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Abstract: This paper looks at the problem of efficient transport operations for Emery Worldwide, a subsidiary of Consolidated Freightways, Inc. Emery is a major international carrier of "hard" air freight, which is the air freight other than envelopes and small packages. The paper focuses on Emery's North American operations, which constitute the majority of Emery's air freight business.

THE TRANSPORT OPERATIONS PROBLEM

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**THE TRANSPORT OPERATIONS PROBLEM**

**Emery Worldwide**

**Term Paper  
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THE TRANSPORT OPERATIONS PROBLEM

Emery Worldwide

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# THE TRANSPORT OPERATIONS PROBLEM

## Introduction

Transport operations are critical to the success of an air freight company. Air freight revenues are dependent on the company's ability to provide quick transit among a large number of demand points. Transport costs make up a large portion of total expenses.

In most cases, volumes between pairs of demand points are not large enough to support a dedicated vehicle between the points. Efficient transport operations therefore depend on identifying a set of routes and a set of assignments of vehicles to routes which will provide both quick transit and low transport cost.

This paper looks at the problem of efficient transport operations for Emery Worldwide, a subsidiary of Consolidated Freightways, Inc. Emery is a major international carrier of "hard" air freight, which is air freight other than envelopes and small packages. The paper focuses on Emery's North American operations, which constitute the majority of Emery's air freight business.

The paper examines techniques for determining the best operational policies for Emery's transport operations. 'Operational policies' means decisions about such questions as:

- \* What demand points should be served
- \* What level of service should be provided to each point
- \* What hub and terminal facilities should be maintained
- \* What routes should be traveled
- \* What vehicles should be used

The focus of the paper is on the applicability of a set of operations research techniques known as 'mathematical programming'. These techniques are discussed in terms of how well they are suited to Emery's specific planning needs. The paper does not survey all of the research which might be applicable to Emery's situation, but does attempt to give a picture of what techniques are available.

A quantitative example is included which illustrates how this class of technique could be applied to Emery's planning problem. The values used in the example are loosely based on real operating values, and are intended to form a realistic scenario. The values are not, however, actual operating values.

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Introduction

The paper contains the following five sections:

The Transport Operations Problem describes Emery's planning problem and serves as a background for the remainder of the discussion.

Related Literature is a brief discussion of some approaches to similar problems which have been reported in the recent operations research literature.

A Simple Model describes a mathematical programming model that deals with a limited part of the Emery planning problem. It presents the formulation and solution of a small example problem.

Extensions to the Simple Model discusses the feasibility of making extensions to the simple model. The extensions which are discussed include increasing the size of the problem and extending the model to consider additional characteristics of the real-world problem.

Alternative Approaches discusses the feasibility of using other types of models in place of, or as an adjunct to, mathematical programming models.

Conclusions presents a summarization and discussion of the other sections.

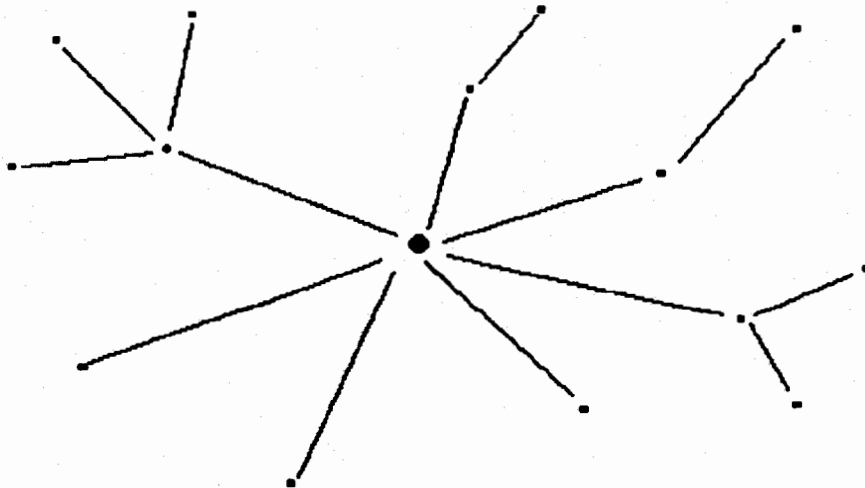
## THE TRANSPORT OPERATIONS PROBLEM

### The Transport Operations Problem

Emery Worldwide is a major international air freight carrier. Emery's North American operations, which are the subject of this paper, include:

- \* Ninety terminal locations
- \* Thirty regularly-scheduled aircraft routes
- \* Fifty regularly-scheduled truck routes
- \* About 25,000 shipments daily

Emery uses a hub-and-spoke network which includes a major North American hub plus several smaller North American hubs. Aircraft and trucks typically travel into the hub and back out again five nights a week.



