ETM
ENGINEERING & TECHNOLOGY MANAGEMENT

Title: Media Allocation Model

Course Title	: Nonlinear Programming
Course Number	: EMGT 510
Instructor	: Dr. Richard F. Deckro
Term	: Fall
Year	: 1992
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ETM OFFICE USE ONLY
Report No.:
Type: Student Project
Note:

Abstract

In this paper, an effort is made to build a model capturing non-linear relationship between frequency and reach. The objective function is having number of exposure, dollar invested in each media and, cost per insertion as variables. Besides using exact non-linear relationship, quantity discount can also be included in the model.

Engineering Management Program
Nonlinear Programming
Term Research Paper
Fall - 1992

Dr. Dick Deckro

"Media Allocation Model"

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NONLINEAR PROGRAMMING
TERM RESEARCH PAPER
EGMT - 510
FALL'92

MEDIA ALLOCATION MODEL

Abstract

There has been a constant effort put-in to develop a model, that involves selection of media vehicles, which maximizes the effectiveness of the advertisement. Variables involved are the set of media vehicles and the number of insertions per vehicle.

Many papers were published in 60's & 70's on this subject. Many of them built linear programming model as an approach to optimal solution. Before the introduction of linear programming model, decisions were made on the basis of experts' opinions, experience and survey data. L.P. model also had the data input from the same source, but complex combinations and wide variety of media vehicles could be considered and analyzed. A scientific and systematic approach was made with the given data.

In this paper, an effort is made to build a model capturing non-linear relationship between frequency and reach. The objective function is having number of exposures, dollar invested in each media and, cost per insertion as variables. Besides using exact non-linear relationship, quantity discount offered can also be included in the model.

A market survey has to be done to establish the relationship between the number of exposures to number of insertions.

MEDIA ALLOCATION MODEL

Introduction

There has been a constant efforts put to develop a model that involves selection of media vehicles which maximize the effectiveness of the advertisement. The variables to be determined are the set of media vehicles and the number of insertions per vehicle. As there were many variables which could not be covered under mathematical formula, media allocation had always been a subjective decision making process. Exports intuition's and knowledge played a critical role in decision making.

There are number of models in existence. Three of the most popular models are *linear programming, simulation (14),* and *mathematical formula model (12)*. Linear programming is an optimizing model, while simulation is non-optimizing. Beta binomial is a formula model. Each model has its own advantages and disadvantages. Linear programming model will be discussed more in detail before looking into non-linear relationships.

Linear Programming

Significant break through was made when linear programming model was used for media selection model. These models were built with the linear objective function(s). In other words, these models were based on an assumption that the value of 'n' exposures to one person was equivalent to one exposure to each of 'n' persons.

Dr. Day , in 1962, led the way in presenting a linear programming model with the objective function maximizing the possible contribution, given a budget constraint (5). The model was a very simple, but novel at that time.

Maximize:	$P_1X_1 + P_2X_2 +$	$+ P_n X_n$
Subject to:	$A_{11} X_1 + A_{12} X_2 +$	$+ A_{1n} X_n < C_1$
	.	
	.	
	$A_{m1} X_1 + A_{m2} X_2 +$	$+ A_{mn} X_n < C_m$
	$X_1, X_2, \ldots\ldots\ldots, X_n > 0$	

where

X's - particular advertising vehicles considered for inclusion in the media schedule

P's - advertising values of corresponding ad. vehicles

C's - maximum possible use of some resources or similar relationship

A's - the rate at which each X uses up that particular scarce factor

n - number of variables

m - number of constraints

Later, following the same lead, many had come with advanced linear programming models. Models were updated and improved constantly. A specific media model (9) unlike Day's general one, was published later. That was closely followed by a specific audience data measuring model with adjusted audience(AA), weighted Audience (WA), and weighted exposure units (WEU) (3).

Valuable advancement took place when linear programming model was extended to Decision Programming (15), Dynamic Programming (16) (17), Goal Programming (2)(18)(19), Fuzzy Linear Programming (20) and to Multiple Objective Integer Programming (21). A methodology with hard constraints (must be satisfied) and soft constraints (want to be satisfied) had been formulated (6).

