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Title: Minimizing Cost in a Crashed Schedule

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Abstract: The purpose of this paper is to find the real engineering application using discrete linear programming approach.

"MINIMIZING COST IN A CRASHED SCHEDULE"

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CONTENTS

I. INTRODUCTION:

This paper is made to fullfil the requirement for EMGT 505 course. The purpose of this paper is to find the real engineering application using discrete linear programming approach.

A. CASE BACKGROUND

A fabrication company, Vessel Fabricator Inc. (VBI) just received a contract to fabricate a production separator to be installed in an offshore production platform. VBI has been a good fabricator of vessels especially for offshore industry.

For fabricating a typical vessel, it usually takes for 30 to 60 days, depending on the design/specification, size, and material of the vessel. The schedule of the vessel fabrication is given in Appendix 1. The durations are given in hours.

However, VBI is requested by the owner to fabricate a vessel within 25 days. Since this time-frame is not possible under normal conditions, VBI must consider ways to reduce the project duration (under a crash schedule) so that the owner's request can be fulfilled. Some possibilities are to assign more workers and pay more to sub-contractors e.g., in-house non-destructive test (NDT). This will increase the cost of the fabrication. The increase cost per task is given in Table 1, according to the management analysis. For example, task number 05 takes 64 hours to accomplish under normal schedule and costs $6,400. Under the crash schedule, task number 05 costs $8,800, an additional cost of $2,400, and has completion of 40 hours. VBI analysis indicates that task number 02 can not be shortened.

VBI wants to determine the minimum cost under a crash schedule. The firm is interested in working under a crash schedule provided the request of 25 days is feasible within their other operational constraints.

TABLE 1

TASK	NORMAL SCHED.		CRASH SCHED.		HRS	ADD'L
				EXTRA		
NO.	HRS	COST	HRS	COST	SAVED	COST/HR
01	16	640	12	300	4	75
02	8	320	-	-	-	-
03	40	2,400	24	2,400	16	150
04	24	1,440	16	800	8	100
05	64	6,400	40	1,800	24	75
06	16	640	12	480	8	60
07	24	1,920	16	1,200	8	150
08	16	1,280	12	1,200	8	150
09	16	1,600	12	800	8	100
10	40	4,000	24	2,000	16	125
11	80	6,400	56	2,400	24	100
12	16	640	12	300	4	75
13	80	8,000	48	4,800	32	150
14	16	960	8	1,600	8	200
15	32	2,560	20	2,400	12	200
16	24	3,360	12	3,000	12	250
17	32	3,840	24	2,000	8	250

B. ASSUMPTIONS:

1. EITHER NO PENALTY FOR LATE SCHEDULE OR NO BONUS FOR EARLY SCHEDULE.
2. THE SCHEDULE SEQUENCE CAN NOT BE ALTERED.
3. THE WORKING HOURS ARE LIMITED TO 8 HOURS PER DAY, WITH NO OVERTIME PROVIDED FOR THIS PROJECT.
4. NO WEEK-END RATE AND NO HOLIDAY OCCURS DURING THE FABRICATION.
5. NEGLECTS ENGINEERS' AND SUPERVISORS' EXTRA PAY AND ADDITIONAL OVERHEAD COSTS.
6. TIME SCALE IS IN 1-HOUR.
7. UNIFORM PAY FOR WORKERS IN ORDER TO DECREASE THE NUMBER OF VARIABLES AND CONSTANTS IN THIS MODEL.
8. MANPOWER AND EQUIPMENT ARE AVAILABLE FOR EITHER NORMAL OR CRASH SCHEDULES.
9. EXTRA COST IS DERIVED FROM MORE WORKERS ALLOCATED, AND MORE PAY TO SUBCONTRACTORS AND INSPECTORS FROM THIRD PARTY.
10. THE EXTRA COST ANALYSIS IS DERIVED BY THE MANAGEMENT ACCORDING TO THE HISTORICAL DATA.
11. THE HYDROTESTING CAN BE IMPLEMENTED WITH NO PROBLEM.
12. REPAIR CAN BE HANDLED WITH MINOR WORKS, AS HAS BEEN EXPERIENCED BY VBI.

C. LINEAR PROGRAMMING MODEL

1. UNDER NORMAL SCHEDULE: NORMAL COST $46,400.

Let: Xi = The starting time for task i, for i = 01, 02, ..., 17.

Xend = The time to complete the entire fabrication.

Starting time is 0.