A Hub-and-Spoke Network



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OBJECTIVE: Maximize the Contribution of Transport Operations

$$\text{Contribution} = \text{Revenue} - \text{Costs}$$

Transport operations affect costs in a straightforward way. Total transport costs are the sum of the component costs. Transport operations also affect revenue. The air freight business depends on the willingness of customers to pay a premium price for quick delivery. Transport operations policy must be based on the combined effects on revenues and costs.

Revenue

Emery does business at the high end of the "hard" freight market. Emery provides a premium service at a premium price for that freight which justifies the price. The services which it offers include:

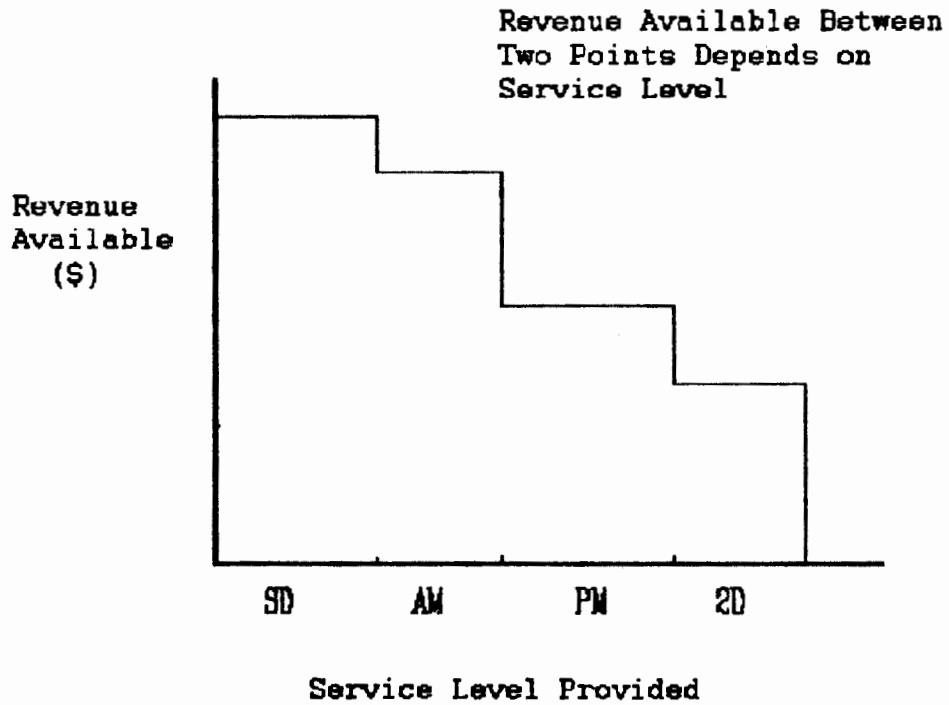
- \* Same Day (SD)
- \* Next Morning (AM)
- \* Next Day (PM)
- \* Second Day (2D)

The revenue per unit shipped is of course higher at higher service levels.

Some level of revenue is potentially available at each service level between any two points. Emery can obtain the potential revenue if it can provide the required level of service between the points.

Service that qualifies at any level also qualifies at all lower levels. If Emery operates a route between two points which meets the 'AM' service level, that same route also meets the 'PM' and '2D' service levels.

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The revenue available between two points is directional. That is, the revenue available from A to B is not generally the same as the revenue available from B to A.

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Costs

Transport costs are incurred in order to earn revenues. Transport costs and service levels (transit times) are affected by the route used and by the type of vehicle used (especially ground vs. air).

Traffic Routing

Shipments can travel between two points either directly or through any series of points. The total revenue which is available from the network is dependent on transit times and on the capacity between points.

Transshipment

Shipments which travel through intermediate points must be transshipped. Transshipping requires several steps at the transship point:

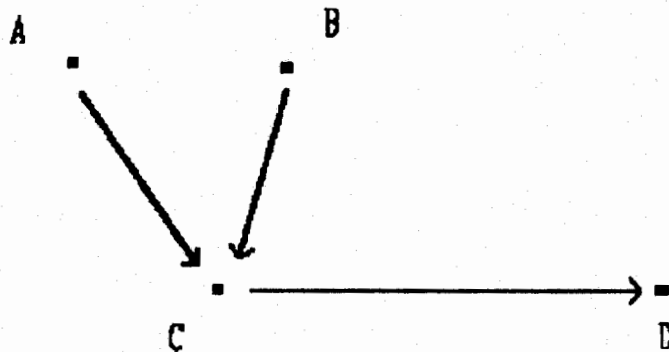
- \* Vehicle reaches the dock
- \* Cargo is unloaded
- \* Cargo is sorted
- \* Cargo is reloaded
- \* Vehicle leaves the dock

Each of these steps has a time and a cost. In addition, each of the steps requires facilities. Each point has a limited capacity for craft arrivals/departures per time period. Each point also has a cargo transshipment capacity per time period. In addition, any point that is used as a transship point has a shift cost plus a cost per unit of cargo transshipped.

