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Title: An Optimal Printed Circuit Board Test Process Capacity Model: Applications of Linear Programming

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Abstract: With this project we define an optimal capacity model for a printed circuit board test process using Linear Programming techniques. An example process was studied and characterized to form constraints for the model with the objective to minimize the Work-in-Process inventory carried within the operation. Sensitivity analysis was performed on the completed model to identify critically limited resources, and possible solutions were identified. Introduction of a new product and the impact on the process were studied. Due to the limitations of Linear Programming modeling, we concluded that Integer or Goal Programming warranted investigation for their applicability to the problem.

# APPLICATIONS OF LINEAR PROGRAMMING
# PRINTED CICUIT BOARD TEST PROCESS CAPACITY MODEL

B. Leichner, H. Wong, S. Walsh

**EMP-P9107**

# APPLICATIONS OF LINEAR PROGRAMMING

# PRINTED CIRCUIT BOARD TEST PROCESS CAPACITY MODEL


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# ABSTRACT

The intent of this project was to define an optimal capacity model for a Printed Circuit Board Test Process using Linear Programming techniques. An example process was studied and characterized to form constraints for the model with the objective to minimize the Work-in-Process inventory carried within the operation. Sensitivity analysis was performed on the completed model to identify critically limited resources and possible solutions were identified. Introduction of a new product and the impact on the process were studied. Due to the limitations of Linear Programming modelling, it was concluded that Integer or Goal Programming warranted investigation for their applicability to the problem.

## EXECUTIVE SUMMARY

The intent of this project was to define an optimal capacity model for a Printed Circuit Board Test Process using Linear Programming techniques. The team selected an example process and studied the characteristics of product flow through each step of the process. From these studies, constraints were developed that defined the main test functions: In-Circuit Test, Environmental Stress and Functional Test. It should be noted that many of the real conditions of the operation we studied could not be easily input using the Linear Programming tools available and certain 'rationalizations' had to be made. Minimizing Work-in-Process inventory was defined as the objective of the model, based on production and test cost of the products. The model was then input into the LINDO software package and a solution generated. Using LINDO, sensitivity analysis was performed on the model and Test 3 was identified as a critical resource with very little slack available for unexpected increases in production demands or tester downtime. Possible solutions to this problem and their effects were studied in a second model. Also of interest to the team was the introduction of a new product to the process. An example product was formulated and inserted in the original model and the effects this had on the test process studied and analyzed. In conclusion, we feel that the limitations of Linear Programming and it's inability to handle the real process constraints would lead us to either develop a Goal Program for this process or use Integer Programming for a more accurate model of the process.

## PROBLEM DEFINITION

The system modeled is a printed circuit assembly (PCA) test process. Boards are built on an assembly line within the factory and then moved into the test process for evaluation and defect identification. This operation tests three board types, identified as A, B and C. The test operations run two shifts per day, four days per week of 10 hours each.

The first test operation is an In-Circuit Test (ICT) which identifies any incorrect open or short circuits, checks electrical devices for the expected correct value and programmed devices for the correct codes. The test time and operation times (time for the operator to input data about the tests) are divided by the yield factor (percent of passing boards) to give a total run time for each board. Test duration times for this step are:

| Board | Test Time | Opn. Time | Yield | Total Time |
|---|---|---|---|---|
| A | 5 min | 1.5 min | 88% | .12 hr |
| B | 7 min | 1.5 min | 95% | .15 hr |
| C | 5.5 min | 1.5 min | 90% | .13 hr |

There is only one In-Circuit Tester and the test is operator attended, so the Right Hand Side (RHS) for this operation's constraint is limited to 80 hours. The constraint multiplied by the volume of boards and limited so it cannot exceed the total time available is:

$$0.12\ A + 0.15\ B + 0.13\ C <= 80$$

The second step in the PCA test area is a Environmental Stress test. This is performed in a chamber that heats and cools the boards very rapidly for a given number of cycles. The number of cycles required for an effective test can vary depending on the maturity of the product's design and the failure rates of the boards over time. Boards A and B require 15 cycles each while board C has been around a while and is a very mature product, displaying few failures over time and only requires 8 cycles.

There is only one chamber and it is limited to about 2400 board/cycles per week. It is common practice that when boards fail further along in the test process they are put back in the chamber for re-stressing. Consequently, we want to reserve a minimum of time for production of new product, yet leave some time for repair work as well, usually about 600 cycles.

The constraints for this step, with coefficients in cycles per board, are:

$$15\,A + 15\,B + 8\,C \geq 1500$$

$$15\,A + 15\,B + 8\,C \leq 1800$$

This allows for a minimum of 1500 production cycles, but a maximum of 1800. It is also good practice to operate this piece of equipment continuously, so the 1500 cycle minimum helps in this criteria as well.