Objective function :

MIN Z = Xend,

such that,

X07 - X01 >= 16	X13 - X03>= 40
X08 - X02 >= 8	X13 - X10 >= 40
X08 - X07 >= 24	X14 - X11 >= 80
X09 - X04 >= 24	X14 - X12 >= 16
X09 - X08 >= 16	X14 - X13 >= 80
X10 - X09 >= 16	X15 - X14 >= 16
X11 - X05 >= 64	X16 - X15 >= 24
X11 - X09 >= 16	X17 - X16 >= 24
X12 - X06 >= 16	Xend- X17 >= 32
X12 - X09 >= 16	

X01, X02, ..., X17, Xend >= 0

2. UNDER CRASH SCHEDULE

The fabrication is requested to be finished within 25 days. The extra costs are given in Table 1.

Ci = Ni + UiTi

where Ci = the cost of task i under crash schedule,

Ni = the cost of task i under normal schedule,

Ui = increase in cost per hour saved, under crash schedule, for task i,

T_i = the number of hours that task i should be compressed,

Xend=the time (in hours) to complete the entire fabrication,

Dend=the time (in days) to complete the entire fabrication.

Objective function : $\quad \text{Min } Z = Z' - 46{,}400$

such that,

$$
\begin{aligned}
Z' &= C01 + C02 + \ldots + C17 \\
&= N01 + N02 + \ldots + N17 + \\
&\quad U01*T01 + U02*T02 + \ldots + U17*T17 \\
&= 46{,}400 + \\
&\quad 75\ T01 + 0\ T02 + 150\ T03 + 100\ T04 + \\
&\quad 75\ T05 + 60\ T06 + 150\ T07 + 150\ T08 + \\
&\quad 100\ T09 + 125\ T10 + 100\ T11 + 75\ T12 + \\
&\quad 150\ T13 + 200\ T14 + 200\ T15 + 250\ T16 \\
&\quad + 250\ T17
\end{aligned}
$$

$Dend \le 25$

$Xend = 8\ Dend$

$$
\begin{array}{ll}
X07 - X01 + T01 \ge 16 & X13 - X03 + T03 \ge 40 \\
X08 - X02 + T02 \ge 8 & X13 - X10 + T10 \ge 40 \\
X08 - X07 + T07 \ge 24 & X14 - X11 + T11 \ge 80 \\
X09 - X04 + T04 \ge 24 & X14 - X12 + T12 \ge 16 \\
X09 - X08 + T08 \ge 16 & X14 - X13 + T13 \ge 80 \\
X10 - X09 + T09 \ge 16 & X15 - X14 + T14 \ge 16 \\
X11 - X05 + T05 \ge 64 & X16 - X15 + T15 \ge 24 \\
X11 - X09 + T09 \ge 16 & X17 - X16 + T16 \ge 24 \\
X12 - X06 + T06 \ge 16 & Xend - X17 + T17 \ge 32 \\
X12 - X09 + T09 \ge 16 &
\end{array}
$$

$$
\begin{array}{lll}
T01 \le 4 & T06 \le 4 & T12 \le 4 \\
T02 \le 0 & T07 \le 8 & T13 \le 32 \\
T03 \le 16 & T08 \le 4 & T14 \le 8
\end{array}
$$

T04 <= 8	T09 <= 4	T15 <= 12
T05 <= 24	T10 <= 16	T16 <= 12
	T11 <= 24	T17 <= 8

X01, X02, ..., X17, Xend, T01, T02, ..., T17 >= 0

Xend, Dend >= 0

T01, T02, ..., T17 are integers.

D. THE RESULTS:

The duration for the normal schedule is 36 days with total cost of $46,400, where the duration for crashed schedule is 25 days with the total cost of $61,400, an additional cost of $15,000. VBI should accept the offer for the crashed schedule provided the additional payment for the contract is well above $15,000. In this model, VBI is not to consider the possibility of failure which topic of decision under uncertainties will not be discussed in this paper.

II. COMPUTER SOLUTIONS

The LP model is solved by LINDO software program, and the print-out is attached in Appendix 2.

A. NORMAL SCHEDULE :

Under normal schedule, the critical path is given as: Start --> 01 --> 07 --> 08 --> 09 --> 10 --> 13 --> 14 --> 15 --> 16 --> 17 --> XEND.

All activities in the critical path have the objective coefficient range between -.125 and infinity (note that DEND = .125 XEND). Activities 02, 04, 05, and 06 have slacks of 32, 32, 48, and 160 hours respectively. Those activities can be started as late as on the slack hours. The time duration can be solved by integer or continuous variables. The unit time is 1-hour. It can be lowered to 1/2-hour according to the practice by VBI, but for this model, the unit is chosen to be 1-hour for simplification.