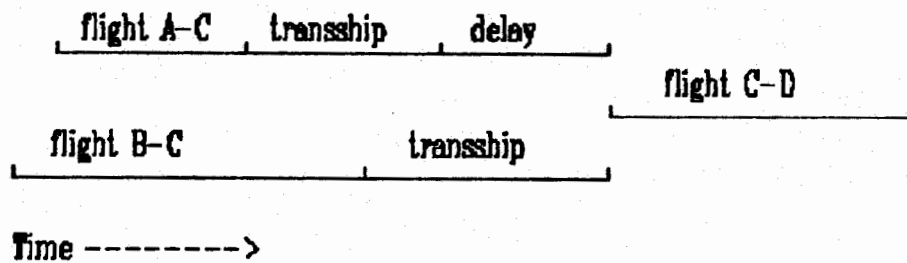
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Transit Times

The transit time for any shipment is the sum of all its point-to-point times plus any time at transship points. Time at transship points can include delays due to contention for facilities and waits for connecting vehicles. In general, a vehicle departing from a transship point will not leave until after the last arriving vehicle has arrived and its cargo has been offloaded and sorted.



A ->D and B ->D are transshipped through C



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Transport Cost

Transport cost depends on the type of vehicle (especially aircraft versus truck) and on contractual or financial arrangements.

In general, dedicated and contract transport have a fixed cost per unit time (dollars per day). In addition, they have a cost per unit of freight (weight and/or volume) and a cost per distance or time travelled.

Spot transport between two points has a cost per unit of freight (weight or volume).

Competitive Environment

The air freight business is highly concentrated, for a number of reasons. These include the rather large fixed costs of maintaining a minimal set of facilities and organization, the less-than-load demands that are available at most demand points, and the large fixed cost of aircraft.

Because of the concentration in this market, any decision to take or relinquish market affects a small number of competitors. Attempts to take market may be resisted in the form of price competition. Relinquishing market may strengthen competitors in ways that are strategically significant.

Major national air freight customers may be affected by a decision to discontinue or degrade service to some points. This may affect Emery's ability to keep a customer's business for those points which Emery continues to serve.

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### Related Literature

The preceding section suggests the many factors that should be considered in developing a transport operations policy for Emery Worldwide. There is no single integrated model which is capable of prescribing the best operating policy for Emery, considering all of the factors which were described in the preceding section. All practical approaches deal with portions of the problem.

Emery's planning problem has many similarities with a large number of other planning problems. This class of problems has been the subject of much research.

One way to simplify the approach is to take a high-level or "strategic" approach to the problem. Hall (1989) takes this approach in his study "Configuration of an Overnight Package Air Network". The first section of Hall's paper looks at the placement for a central North American hub facility. He considers the interactions between hub placement, time zones and travel distances and arrives at a formulation which expresses the effect of hub placement on the minimum time window available for overnight shipments. In the second section of his paper, Hall looks at how hub placement affects the time pattern of arriving flights and the sortation capacity which must be available to support the arriving shipments within the time window. In the third section of his paper, he looks at routing strategies which use more than one hub. He examines how different configurations affect the ability to meet time constraints and how they affect the number of routes which must be flown.

By focussing on major questions, Hall's approach provides guidance on major or "strategic" aspects of the transportation operations policy. It does this without requiring a complex mathematical problem formulation and an elaborate mathematical programming technique to determine a solution. This approach does give guidance on major questions such as how many hubs should be operated, where they should be placed, and what sortation capacity may be needed. It does not, however, answer more detailed questions such as which specific routes should exist, what vehicles should be used, and what demand should be served.

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Related Literature

Leung, Magnanti and Singhal (1990) present a more-specific model in their article "Routing in Point-to-Point Delivery Systems: Formulations and Solution Heuristics". Leung et al. consider a problem with the following characteristics:

- \* Point-to-point demand is fixed and known.
- \* A transportation network exists which consists of terminals and distribution centers. ("Distribution centers" are comparable to "hubs".)
- \* Transport between points in the network is done with a homogeneous fleet of trailers. There is a fixed cost for dispatching a trailer across a given link of the network, independent of volume.
- \* Each distribution center (hub) has a characteristic processing cost function. The cost function is stepwise linear and gives increasing costs per unit when the quantity of goods shipped through the distribution center exceeds a nominal 'capacity'.
- \* All freight between a given pair of points must follow the same route through the network.