The objective function was the focal point and, it was changed to suit different ideas and new invention of:

Nonlinear integer programming models

Dynamic programming models

Reach and frequency (dual) model (4)

Analysis of Linear Programming:

Some of the major advantages include

- a mathematical basis for decision making which otherwise would have been highly subjective.
- a systematic approach for considering various options
- a quick and easy way to analyze the available alternative solutions

Some of the major disadvantages that outweigh the advantages

- the model does not take into account of reach and frequency
- Audience overlap (11) (22)
- discount structure of media vehicles
- timing of insertions
- effect of one or more type of advertisements placed in the same media
- does not address the decrease in response with the increase in advertisements
- non-integer solution values
- forgetting and carry-over effects are not considered

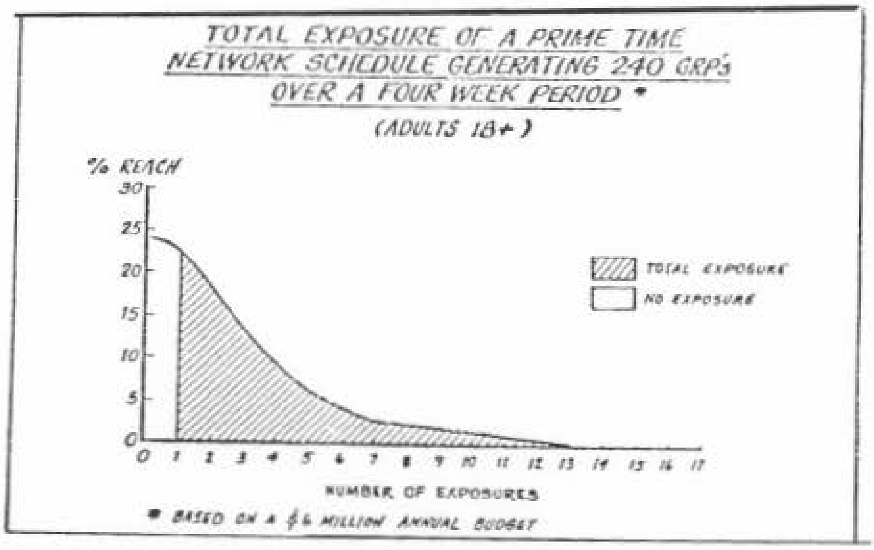
Various Operational constraints other than budget one are introduced to avoid LP loading over one vehicle. These restraints are imposed by decision maker's judgement. With decision makers' intuition, the model was led to suit the decision makers' likings. The result does not depend much on the constraints(results does not change much) and the solution by LP does not differ very much from judgmental values. Linear model is a general model and the answers should be a base for further analysis.

A model with linear objective function including number of insertions, effectiveness rating coefficient (subjective,survey), past ability to produce successful advertising readership(9), the appropriateness of the editorial climate of the medium(9), effectiveness of the size of ads., timing & space consideration (10), color or black & white print has brought Linear Programming to its saturation point.

Nonlinear response function

The hidden fact is that the multiple exposure may not have the same effects . The effectiveness keeps declining and thus forming nonlinear benefit curves. For a particular total budget constraint, there exists a trade-off relationship between optimal reach and frequency. A trade-off curve can be developed between reach and frequency for various budget constraint levels.

Figure 1



Reproduced from the book "*Effective Frequency: The Relationship between Frequency and Advertising Effectiveness*" (17).

As the aggregate response to promotional inputs is nonlinear function, the response curve is broken into various segments, then each segment is approximated to linear equation (7). Nonlinear benefit curves incorporated makes the model a nonlinear one (1).

From the figure 1, it can be noticed that the curve formed is of exponential nature with the number of Exposures in X-axis and the amount of Reach in Y-axis.

$$R = e^{-tE}$$

where

R - amount of Reach

E - number of exposures

Model Formulation

Maximize: $P_1 X_1 + P_2 X_2 + \qquad + P_n X_n$

where

P - Prospects reached by each media vehicle per dollar invested

= Reach / Cost per insertion

$= R / C = e^{-tE} / C$

X - Dollar invested in each media

Substituting 'P' in terms of 'E' (number of exposures),

$$\text{Maximize: } \frac{e^{-tE1} X_1}{C_1} + \frac{e^{-tE2} X_2}{C_2} + \qquad + \frac{e^{-tEn} X_n}{C_n}$$

where

E - Number of Exposures of a media vehicle

X - Dollar invested in each media vehicle

C - Cost per insertion

Subject to: $X_1 + X_2 + \qquad + X_n = B$

$E_1, E_2, \ldots\ldots, E_n > 3$

$E_1, E_2, \ldots\ldots, E_n < 9$

where

B - Total budget for advertising

Exposures are limited between 3 and 9 for them to be effective (17, pp- 56)

For this model to hold good, there has to be a relationship established between number of Exposures to number of Insertions. A market survey needs to be done to establish a relationship between number of Exposures to number of advertisements.

Analysis on the Model

The model presented is capturing the nonlinear response in the objective function. The cumulative Reach will be a negative exponential function. In general, the equation between the frequency(number of exposures) and the reach is considered as linear. The number of Gross Rating Point (GRP) will be equal to the product of reach and frequency, i.e., the average number of exposures.

