit-price and the market-share functions will suffer negligible loss of accuracy if they were linearized.[86] The author's testing implied the superiority of the solution from the goal programming model (versus the solution from various single objective models), especially if the decision maker has established the priorities of several goals.

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[83]Mitra and Patankar, p. 414.

[84]Mitra and Patankar, pp. 414-15.

[85]Mitra and Patankar, p. 415. The elasticities are often nonlinear functions which can also be handled using the goal programming approach.

[86]Mitra and Patankar, p. 418.

MARKOVIAN METHOD

In addition to the above methods to estimate warranty reserves, Balachandran et al[87] developed a method based on the use of Markov chains. As an example problem, they assumed that the manufacturer's policy is to repair a component on the first two breakdowns and replace the component on its third breakdown. They further assumed a product with three independent components. They then described the failure-repair process of the product as a Markov chain.[88] The state of the Markov chain is specified as a three-dimensional vector to correspond to the three components of the product. The elements of the vector denote the number of failures in each component up to the present time.

The process of transition between states is assumed to follow a Markov chain. That is, given the present state of the process, the probability of reaching another state in the future is independent of the past states.[89] A Poisson distribution was used to characterize the transitions from one state to another.

Any component that fails will be repaired or replaced immediately so that the product will be in full service without material delay. Thus, the duration of time needed to repair or replace a failed component is assumed to be negligible in comparison to the warranty period.[90]

The assumed policy states that when a component fails for the third time, it is replaced by a new component. There is a

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[87]Balachandran, K.R., Maschmeyer, Richard A. and Livingstone, J. Leslie, "Product Warranty Period: A Markovian Approach to Estimation and Analysis of Repair and Replacement Costs," The Accounting Review, Vol. LVI, No. 1, January 1981, pp. 115-124.

[88]Balachandran et al., p. 118.

[89]Balachandran et al., p. 119.

[90]Balachandran et al., p. 119.

cost involved corresponding to each transition. The cost is dependent on whether the component is repaired or replaced. Two cost matrices are given, one for repair and one for replacement.[91]

Let c_{ij} denote the cost due to transition from state i to state j. The expected warranty cost over the next n periods, denoted by $C_i^{(n)}$, given the current state is i, is given then by:

$$C_i^{(n)} = C_i^{(1)} + \sum_{j=1} P_{ij} C_j^{(n-1)}$$

where k is the number of possible states. This equation can be solved recursively by letting n = 1, 2,...N.[92]

While this method was shown to work in the sample problem, it can quickly become unwieldy, especially if more components are included in the analysis. For example, in the small sample problem with only three components and three possible states, both of the cost matrices and the transition matrix were each 27 x 27 matrices. In addition, this method suffers from some of the same problems as the above methods, as in how to accurately determine the transition probabilities between the states.

COMPARISON OF THE METHODS

The majority of the literature references are concerned with the use of general probabilistic methods, including renewal theory to determine warranty reserves. Most of these methods are concerned with nonrepairable products, although in practice, many items can be repaired at a cost lower than that of full replacement. If repair is possible, the cost of this repair would also appear to be stochastic. However, this problem is not addressed in most of the methods surveyed.

The fundamental difficulty with the use of statistical

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[91]Balachandran et al., p. 119.

[92]Balachandran et al., p. 122.

methods to determine warranty reserves is the determination of the proper failure rate distribution. Several common situations make this determination particularly difficult. First, as mentioned previously, determination of the failure distribution may be difficult in the case of new products where there is not very much historical data. Second, most statistical functions assume a relatively large sample size, so that products made in smaller quantities may not be properly estimated by statistical functions. Finally, most of the methods assume a constant failure rate during the operating lifetime, which may not be a valid assumption in many situations, especially if there are a multiple competing failure modes.

Renewal theory, which shares many of the advantages of statistical methods, is useful in determining the variability of the warranty reserve costs. Once again, the underlying failure distribution must be determined accurately to assure the accuracy of the estimate. Without this type of information, the renewal function must be estimated, but the estimation techniques are often difficult to apply.

Goal programming techniques do not appear to have gained a wide acceptance, in spite of their ability to consider various goals. Once again, the underlying failure distribution is important, as well as the parameters chosen to represent stochastic variables. Since fixed parameters must be used to represent these random variables, the solution may also be suspect.

The method that appears to be the most difficult to use in real situations is the Markovian method. First, the transition matrix grows quickly as the numbers of components and states grow, making the method computationally impractical. Second, the probability distributions between each pair of states must be estimated. Third, costs must be collected or estimated for each transition. These difficulties severely hamper the use of this method.

A method that has not received a great deal of attention is the use of simulation to estimate warranty reserves. Since the failure rate, the cost to repair/replace and other parameters are random, simulation of these parameters may helpful in evaluating the required warranty reserves. A combination of the statistical methods with simulation techniques may provide additional insights into optimal warranty reserve policies.

CONCLUSION

The importance of accurate estimate of warranty reserves cannot be overstated. While there are a number of methods and models used for these estimates, all of the models are dependent on an accurate determination of the underlying failure distribution. Therefore, it is imperative that manufacturers keep good records of past failures and warranty claim so that this failure distribution can be known as well as possible. As consumers look to manufacturers for better quality and better service levels, warranty reserves (for replacement and repair) may become an even more important financial planning consideration.

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