This problem formulation does not require that the flow of trailers into a point must equal the flow out of a point.

Their procedure solves this problem to minimize the sum of trailer costs and distribution center processing costs. It results in a set of assignments of terminals to distribution centers and a set of routes among the distribution centers.

This problem is a mixed-integer program with nonlinear constraints. The authors present a solution procedure which is specific to this problem formulation. The solution procedure is not an optimizing procedure but is rather a heuristic procedure which finds a "good" solution with no guarantee that it will find the best solution possible. The procedure relies on breaking the problem into two related sub-problems. The solution of one sub-problem feeds the other. Then the solution of the second sub-problem becomes input to the first, and so on until no further improvement can be made.

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Related Literature

One of the two sub-problems consists of determining which distribution center should be assigned to each terminal. The answer to this depends on the minimum-cost routing from each distribution center to the other distribution centers.

The other sub-problem consists of determining the minimum-cost routing from each distribution center to the other distribution centers. The answer to this depends on the assignment of terminals to distribution centers.

The solution procedure starts with a reasonable set of assignments of terminals to distribution centers and a reasonable set of routes between distribution centers. It then iterates back and forth between the two sub-problems. In this way the original reasonable solution is improved. This procedure does not guarantee that the best possible solution will be found. In fact, the final answer depends on the original reasonable solution.

Solomon and Desrosiers (1988) present a survey of problem formulations and solution procedures to a related set of problems in their paper "Time Window Constrained Routing and Scheduling Problems". This class of problems is similar to the Emery problem in that it considers routing of vehicles within time windows. It differs from Emery's problem in the important respect that it considers the problem to be cost minimization given a fixed demand, whereas the Emery problem includes the choice of what points to serve and which service levels to carry. The survey results do point out that problems of practical size and complexity must be handled with special heuristic procedures rather than with off-the-shelf optimizing programs.



## THE TRANSPORT OPERATIONS PROBLEM

### A Simple Model

This section of the paper presents a rather simple mathematical programming model which is specifically designed to deal with Emery's planning needs. In an earlier section I summarized Emery's "planning problem". That problem is very complex. The model which will be presented in this section deals with a subset of the problem.

The sub-problem deals with considerations which are relevant on a medium-range planning horizon, say one to two years. In the medium-range problem, ground facilities can be considered as fixed, while the choice of markets to serve, the fleet of vehicles to be used, and the vehicle routes to be traveled can be considered as decision variables.

The model is based on the following assumptions:

- \* There is a known amount of cargo available between any two points. The cargo is available to Emery if it can be delivered quickly enough. The available cargo falls into multiple service tiers, and the cargo with stricter service requirements carries a higher revenue per unit. Emery can choose to carry any amount of the available cargo, from none to all.
- \* The possible vehicle routes between points can be specified. The cost for each vehicle route includes a fixed cost (per movement) plus a cost per unit of cargo carried on the vehicle.
- \* The feasible cargo routes (from demand point to demand point) can be specified in terms of the vehicle movements. For example, suppose there is a vehicle route between A and B, and another vehicle route between B and C. The assumption is that it can be specified whether the combined route A-B-C can meet the service time requirements at the different service tiers.
- \* Transshipment costs and capacities can be ignored, or can be considered indirectly through the specification of feasible cargo routes.

The remainder of this section presents the mathematical formulation of the model, briefly discusses the solution procedure, and then discusses the types of information which the solution provides. A sample problem is presented to give a concrete illustration. The sample problem uses parameters which

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A Simple Model

are intended to provide a realistic scenario, but are not based on actual values. A complete statement of the sample problem formulation and solution are included in the appendix.

Mathematical Formulation of the Model

Five different types of mathematical relationships make up this model. These are:

1. Cargo availability between points
2. Cargo routing options
3. Route capacity limits
4. Vehicle used to serve each route
5. Contribution (revenue minus costs)

1. Cargo Availability Between Points

In this model, the cargo available between any two points is fixed and known, and there are different amounts available at different service levels. This can be expressed in a table like the one which follows. In the table, the demand points are A, B, D and E, and there are two levels of service, AM and PM.