The second and third operations actually test the board's functionality and can be quite long in duration. These tests are operator attended and have only one station per board type available. Test times are calculated similar to the ICT tests, factoring in the board yield to allow for the failure rate; the coefficients are in hours of test per board.

Constraints for Tests 2 and 3 are:

Test 2: $0.55\,A + 0.85\,B + 0.75\,C \leq 80$

Test 3: $0.7\,A + 0.45\,B + 0.35\,C \geq 30$

$0.7\,A + 0.45\,B + 0.35\,C \leq 80$

Test station 3 is often used to evaluate failures in the process as well as customer returns from the field and is often impacted heavily by these demands. To ensure that new production boards always have priority, a lower limit has been established to guarantee that a certain level of production boards are tested on these machines.

Board volume is preset by forecasting the demand for the coming period. This is usually given with two figures, an 'at least' number and a 'not more than' number. The current forecast numbers for boards A, B and C are:

30 >= A <= 45

45 >= B <= 50

25 >= C <= 30

When demand reaches or exceeds these upper limits, the factory is allowed to work overtime to achieve the demand.

The objective of the model is to minimize Work-in-Process (WIP) inventory yet still meet the factory constraints and forecast expectations. This is dependent on the manufacture and test cost of the boards; A = $775, B = $650, C = $525.

Which translates to the following objective function:

MIN 775 A + 650 B + 525 C

PROBLEM MODEL

The finished model input in LINDO format and solved using LINDO on a desktop computer:

```
MIN      775 A + 650 B + 525 C
SUBJECT TO
       2)   0.12 A + 0.15 B + 0.13 C <=   80
       3)   15 A + 15 B + 8 C >=   1500
       4)   15 A + 15 B + 8 C <=   1800
       5)   0.55 A + 0.85 B + 0.75 C <=   80
       6)   0.7 A + 0.45 B + 0.35 C >=   30
       7)   0.7 A + 0.45 B + 0.35 C <=   80
       8)   A >=   30
       9)   A <=   45
      10)   B >=   45
      11)   B <=   50
      12)   C >=   25
      13)   C <=   30
END

LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)     74631.9500

 VARIABLE        VALUE          REDUCED COST
        A      41.388890           .000000
        B      45.277770           .000000
        C      25.000000           .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)      64.991670           .000000
       3)        .000000        -66.944440
       4)     300.000000           .000000
       5)        .000000        416.666700
       6)      28.097220           .000000
       7)      21.902780           .000000
       8)      11.388890           .000000
       9)       3.611106           .000000
      10)        .277773           .000000
      11)       4.722228           .000000
      12)        .000000       -301.944500
      13)       5.000000           .000000
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

| VARIABLE | CURRENT COEF | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| A | 775.000000 | INFINITY | 125.000000 |
| B | 650.000000 | 125.000000 | INFINITY |
| C | 525.000000 | INFINITY | 301.944500 |

RIGHTHAND SIDE RANGES

| ROW | CURRENT RHS | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| 2 | 80.000000 | INFINITY | 64.991670 |
| 3 | 1500.000000 | 2.272685 | 38.636410 |
| 4 | 1800.000000 | INFINITY | 300.000000 |
| 5 | 80.000000 | 1.416668 | .083332 |
| 6 | 30.000000 | 28.097220 | INFINITY |
| 7 | 80.000000 | INFINITY | 21.902780 |
| 8 | 30.000000 | 11.388890 | INFINITY |
| 9 | 45.000000 | INFINITY | 3.611106 |
| 10 | 45.000000 | .277773 | INFINITY |
| 11 | 50.000000 | INFINITY | 4.722228 |
| 12 | 25.000000 | .182478 | 3.102193 |
| 13 | 30.000000 | INFINITY | 5.000000 |

The solution implies that our test line should average 41.4 units of board A, 45.3 units of board B and 25 units of board C each week for an average WIP value of $74,631.95.

This solution requires testing towards the high end of the forecast for board A, nearly the minimum of board B and the absolute minimum of board C. There will probably be excess capacity to handle the possible 4 or 5 unit increase in demand for board A, but the model should be examined for sensitivity to increases in demand for the other boards and resource limitations that might impair the test process capability to meet higher demands.

SENSITIVITY ANALYSIS

The most interesting of our resources are the ICT and Test 2 resources which do not have upper and lower bounds that restrict us from changing much.

| ROW | SLACK OR SURPLUS | DUAL PRICES |
|---|---|---|
| 2) | 64.991670 | .000000 |
| 5) | .000000 | 416.666700 |

RIGHTHAND SIDE RANGES

| ROW | CURRENT RHS | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| 2 | 80.000000 | INFINITY | 64.991670 |
| 5 | 80.000000 | 1.416668 | .083332 |

When we examine Resource 2 in the LINDO output, corresponding to ICT, we note that there is a significant amount of slack available ( 65 hr). With an average of only .13 hr required to test a board, there is probably enough slack to handle a production increase of more than 450 boards per week.

The absence of any slack on Resource 5 (corresponding to Test 2) and the high dual price for the constraint indicate a unit (hour) increase is worth investigating because it can reduce the objective function cost by $416. When we check the RHS range, we see that this decrease is good only for the first 1.42 hours added, to the range limit of 81.42 hours. Because these tests are performed on specialized pieces of custom built equipment, the options are to add an additional tester or to find some way of incrementally increasing the test capacity. Another test station could be built, doubling capacity, but this cost would be prohibitive and would result in a significant amount of idle capacity at a very high cost.

Even though we cannot justify doubling the capacity of Test 2, we cannot ignore the problem of zero slack. Serious capacity issues will result if there is any unexpected increase in production schedules, board defect rates or tester downtime. Any increases here move us immediately into the solution of adding overtime to the test area. One alternative is to increase the capability of the existing test station so that the tests will run faster. For example, increase computing power on the system to speed the tests up by 25%. In the following LINDO problem output, we have shown the effect of this enhancement:

```
MIN      775 A + 650 B + 525 C
SUBJECT TO
        2)    0.12 A + 0.15 B + 0.13 C <=   80
        3)    15 A + 15 B + 8 C >=    1500
        4)    15 A + 15 B + 8 C <=    1800
        5)    0.4 A + 0.65 B + 0.55 C <=   80   <-- 25% less
        6)    0.7 A + 0.45 B + 0.35 C >=   30     than original
        7)    0.7 A + 0.45 B + 0.35 C <=   80     problem
        8)    A >=    30
        9)    A <=    45
       10)    B >=    45
       11)    B <=    50
       12)    C >=    25
       13)    C <=    30

LP OPTIMUM FOUND AT STEP       4
```

OBJECTIVE FUNCTION VALUE

1) 74041.6600

| VARIABLE | VALUE | REDUCED COST |
|---|---|---|
| A | 36.666670 | .000000 |
| B | 50.000000 | .000000 |
| C | 25.000000 | .000000 |

While this enhancement to Test 2 only reduces weekly WIP by $600, note the increase in slack for constraint 5:

| ROW | SLACK OR SURPLUS | DUAL PRICES |
|---|---|---|
| 5) | 19.083330 | .000000 |

We now have sufficient capacity to handle an unexpected increase of about 26 boards per week without requiring overtime, based on the average test times for all boards. The costs of adding this additional capacity would have to balance favorably with those of the expected overtime that may be required if 26 more boards had been added to the original solution.

## PARAMETRIC ANALYSIS

We discovered that changing the objective function coefficients does not change the basis. The value of the objective function is directly and linearly proportional to the objective function coefficients. Parametric programming of these coefficients is therefore uninteresting.

The same applies to the ICT test. Some interesting information that can be extracted out of parametrically programming this constraint is the large tolerance of variation in board yields. There was so much slack in the machine constraint that yields of less than 10% had no effect on the basis.

Parametric analysis performed on the constraint equations for the Environmental Stress cycles (Row 3) reveals some interesting facts. As shown in the LINDO output, there is no surplus for this constraint; the 1500 cycles must be completely used. The cycle times for boards A, B, and C cannot be decreased from their present values without the solution becoming infeasible. As shown in Table 1, increasing the number of cycles for one board while keeping the other two constant tends to lower the required number of boards and the overall WIP value. Beyond a certain point, increasing the number of cycles has no effect on the solution. These values are, 20 for Board A, 18 for Board B, and 13 for Board C. If the cycle counts are increased a certain amount beyond this point, the upper bound of this process (Row 4) becomes a factor and causes the solution to become infeasible. This point is 30 cycles for Board A, 25 cycles for Board B, and 27 cycles for Board C.

Table 1 - Parametric Programming

Number of Cycles per board for Environmental Stress Constraint 3

| | -1 | 0 | 3 | 5 | 10 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|
| Board A | | | | | | | | |
| # Boards | * | 41.4 | 30 | 30 | 30 | 30 | 30 | 30 |
| Objective | * | 74631 | 69284 | 66593 | 65625 | 65625 | 65625 | 65625 |
| Board B | | | | | | | | |
| # Boards | * | 45.3 | 47.2 | 45 | 45 | 45 | 45 | * |
| Objective | * | 73631 | 67069 | 65625 | 65625 | 65625 | 65625 | * |
| Board C | | | | | | | | |
| # Boards | * | 25 | 27.3 | 26.7 | 25 | 25 | 25 | * |
| Objective | * | 74631 | 70068 | 66562 | 65625 | 65625 | 65625 | * |

Note: infeasibility denoted by "*"

The cycle count values beyond which the solution is unchanging are 20 for board A, 18 for board B, and 13 for Board C. The solutions become unfeasible when the count exceeds 30 for Board A, 25 for Board B, and 27 for Board C

Parametric analysis performed on constraint equations for the Test 2 shows changes in the amount of test time do not have an effect on the solution when lowered, but can make the problem infeasible if raised. The problem with raising the technological coefficients in this test was that the number of board type C would soon go to zero if the test time was increased (Reference Table 2).

Parametric analysis performed on the technological coefficients for constraints 6 and 7 (Test 3) showed that variation by +/- 100% in this test had very little effect on the solution other than to cause it to become infeasible. Test 3 for Board A could vary about +/- 50% without causing the solution to become infeasible. The basis does not change over this range. Any variation greater than this causes the problem to become infeasible. The test times for both Board B and C could vary by +/- 100% without causing the solution to become infeasible or the basis to change.

Table 2 - Parametric Programming

Percentage Change in Test Time for Test 2 - Constraint 5

| | -100% | -50% | -25% | 0% | 25% | 50% | 100% |
|---|---|---|---|---|---|---|---|
| Board A | | | | | | | |
| # Boards | 36 | 36 | 36 | 41 | 45 | 45 | * |
| Objective | 74941 | 74941 | 74941 | 74631 | 72160 | 67727 | |
| Board B | | | | | | | |
| # Boards | 50 | 50 | 50 | 45 | * | * | * |
| Objective | 74941 | 74941 | 74941 | 74631 | | | |
| Board C | | | | | | | |
| # Boards | 36 | 36 | 36 | 25 | 17 | * | * |
| Objective | 74941 | 74941 | 74941 | 74631 | 73755 | | |

Note: infeasibility denoted by "*"

EXTENSIONS

The addition of a new product to the model was also investigated. Typically, new products are initially built at lower volumes and have slightly higher production costs than more mature products. For these reasons, the cost of board D is set at $895. Also, overall test times can run longer due to the lower yields expected from an immature product. Test times for ICT, Test 2 and Test 3 for board D are set at .15 hours, .7 hours and .65 hours respectively. Also, the new board will require many more Environmental Stress cycles, this value is set to 24. The forecast for the new board is expected to be at least 7 units per week but not to exceed 15. This results in our new model:

```
MIN      775 A + 650 B + 550 C + 900 D
SUBJECT TO
        2)   0.12 A + 0.15 B + 0.13 C + 0.15 D <=   80
        3)   15 A + 15 B + 8 C + 24 D >=   1500
        4)   15 A + 15 B + 8 C + 24 D <=   1800
        5)   0.55 A + 0.85 B + 0.75 C + 0.7 D <=   80
        6)   0.7 A + 0.45 B + 0.35 C + 0.65 D >=   30
        7)   0.7 A + 0.45 B + 0.35 C + 0.65 D <=   80
        8)   A >=   30
        9)   A <=   45
       10)   B >=   45
       11)   B <=   65
       12)   C >=   25
       13)   C <=   40
       14)   D >=   7
       15)   D <=   15
```

Four board model solution:

OBJECTIVE FUNCTION VALUE

1) 72812.5000

| VARIABLE | VALUE | REDUCED COST |
|---|---|---|
| A | 30.000000 | .000000 |
| B | 45.000000 | .000000 |
| C | 25.000000 | .000000 |
| D | 7.291667 | .000000 |

The new objective function value is actually slightly lower than the original optimal value of $74,631.95, which implies that the introduction of the new board actually reduces the overall WIP cost for the operation. This is because 7.3 units of the new board consume more time in the Environmental Stress and on Test 3, resources with lower bounds, and take up the time that the large amount of the relatively inexpensive board A did previously (down to 30 units from 41.4 units). A much more careful look at resource capacity should be done if the new board is to be introduced, because all of the former products' volumes are set at the lower limits of production now and there is potential for problems if any of these demands come in higher than this.