B. CRASHED SCHEDULE :

The crashed schedule has two critical paths, one path is the same path as in the normal schedule, the other path is :

Start --> 05 --> 11 --> 14 --> 15 --> 16 --> 17 --> XEND.

The activities being crashed are :

Activity #	Hours saved	Cost/hour	Extra Cost
01	4	75	300
05	12	75	900
07	8	150	1,200
08	4	150	600
09	4	100	400
10	16	125	2,000
11	8	100	800
13	32	150	4,800
14	8	200	1,600
15	12	200	2,400
TOTAL =	108		15,000

Total time saved is 108 hours which equivalent to 13.5 days. On the other hand, the number of days saved in the total schedule is 11 days, from 36 days in the normal schedule to 25 days in the crashed schedule. The 2.5 days difference is the number of days saved in the new critical path which occurs in activities 05 and 11 with 1.5 days and 1 day saved respectively.

III. DISCUSSION ON THE LP MODEL

A. IMPLEMENTATION

The model is a fairly simple one and very straight forward. Yet, it is very helpful and practical for the management in decision making. Assumptions are made by making aggregations and simplifications so that quick decision can be made.

B. STRENGTHS AND WEAKNESSES

There are some strengths and weaknesses for the model. The strengths and weaknesses are given as follows:

STRENGTH:

> The manager can make a quick decision with the help of a simple software such Lindo.

> The company's standard rates for labor/equipment costs should be easily available for the managers. For example,

the aggregate labor cost for fabricating vessel is $20 per hour. (Note, this is only a fictive example).

> The result gives the manager indication on which activities are to be considered for crashing, rather than blindly manages the project to be finished within the crashed schedule.

WEAKNESSES:

> There is a risk of failure in the crashing project since no guarantee that the critical activities such as repair and hydrotesting can be successful in a hasty condition. Even though it is possible, the possibility of failure is high.

> The aggregation on labor unit cost and sub-contractors extra costs can be not very accurate.

> The assumption on unlimited resources does not happen in the real practice in contracting firms.

> Week-end and holiday rates for workers are actually higher according to labor laws.

> The additional cost per hour is not as simple as the linear constraint.

> Overtime should happen in case things turn-up differently, and overhead costs should incur due to crash condition, e.g., extra scaffolding including setting time for additional workers.

There are more weaknesses than strengths in the model. However, compared with the usual practice in VBI, before the linear programming is introduced, the model is much better than the previous practice which used pure intuition for the decision making.

C. AVAILABILITY OF DATA

The analysis for each additional cost is made based upon additional workers to be allocated with labor cost of $20 per hour and a lump sum cost charged by the in-house sub-contractor i.e. the Non-Destructive Test (NDT) firm (by approximations and assumptions). Other data such as availability of resources (work force and equipment), overhead costs, equipment costs, and labor costs are not available so that simplifications are made.

In order to make the model more accurate, those data should be available and the model should be extended with more sophisticated linear programming as will be discussed in the section.

IV. EXTENSIONS ON THE LINEAR PROGRAMMING

As has been mentioned in the last section, the model does not consider resource constrained for the normal and crashed schedule. Deckro and Hebert [2] developed a resource critical crashing model which is based upon the model developed earlier by Pitsker, Watters, and Wolfe (PWW).

The model is defined as follows :

Xij= 1 if activity is started in period j; zero otherwise;
Xt = 1 if the project is completed in period t; zero otherwise;
g = project due data;
ei = the earliest possible period in which activity i can be started;
li = the latest possible period in which activity i can be started which will still allow the project to be completed by g;
Ti = time required to perform activity i;;
k = resource index; k = 1,2,...,K;
rik= amount of resource k used per period by activity i;
Rkj= amount of resource k available in period j.