Cargo Availability and Rates

Org	Dst	AM Weight Available	AM Rev./lb.	PM Weight Available	PM Rev./lb.
A	B	3388	\$1.10	5288	\$0.80
A	D	4525	1.84	6654	1.34
A	E	2378	2.39	4391	1.74
B	A	3336	1.10	5347	.80
B	D	5941	2.56	10521	1.86
B	E	4493	3.16	6674	2.30
D	A	4465	1.84	6763	1.34
D	B	6415	2.56	10302	1.86
D	E	5916	.60	8832	.44
E	A	3153	2.39	3905	1.74
E	B	4294	3.16	6723	2.30
E	D	3149	.60	5101	.44

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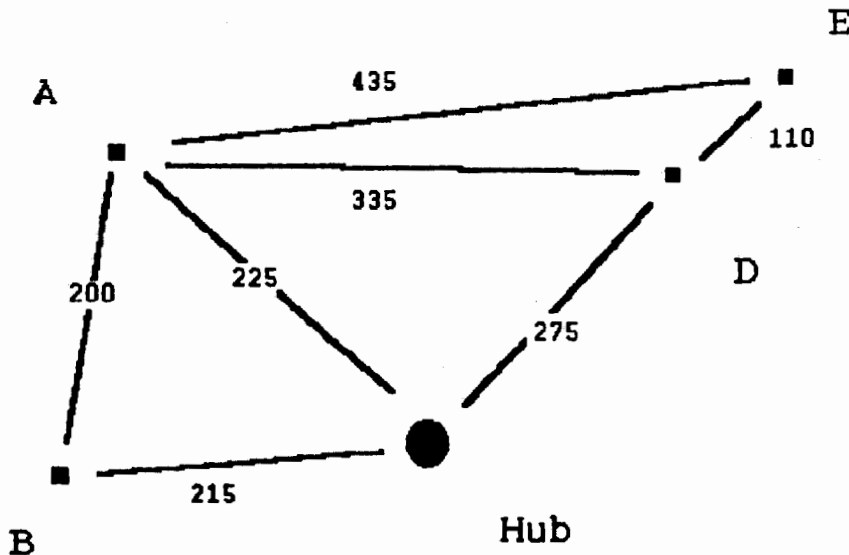
In the model, cargo availability is handled by a set of inequalities. For example, the statement

$$WABAM \leq 3388$$

says that the AM weight shipped from A to B must be less than or equal to 3,388 pounds. The 'Weight Available' columns in the preceding table generate 24 such equations.

2. Cargo routing options

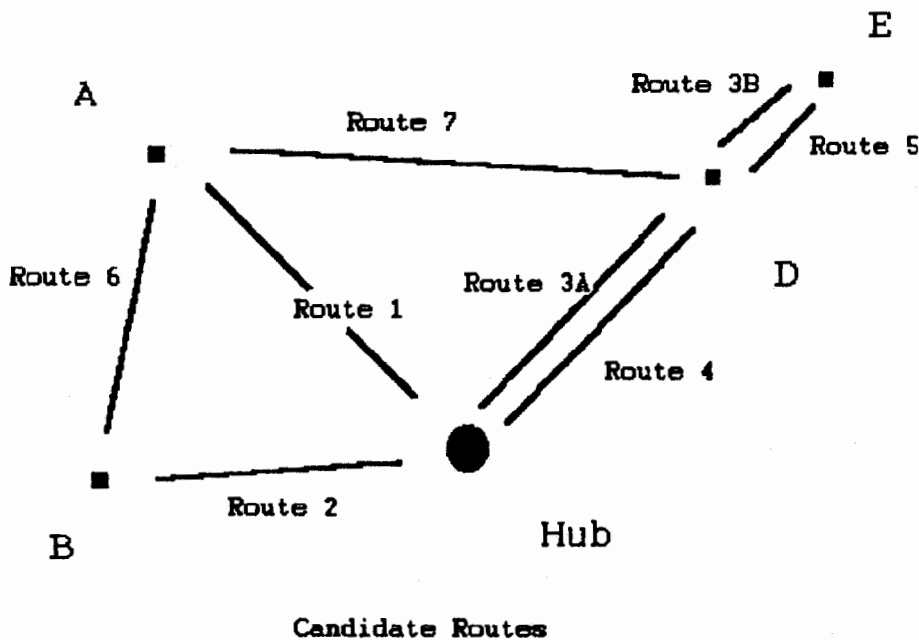
The sample problem is based on a transportation network which includes the four demand points A, B, C, D and a hub H which has no demand. The diagram below shows these five points and the distances between them.



Demand Points and Hub Location  
(Numbers indicate distance)

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A Simple Model

While routes between any set of points are feasible, the sample problem considers just seven routes, as shown on the next diagram.