Table 1

The concept of Total Exposure or Gross Rating Point (GRP's)

Gross Rating Point = Reach X Frequency

Gross Rating Point = Total Exposure Value

where

Reach = % of population exposed to commercial at least once in a given period of time

Frequency = Average number of times the population is exposed to a commercial during that period of time

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In reality, this inversely proportional relationship between reach and number of exposure is not true. As shown before in Fig. 1, it is an exponential function curve. The curve will be of exponential in nature with negative constant, when cumulative reach is considered with respect to number of exposures. This nonlinear relation is captured in the objective equation as it is. Another major advantage of this model is that the term cost per insertion appearing in the objective function equation, which will effectively be utilized in the case quantity discount. Each quantity discount rate can be entered separately (C_i) with the corresponding number of exposures.

Exposures are to be limited to 3 at the lower bound and 9 on the upper bound(17). Less than 3 exposure might not induce enough interest in the prospect and, at the same time more than 9 exposure will loose the capacity to draw any additional attention of the audience. What needs to be established in this model is the relation between number of exposures to number of insertions.

Figure 2

TABLE 12.4 *Weighted* **Media Evaluation Factors Obtained from $500,000 Expenditures in Major Advertising Media Categories**

Target: Female Homemakers Who Are Users of Cold Breakfast Cereals (N = 67,502,000)

Media Category	Number of Ads	Reach (1+)	Reach (3+)	Fre-quency	GRPs	CPM	CPP	CPEP
1. Local Newspapers	6	26.5	14.0	2.9	76.6	$58.82	$ 7343	$40116
2. National Newspapers	16	11.9	3.3	2.1	25.1	30.73	20742	158474
3. Newspaper Supplements	4	11.6	2.3	1.7	19.9	38.19	25780	226724
4. Magazines (Ltd. Data)	9	47.9	19.1	2.3	112.1	7.10	4792	28135
5. Network TV (Prime time)	7	29.9	11.7	2.5	74.5	11.25	7593	48300
6. Network TV (Daytime)	29	42.3	17.9	2.6	111.7	6.75	4558	28430
7. Spot TV (Prime time)	13	48.9	19.7	2.6	125.4	31.87	4118	21269
8. Spot TV (Daytime)	81	47.8	38.5	5.9	283.5	13.67	1766	13001
9. Cable TV	680	38.2	18.3	3.4	130.9	5.66	3820	27386
10. Network Radio	248	23.0	12.8	4.7	107.6	6.90	4659	39107
11. Spot Radio	546	28.7	18.7	5.6	160.5	24.18	3125	26824
12. Outdoor	30	37.4	30.4	11.8	440.3	8.51	1100	15917
13. Direct Mail	1	9.2	0.0	1.0	9.2	148.94	103838	N/A
14. Magazines (Ltd. Data)& Network Television (Ratio = .9524)	16	62.8	32.4	3.0	186.6	8.76	5910	34030

Note: The results presented in this table assume the following vehicle weights for the various media categories: Magazines, 52.5%; Network TV, Prime time, 80.0%; Network TV, Daytime, 55.0%; Spot TV, Prime time, 72.5%; Spot TV, Daytime, 50.0%; Cable TV, 62.5%; Network Radio, 40.0%; Spot Radio, 37.5%; Newspapers and Supplements, 35.0%; Outdoor Posters, 47.5%; Direct Mail, 60%. Local and spot media have been evaluated within the Top Ten U.S. Markets as defined by the *Leo Burnett 1984 Media Costs and Coverage* guide. Advertising costs are based on data presented in the *Leo Burnett* cost guide assuming 30-second broadcast commercials, full-page and four-color magazine and newspaper supplement ads and 85 SAU column inch newspaper ads. Audience data were obtained from Mediamark Research, Inc., Volume P-15, 1984, and Simmons Market Research Bureau, *1983 Survey of Media and Markets*, Volume P-20. The following individuals deserve recognition for achieving maximum effective reach, given the budget constraint, in the various media categories: Patricia J. Eckle, Edward C. Gold, Jeff A. Lamb, Sylvia M. Carlton, Lisa B. Kizer, Deborah C. Rojek and Tracey L. Tharp.

Figure 3 (13, pp.222) is the reproduction of table given in the book " Strategic Media Planning" written by Kent M. Lancaster and Helen E. Katz.

Research Areas

More complex analytical model with time effect, individual behavior factor such as exposure probabilities, purchaser's response to exposures and the effect of past purchase experiences, forgetting and recollecting effects is left open for the future research work.

Media decision makers rely on certain qualitative criteria which, may be, are related to characteristics and polices of media agency, the profile of target audience sought, and the nature of advertising product and service. This aspect of judgmental values can not be removed while taking the decision.

Conclusions

A methodology has been proposed to study the nonlinear objective function with respect to frequency and reach. The optimal solution will present the media to be advertised and the numbers of insertions, considering the quantity discount. However, the results of the model is far away from the ideal solution because of the factors that are explained under Research Areas were not considered. The model gives a back-ground idea and assists in decision making process.

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