The constraints are:

1. Activity start : each activity may be started only once: $\sum_{j=e_i}^{l_i} X_{ij} = 1$ for all i;

2. Sequencing constraints: an activity can not be started until all of its predecessors have been completed: $\sum_{j=e_m}^{l_m} j * X_{mj} - \sum_{s=e_n}^{l_n} s * X_{ns} \leq -T_m$

3. Project completion: there will be one project completion variable, Xt:

$$\sum_{i \in N_t} \sum_{j=e_i}^{l_i} X_{ij} - \| N_t \| X_t \geq 0 \text{ for all } t \in F$$

$$\sum_{t \in F} X_t = 1$$

where,

Xt = 1 if the project is completed in period t and zero otherwise;
F = is the set of all possible completion times of the project;

4. Resource constraints: is developed for each time

period within the project horizon for each type of available resource :

$$\sum_{i=1}^{n} \sum_{j=\ell i}^{\ell i} rik*Xij \leq Rkj \quad \text{for all k and j in the horizon}$$

As compared with the model for VBI, extensions for the resource constraints have been made so that the model is more accurate relative to the real application.

Deckro and Herbert extended the model for more real application, where insufficient resources are available in one or more periods.

The corresponding constraints are:

$$\sum_{i=1}^{n} \sum_{j=\ell i}^{\ell i} rik*Xij \leq Rkj + Wkj \quad \text{for all k and j}$$

and

$$Wkj \leq ukj \quad \text{for all k and j}$$

with the objective function:

$$\text{Min } Z = \sum_{k} \sum_{j} Ckj\ Wkj$$

where,

Wkj= amount of additional resource of type k made available in period j;
Ckj= the cost/unit of additional resource type k made available in period j;
ukj= the maximum amount of additional resource type k which can be obtained in period j.

Extension is made for the bonus or penalty to be considered :

$$\text{Min } Z = \sum_{k} \sum_{j} Ckj*Wkj - \sum_{t=EF}^{d} Bt*Xt + \sum_{t=d+1}^{g} Pt\ Xt$$

where,

d = the target due date which will trigger penalty or bonus payments;

B_t = bonus available for completing the project early in period t, EF $\leq t \leq d$;
P_t = penalty due for completing the project late in period t, $d + 1 \leq t \leq g$;
EF = earliest possible finish of the project if all possible additional resources were utilized.

As Deckro, Hebert, and Verdini [1] introduced "the cost based/resource allocation model for the assignment of resources throughout the project. Resources will be distributed to allow for the most cost effective allocation considering all cost elements. And "by allowing the cost to vary from period to period, actual conditions can be more accurately presented."

Moreover Deckro, Hebert, Verdini, Winkofsky, and Gagnon [1, 7] mentioned other approaches that give solution to the real application such as decomposition in multi-project scheduling. They are still pursuing the solution approach on this particular application.

V. CONCLUSION

This paper can be extended more by literature search that the articles or materials might be available in the libraries. To get more accurate and thorough results, time should be provided and more efforts should be carried out.

The model given is applicable for fabrication companies that generally encounter crashed schedule. However, this model, and of course with more extensions as discussed before, can be applied in other applications such as for production, flexible manufacturing, sales delivery, or even war fare practices.