In the figure above, Route 1 is a closed loop route from point A to the hub and back to point A. Routes 3A and 3B represent a single closed loop that travels from point E to point D, point D to the hub, hub to point D, and point D back to point E. Although all the routes considered in the sample problem are closed loops, the model allows one-way routes.

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Consider cargo moving from point A to point E. Suppose that A-E cargo will be delivered on time if it travels from A to D to E or if it travels from A to H to D to E, but that it won't be delivered on time if it travels from A to B to H to D to E. Then the feasible routings for A-E cargo are as follows:

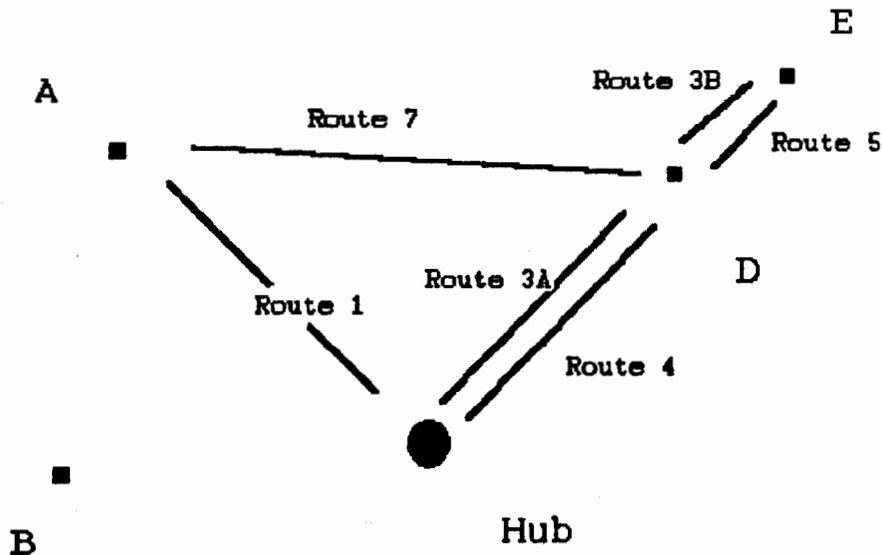
Routings from A to D to E:

Route 7, then Route 3B  
Route 7, then Route 5

Routings from A to H to D to E:

Route 1, then Route 3A, then Route 3B  
Route 1, then Route 3A, then Route 5  
Route 1, then Route 4, then Route 3B  
Route 1, then Route 4, then Route 5

The routing options for A-E cargo are shown in the diagram which follows:



Feasible Routes for A-E Traffic

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The routing options for AM traffic from point A to point E are specified in the sample problem as follows:

- 1)  $WAEAM = SAEAMR1 + SAEAMR7$
- 2)  $SAEAMR1 = SAEAMR3A + SAEAMR4$
- 3)  $SAEAMR7 + SAEAMR3A + SAEAMR4 = SAEAMR3B + SAEAMR5$

Equation 1) states that the AM weight shipped from point A to point E is the sum of the AM weight from point A to point E shipped via Route 1 and the weight shipped via Route 7.

Equation 2) states that the AM weight shipped from A to E via Route 1 equals the sum of the weight shipped via Route 3A and the weight shipped via Route 4. This equation simply states that all of this type of shipment which is shipped into the hub (via Route 1) must also be shipped out of the hub.

Equation 3) is similar to equation 2), except that it deals with AM shipments from point A to point E shipped into and out of point D.

A set of equations like these three is needed to define the feasible routes for each point pair and each service level.

### 3. Route capacity limits

In this model, a route is considered to be a vehicle travelling between a set of points. A route therefore has a capacity and a cost. The candidate routes which were considered in the sample problem are listed in the table on the next page. In the sample problem, the cost/pound in one direction on a route is set the same as the cost/pound in the other direction. The model will, however, allow different costs in different directions.