# CONCLUSIONS

While examining the real production system and translating that into the Linear Program model, reality often dictated 'or' conditions and 'if-then' conditions that were impossible to model using linear programming techniques. The Environmental Stress chamber was inoperable with certain mixes of boards. Some boards could not be tested without another particular board in the system at the same time. In these cases, the hard facts of reality had to be changed or ignored to fit the ability of the tools to handle the constraints. In order to stay closer to reality, Integer Programming might have been a better choice than Linear Programming in this case.

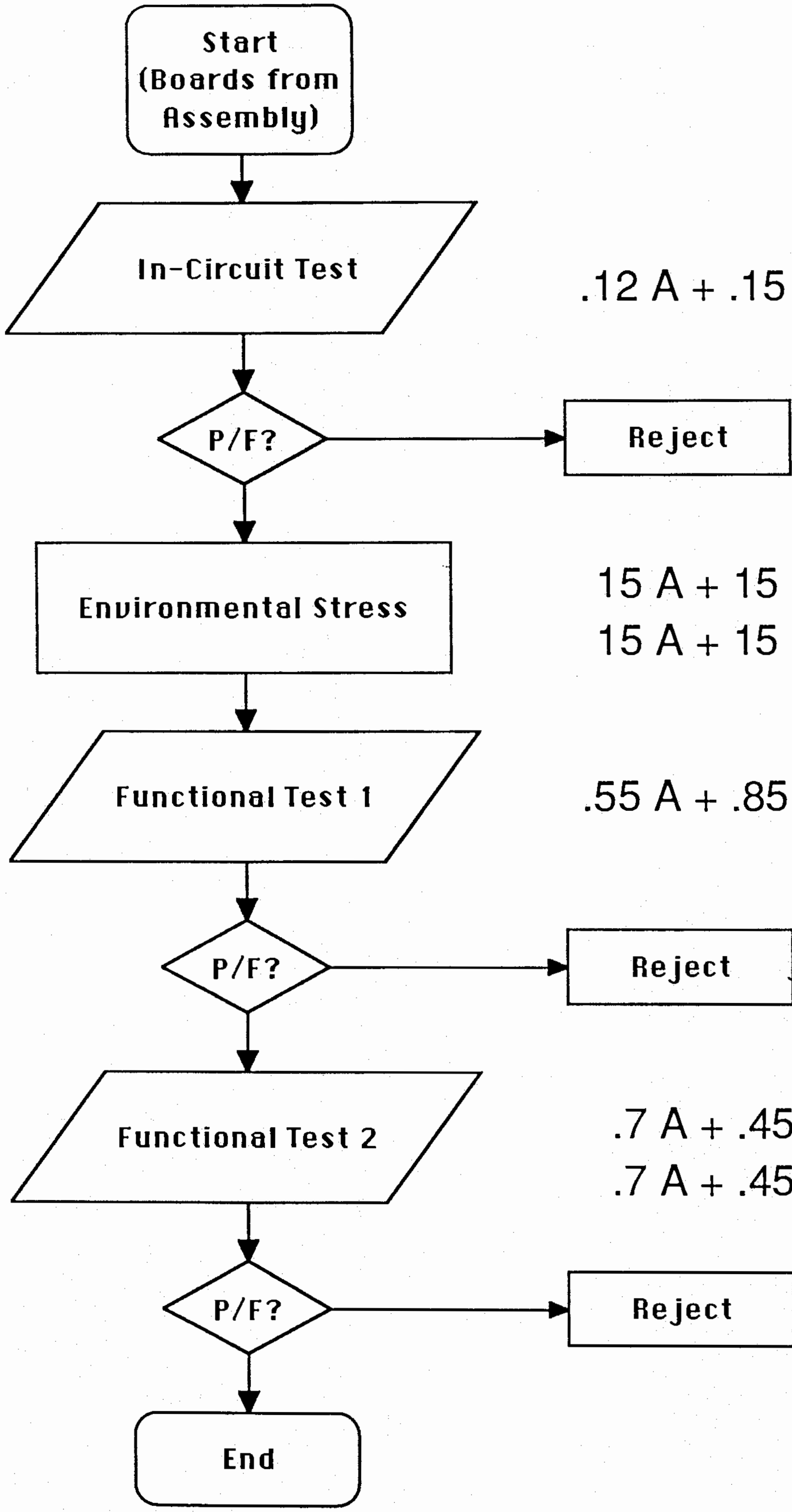
Another possible solution might have been to use Goal Programming. This method might have been better suited to actual manufacturing and production decisions. Using Goal Programming would have showed a range of acceptable solutions which could be selected from based on the immediate production conditions. This type of information is often preferred in the world of manufacturing, where the optimal solution in planning can easily fall apart when shortages occur or expedited orders are introduced.

# APPENDIX A

Flowchart of PCA test operation.

3 Board types:

A , B , C




In-Circuit Test:

$$.12\,A + .15\,B + .13\,C \;<=\; 80$$

Environmental Stress:

$$15\,A + 15\,B + 8\,C \;<=\; 1800$$
$$15\,A + 15\,B + 8\,C \;>=\; 1500$$

Functional Test 1:

$$.55\,A + .85\,B + .75\,C \;<=\; 80$$

Functional Test 2:

$$.7\,A + .45\,B + .35\,C \;<=\; 80$$
$$.7\,A + .45\,B + .35\,C \;>=\; 30$$

# APPENDIX B

LINDO input files and problem solution output.

LINDO 'take' file for generating original problem analysis