VI. LITERATURE SEARCH :

1. Deckro, R.F., Herbert, J.E. and Verdin, W.A. (1991) "Cost Based Allocation of Resources in Project Planning," PICMET 91 CONFERENCE, Portland, pp. 278-283.
2. Deckro, R.F. and Herbert, J.E. (1989) "Resource Constrained Project Crashing," OMEGA, Vol, 17, No. 1, pp. 69-79.
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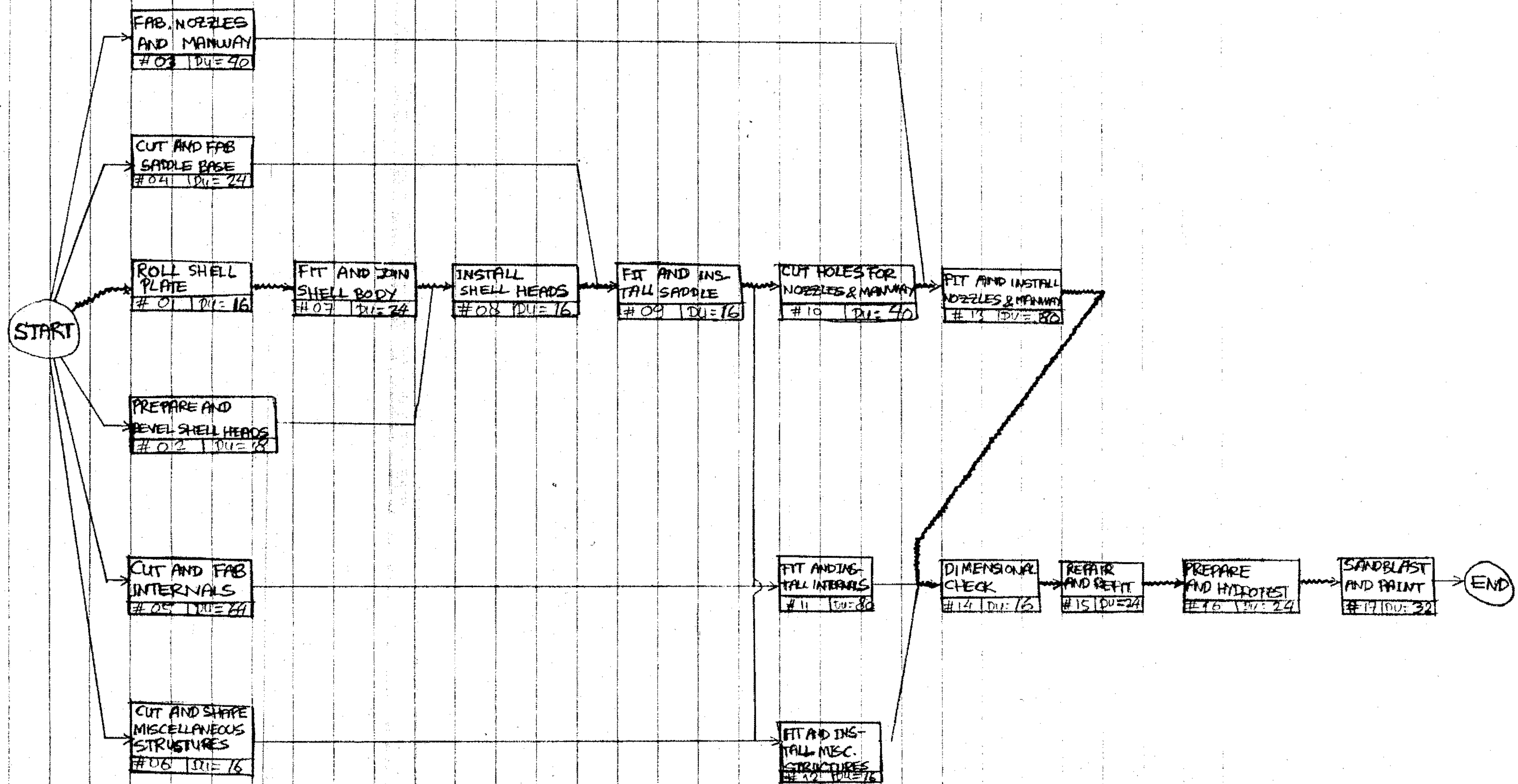
7. Deckro, R.F., E.P. Winkofsky, J.E. Hebert, and R.J. Gagnon (1991) "A Decomposition Approach to Multi-Project Scheduling," European Journal of Operational Research. Vol. 51, No. 1, pp. 167-168.

APPENDIX 1: VESSEL FABRICATION UNDER NORMAL CONDITIONS



NOTE: ~~~ = CRITICAL PATH UNDER NORMAL SCHEDULE

ROW SLACK OR SURPLUS DUAL PRICES

2)	.000000	-1.000000
3)	.000000	-1.000000
4)	.000000	-1.000000
5)	.000000	2000.000000
6)	.000000	250.000000
7)	.000000	-175.000000
8)	20.000000	.000000
9)	.000000	-175.000000
10)	16.000000	.000000
11)	.000000	-175.000000
12)	.000000	-150.000000
13)	.000000	-75.000000
14)	.000000	-25.000000
15)	92.000000	.000000
16)	56.000000	.000000
17)	.000000	.000000
18)	.000000	-150.000000
19)	.000000	-100.000000
20)	.000000	.000000
21)	.000000	-150.000000
22)	.000000	-250.000000
23)	.000000	-250.000000
24)	.000000	-250.000000
25)	.000000	-250.000000

--MORE--

repeated

23)	.000000	-250.000000
24)	.000000	-250.000000
25)	.000000	-250.000000

--MORE--

26)	.000000	100.000000
27)	16.000000	.000000
28)	8.000000	.000000
29)	12.000000	.000000
30)	4.000000	.000000
31)	.000000	25.000000
32)	.000000	25.000000
33)	.000000	75.000000
34)	.000000	25.000000
35)	16.000000	.000000
36)	4.000000	.000000
37)	.000000	.000000
38)	.000000	50.000000
39)	.000000	50.000000
40)	12.000000	.000000
41)	8.000000	.000000

NO. ITERATIONS= 29

DO RANGE(SENSITIVITY) ANALYSIS?
? Y