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Candidate Routes

Route 1: A-H-A	Vehicle Type: A1 Capacity: 78000	Route Distance: 900 Fixed Cost: 19,000.00
	Leg: A-H Miles: 450	Cost/pound: .23
	Leg: H-A Miles: 450	Cost/pound: .23
Route 2: B-H-B	Vehicle Type: A2 Capacity: 62000	Route Distance: 860 Fixed Cost: 15,600.00
	Leg: B-H Miles: 430	Cost/pound: .30
	Leg: H-B Miles: 430	Cost/pound: .30
Route 3: E-D-H-D-E	Vehicle Type: A2 Capacity: 62000	Route Distance: 1540 Fixed Cost: 22,400.00
	Leg: E-D Miles: 220	Cost/pound: .15
	Leg: D-H Miles: 550	Cost/pound: .39
	Leg: H-D Miles: 550	Cost/pound: .39
	Leg: D-E Miles: 220	Cost/pound: .15
Route 4: D-H-D	Vehicle Type: A2 Capacity: 62000	Route Distance: 1100 Fixed Cost: 18,000.00
	Leg: D-H Miles: 550	Cost/pound: .39
	Leg: H-D Miles: 550	Cost/pound: .39
Route 5: E-D-E	Vehicle Type: T1 Capacity: 45000	Route Distance: 440 Fixed Cost: 6,600.00
	Leg: D-E Miles: 220	Cost/pound: .05
	Leg: E-D Miles: 220	Cost/pound: .05
Route 6: A-B-A	Vehicle Type: A2 Capacity: 62000	Route Distance: 800 Fixed Cost: 15,000.00
	Leg: A-B Miles: 400	Cost/pound: .28
	Leg: B-A Miles: 400	Cost/pound: .28
Route 7: A-D-A	Vehicle Type: A2 Capacity: 62000	Route Distance: 1340 Fixed Cost: 20,400.00
	Leg: A-D Miles: 670	Cost/pound: .47
	Leg: D-A Miles: 670	Cost/pound: .47

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A Simple Model

Although the sample problem generally considers only one vehicle route between a given set of points, the model has the capability to consider alternative vehicles. It does this by defining additional routes which share the same demand points but which have different capacities and costs.

The capacity constraint for a route is modeled with an equation such as the following:

$$\begin{aligned} & \text{SABAMR1} + \text{SABPMR1} + \\ & \text{SADAMR1} + \text{SADPMR1} + \\ & \text{SAEAMR1} + \text{SAEPMR1} \leq 78000 \end{aligned}$$

This equation says that the total weight on the A to H leg of Route 1 can not be greater than 78,000 pounds. A similar equation is needed for each leg of each route.

4. Vehicle used to serve each route

If any weight is carried on any leg of a route, the fixed cost for the vehicle which serves that route is incurred. A set of equations is needed to indicate which of the available routes are used:

$$\begin{aligned} & \text{SABAMR1} + \text{SABPMR1} + \\ & \text{SADAMR1} + \text{SADPMR1} + \\ & \text{SAEAMR1} + \text{SAEPMR1} + \\ & \text{SBAAMR1} + \text{SBAPMR1} + \\ & \text{SDAAMR1} + \text{SDAPMR1} + \\ & \text{SEAAMR1} + \text{SEAPMR1} - 156000 \text{ USE1} \leq 0 \end{aligned}$$

In this equation, USE1 is a variable which can either have the value 0 or the value 1. If any of the S... variables is greater than 0, this equation forces USE1 to have the value 1. The value '156000' is the total weight which can be carried on both legs of Route 1. There is one such equation for each defined route.



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5. Contribution (revenue minus costs)

The four types of equations which are discussed above define which combinations of demand, routes and vehicles are feasible. The final equation needed is one which defines which of the combinations are more desirable. This final equation (or objective function) calculates the contribution (revenue minus costs) of the combination. The objective function for this model takes the following form:

$$\text{Contribution} = \text{Revenue} - \text{Route fixed costs} - \text{Costs/pound}$$

Revenue is equal to weight shipped times dollars per pound. For the sample problem, this is:

$$\begin{aligned} &1.10*WABAM + 0.80*WABPM + \\ &1.84*WADAM + 1.34*WADPM + \\ &\dots \end{aligned}$$

The numbers (1.10, 0.80, etc.) are taken from the chart above, "Cargo Availability and Rates".

Route fixed costs are the fixed vehicle costs which are incurred if a route is used. For the sample problem, the expression for route fixed costs is:

$$\begin{aligned} &19000*USE1 + \\ &15600*USE2 + \\ &\dots \end{aligned}$$

The numbers (19000, 15600, etc.) are taken from the chart above, "Candidate Routes".

Costs/pound are the additional shipping costs which depend on the vehicle used and the amount of weight shipped. For the sample problem, the expression for costs/pound is:

$$\begin{aligned} &0.23*SABAMR1 + 0.23*SABPMR1 + \\ &0.23*SADAMR1 + 0.23*SADAMR1 + \\ &\dots \end{aligned}$$

The numbers (0.23, etc.) are taken from the chart above, "Candidate Routes".

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The total objective function, then, looks like the following:

$$\begin{aligned} \text{Contribution} = & 1.10*WABAM + 0.80*WABPM + \\ & 1.84*WADAM + 1.34*WADPM + \\ & \dots \\ & - 19000*USE1 \\ & - 15600*USE2 \\ & \dots \\ & - 0.23*SABAMR1 - 0.23*SABPMR1 \\ & - 0.23*SADAMR1 \quad 0.23*SADAMR1 \\ & \dots \end{aligned}$$

The complete formulation of the sample problem appears in the Appendix, beginning with page 33.