```
MIN      775 A + 650 B + 525 C
SUBJECT TO
       2)    0.12 A + 0.15 B + 0.13 C <=    80
       3)    15 A + 15 B + 8 C >=    1500
       4)    15 A + 15 B + 8 C <=    1800
       5)    0.55 A + 0.85 B + 0.75 C <=    80
       6)    0.7 A + 0.45 B + 0.35 C >=    30
       7)    0.7 A + 0.45 B + 0.35 C <=    80
       8)    A >=    30
       9)    A <=    45
      10)    B >=    45
      11)    B <=    50
      12)    C >=    25
      13)    C <=    30
END
LEAVE
```

```
MIN      775 A + 650 B + 525 C
SUBJECT TO
       2)   0.12 A + 0.15 B + 0.13 C <=    80
       3)   15 A + 15 B + 8 C >=   1500
       4)   15 A + 15 B + 8 C <=   1800
       5)   0.55 A + 0.85 B + 0.75 C <=    80
       6)   0.7 A + 0.45 B + 0.35 C >=   30
       7)   0.7 A + 0.45 B + 0.35 C <=   80
       8)   A >=   30
       9)   A <=   45
      10)   B >=   45
      11)   B <=   50
      12)   C >=   25
      13)   C <=   30
END

LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)     74631.9500

 VARIABLE        VALUE          REDUCED COST
        A        41.388890          .000000
        B        45.277770          .000000
        C        25.000000          .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)        64.991670          .000000
       3)          .000000       -66.944440
       4)       300.000000          .000000
       5)          .000000       416.666700
       6)        28.097220          .000000
       7)        21.902780          .000000
       8)        11.388890          .000000
       9)         3.611106          .000000
      10)          .277773          .000000
      11)         4.722228          .000000
      12)          .000000      -301.944500
      13)         5.000000          .000000

NO. ITERATIONS=       6
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

| VARIABLE | CURRENT COEF | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| A | 775.000000 | INFINITY | 125.000000 |
| B | 650.000000 | 125.000000 | INFINITY |
| C | 525.000000 | INFINITY | 301.944500 |

RIGHTHAND SIDE RANGES

| ROW | CURRENT RHS | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| 2 | 80.000000 | INFINITY | 64.991670 |
| 3 | 1500.000000 | 2.272685 | 38.636410 |
| 4 | 1800.000000 | INFINITY | 300.000000 |
| 5 | 80.000000 | 1.416668 | .083332 |
| 6 | 30.000000 | 28.097220 | INFINITY |
| 7 | 80.000000 | INFINITY | 21.902780 |
| 8 | 30.000000 | 11.388890 | INFINITY |
| 9 | 45.000000 | INFINITY | 3.611106 |
| 10 | 45.000000 | .277773 | INFINITY |
| 11 | 50.000000 | INFINITY | 4.722228 |
| 12 | 25.000000 | .182478 | 3.102193 |
| 13 | 30.000000 | INFINITY | 5.000000 |

LINDO 'take' file for generating second problem analysis with 25% improved efficiency on resource 5 (Test 3)

---

```
MIN      775 A + 650 B + 525 C
SUBJECT TO
       2)    0.12 A + 0.15 B + 0.13 C <=    80
       3)    15 A + 15 B + 8 C >=    1500
       4)    15 A + 15 B + 8 C <=    1800
       5)    0.4 A + 0.65 B + 0.55 C <=    80
       6)    0.7 A + 0.45 B + 0.35 C >=    30
       7)    0.7 A + 0.45 B + 0.35 C <=    80
       8)    A >=    30
       9)    A <=    45
      10)    B >=    45
      11)    B <=    50
      12)    C >=    25
      13)    C <=    30
END
LEAVE
```

```
MIN      775 A + 650 B + 525 C
SUBJECT TO
       2)    0.12 A + 0.15 B + 0.13 C <=    80
       3)    15 A + 15 B + 8 C >=   1500
       4)    15 A + 15 B + 8 C <=   1800
       5)    0.4 A + 0.65 B + 0.55 C <=    80
       6)    0.7 A + 0.45 B + 0.35 C >=    30
       7)    0.7 A + 0.45 B + 0.35 C <=    80
       8)    A >=   30
       9)    A <=   45
      10)    B >=   45
      11)    B <=   50
      12)    C >=   25
      13)    C <=   30
END

LP OPTIMUM FOUND AT STEP       4

       OBJECTIVE FUNCTION VALUE

       1)     74041.6600

 VARIABLE        VALUE          REDUCED COST
        A       36.666670          .000000
        B       50.000000          .000000
        C       25.000000          .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)       64.850000          .000000
       3)         .000000       -51.666670
       4)      300.000000          .000000
       5)       19.083330          .000000
       6)       26.916670          .000000
       7)       23.083330          .000000
       8)        6.666667          .000000
       9)        8.333333          .000000
      10)        5.000000          .000000
      11)         .000000       125.000000
      12)         .000000      -111.666700
      13)        5.000000          .000000

NO. ITERATIONS=       4
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

| VARIABLE | CURRENT COEF | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| A | 775.000000 | 209.375000 | 125.000000 |
| B | 650.000000 | 125.000000 | INFINITY |
| C | 525.000000 | INFINITY | 111.666700 |

RIGHTHAND SIDE RANGES

| ROW | CURRENT RHS | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| 2 | 80.000000 | INFINITY | 64.850000 |
| 3 | 1500.000000 | 125.000000 | 99.999990 |
| 4 | 1800.000000 | INFINITY | 300.000000 |
| 5 | 80.000000 | INFINITY | 19.083330 |
| 6 | 30.000000 | 26.916670 | INFINITY |
| 7 | 80.000000 | INFINITY | 23.083330 |
| 8 | 30.000000 | 6.666667 | INFINITY |
| 9 | 45.000000 | INFINITY | 8.333333 |
| 10 | 45.000000 | 5.000000 | INFINITY |
| 11 | 50.000000 | 6.666667 | 5.000000 |
| 12 | 25.000000 | 5.000000 | 15.625000 |
| 13 | 30.000000 | INFINITY | 5.000000 |

LINDO 'take' file for generating third problem analysis with additional board "D" included

---

```
MIN      775 A + 650 B + 525 C + 895 D
SUBJECT TO
        2)    0.12 A + 0.15 B + 0.13 C + 0.15 D <=    80
        3)    15 A + 15 B + 8 C + 24 D >=    1500
        4)    15 A + 15 B + 8 C + 24 D <=    1800
        5)    0.55 A + 0.85 B + 0.75 C + 0.7 D <=   80
        6)    0.7 A + 0.45 B + 0.35 C + 0.65 D >=   30
        7)    0.7 A + 0.45 B + 0.35 C + 0.65 D <=   80
        8)    A >=   30
        9)    A <=   45
       10)    B >=   45
       11)    B <=   50
       12)    C >=   25
       13)    C <=   30
       14)    D >=   7
       15)    D <=   15
END
LEAVE
```

```
MIN      775 A + 650 B + 525 C + 895 D
SUBJECT TO
        2)    0.12 A + 0.15 B + 0.13 C + 0.15 D <=    80
        3)    15 A + 15 B + 8 C + 24 D >=   1500
        4)    15 A + 15 B + 8 C + 24 D <=   1800
        5)    0.55 A + 0.85 B + 0.75 C + 0.7 D <=   80
        6)    0.7 A + 0.45 B + 0.35 C + 0.65 D >=   30
        7)    0.7 A + 0.45 B + 0.35 C + 0.65 D <=   80
        8)    A >=   30
        9)    A <=   45
       10)    B >=   45
       11)    B <=   50
       12)    C >=   25
       13)    C <=   30
       14)    D >=   7
       15)    D <=   15
END

LP OPTIMUM FOUND AT STEP       7

       OBJECTIVE FUNCTION VALUE

       1)     72151.0400

 VARIABLE        VALUE          REDUCED COST
        A        30.000000           .000000
        B        45.000000           .000000
        C        25.000000           .000000
        D         7.291667           .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)        65.306250           .000000
       3)          .000000        -37.291670
       4)       300.000000           .000000
       5)         1.395832           .000000
       6)        24.739580           .000000
       7)        25.260420           .000000
       8)          .000000       -215.625000
       9)        15.000000           .000000
      10)          .000000        -90.625000
      11)         5.000000           .000000
      12)          .000000       -226.666700
      13)         5.000000           .000000
      14)          .291667           .000000
      15)         7.708333           .000000

NO. ITERATIONS=       7
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

| VARIABLE | CURRENT COEF | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| A | 775.000000 | INFINITY | 215.625000 |
| B | 650.000000 | INFINITY | 90.625000 |
| C | 525.000000 | INFINITY | 226.666700 |
| D | 895.000000 | 145.000000 | 895.000000 |

RIGHTHAND SIDE RANGES

| ROW | CURRENT RHS | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| 2 | 80.000000 | INFINITY | 65.306250 |
| 3 | 1500.000000 | 47.857090 | 7.000000 |
| 4 | 1800.000000 | INFINITY | 300.000000 |
| 5 | 80.000000 | INFINITY | 1.395832 |
| 6 | 30.000000 | 24.739580 | INFINITY |
| 7 | 80.000000 | INFINITY | 25.260420 |
| 8 | 30.000000 | .466667 | 12.333330 |
| 9 | 45.000000 | INFINITY | 15.000000 |
| 10 | 45.000000 | .466667 | 12.333330 |
| 11 | 50.000000 | INFINITY | 5.000000 |
| 12 | 25.000000 | .875000 | 23.125000 |
| 13 | 30.000000 | INFINITY | 5.000000 |
| 14 | 7.000000 | .291667 | INFINITY |
| 15 | 15.000000 | INFINITY | 7.708333 |

LINDO 'take' file for generating fourth problem analysis with new product "D" added and increased capacity for resource 5

---

```
MIN      775 A + 650 B + 525 C + 895 D
SUBJECT TO
        2)    0.12 A + 0.15 B + 0.13 C + 0.15 D <=    80
        3)    15 A + 15 B + 8 C + 24 D >=    1500
        4)    15 A + 15 B + 8 C + 24 D <=    1800
        5)    0.4 A + 0.65 B + 0.55 C + 0.55 D <=    80
        6)    0.7 A + 0.45 B + 0.35 C + 0.65 D >=    30
        7)    0.7 A + 0.45 B + 0.35 C + 0.65 D <=    80
        8)    A >=    30
        9)    A <=    45
       10)    B >=    45
       11)    B <=    50
       12)    C >=    25
       13)    C <=    30
       14)    D >=    7
       15)    D <=    15
END
LEAVE
```

```
MIN      775 A + 650 B + 525 C + 895 D
SUBJECT TO
        2)   0.12 A + 0.15 B + 0.13 C + 0.15 D <=    80
        3)   15 A + 15 B + 8 C + 24 D >=    1500
        4)   15 A + 15 B + 8 C + 24 D <=    1800
        5)   0.4 A + 0.65 B + 0.55 C + 0.55 D <=    80
        6)   0.7 A + 0.45 B + 0.35 C + 0.65 D >=    30
        7)   0.7 A + 0.45 B + 0.35 C + 0.65 D <=    80
        8)   A >=    30
        9)   A <=    45
       10)   B >=    45
       11)   B <=    50
       12)   C >=    25
       13)   C <=    30
       14)   D >=    7
       15)   D <=    15
 END

 LP OPTIMUM FOUND AT STEP        7

        OBJECTIVE FUNCTION VALUE

        1)      72151.0400

  VARIABLE        VALUE          REDUCED COST
         A        30.000000           .000000
         B        45.000000           .000000
         C        25.000000           .000000
         D         7.291667           .000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        65.306250           .000000
        3)          .000000       -37.291670
        4)       300.000000           .000000
        5)        20.989580           .000000
        6)        24.739580           .000000
        7)        25.260420           .000000
        8)          .000000      -215.625000
        9)        15.000000           .000000
       10)          .000000       -90.625000
       11)         5.000000           .000000
       12)          .000000      -226.666700
       13)         5.000000           .000000
       14)          .291667           .000000
       15)         7.708333           .000000

 NO. ITERATIONS=         7
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

| VARIABLE | CURRENT COEF | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| A | 775.000000 | INFINITY | 215.625000 |
| B | 650.000000 | INFINITY | 90.625000 |
| C | 525.000000 | INFINITY | 226.666700 |
| D | 895.000000 | 145.000000 | 895.000000 |

RIGHTHAND SIDE RANGES

| ROW | CURRENT RHS | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
|---|---|---|---|
| 2 | 80.000000 | INFINITY | 65.306250 |
| 3 | 1500.000000 | 185.000000 | 7.000000 |
| 4 | 1800.000000 | INFINITY | 300.000000 |
| 5 | 80.000000 | INFINITY | 20.989580 |
| 6 | 30.000000 | 24.739580 | INFINITY |
| 7 | 80.000000 | INFINITY | 25.260420 |
| 8 | 30.000000 | .466667 | 12.333330 |
| 9 | 45.000000 | INFINITY | 15.000000 |
| 10 | 45.000000 | .466667 | 12.333330 |
| 11 | 50.000000 | INFINITY | 5.000000 |
| 12 | 25.000000 | .875000 | 23.125000 |
| 13 | 30.000000 | INFINITY | 5.000000 |
| 14 | 7.000000 | .291667 | INFINITY |
| 15 | 15.000000 | INFINITY | 7.708333 |