#### Solution Procedure

This model is a mixed-integer programming model. The 'USE' variables are (0,1) integer variables. The problem can be solved with standard off-the-shelf mixed-integer-programming software.

The sample problem, which contains 100 constraints, was solved beginning with the LP relaxation after 25 branches and 599 pivots.

#### Solution Results

Because the model is a mixed-integer programming problem, the solution results are optimum, given the assumptions of the model and the accuracy of the model's parameters. The results which are provided in the model solution include:

- \* Which routes should be operated
- \* Which routes should be used for each type of shipment
- \* Surplus capacity of each vehicle
- \* The contribution which would result from additional cargo for each of the demand point pairs, given the routes that are selected in the optimal solution

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For the sample problem, the results are:

\* Routes which should be operated:

Route 2  
Route 4  
Route 5  
Route 7

\* Routes which should be used for each type of shipment:

<u>Demand Pair</u>	<u>Service</u>	<u>Routing</u>
A-B	AM	None
A-B	PM	None
A-D	AM	R7
A-D	PM	R7
A-E	AM	R5-R7
A-E	PM	R5-R7
B-A	AM	None
B-A	PM	None
B-D	AM	R2-R4
B-D	PM	R2-R4
B-E	AM	R2-R4-R5
B-E	PM	R2-R4-R5
D-A	AM	R7
D-A	PM	R7
D-B	AM	R4-R2
D-B	PM	R4-R2
D-E	AM	R5
D-E	PM	R5
E-A	AM	R7-R5
E-A	PM	R7-R5
E-B	AM	R5-R4-R2
E-B	PM	R5-R4-R2
E-D	AM	R5
E-D	PM	R5

\* Surplus Capacity of Each Vehicle:

Route 2, B-H	34,371
Route 2, H-B	34,266
Route 4, D-H	34,266
Route 4, H-D	34,371
Route 5, D-E	12,316
Route 5, E-D	25,733
Route 7, A-D	44,052
Route 7, D-A	43,714

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\* Contribution from Additional Cargo (dollars/pound), given  
the selected routes:

A-B AM	None
A-B PM	None
A-D AM	1.37
A-D PM	.87
A-E AM	1.87
A-E PM	1.22
B-A AM	None
B-A PM	None
B-D AM	1.87
B-D PM	1.17
B-E AM	2.42
B-E PM	1.56
D-A AM	1.84
D-A PM	1.34
D-B AM	1.87
D-B PM	1.17
D-E AM	.55
D-E PM	.39
E-A AM	1.92
E-A PM	1.27
E-B AM	2.42
E-B PM	1.56
E-D AM	.55
E-D PM	.39

## THE TRANSPORT OPERATIONS PROBLEM

### Extensions to the Simple Model

The model which was presented in the preceding section is targeted to address medium-range planning issues, such as which routes to operate and what vehicles to use. The model provides an optimal operating policy given the assumptions which are built into the model and given the accuracy of the parameters used to build the model. Two short-comings of this model are its failure to handle transshipment requirements explicitly and its size.

#### Transshipment Extensions

Transshipments are significant in this problem in a couple of ways. First, transshipment capacity can be a constraint on the amount of cargo which can pass through a point. Second, transshipment has a cost. The transshipment cost includes a facility cost. This cost may be considered as sunk for the medium-range view taken in this model. The transshipment cost also includes a shift cost, which depends on whether a point is used as a transship point, and a cost per unit of cargo transshipped, which depends on how much cargo is transshipped there.

The model which was presented in the previous section can deal with transshipment capacity constraints in a partial way through the model formulation process. Routes which would require transshipment at a point that can not provide it should not be defined as feasible routes. The model does not, however, deal with variable transshipment costs.

The model could be extended to deal with variable transshipment cost and with transshipment capacity constraints in an explicit way. Equations would be added to the model which would identify which points are used as transship points and which cargo is being transshipped. This would allow transshipment capacity constraints to be applied, and would also allow variable transship costs to be considered in calculating the contribution.

Referring back to the sample problem, if any A-E cargo were shipped through point D, this would make point D a transship point, and a transship shift cost would be incurred as a result. The total weight transshipped at point D would determine the transship processing costs at point D. An equation would be defined for each potential transship point which would indicate whether the point was indeed used for transshipment. These equations would look like the route use equations which were explained in the preceding section.

THE TRANSPORT OPERATIONS PROBLEM  
Extensions to the Simple Model

Model Size

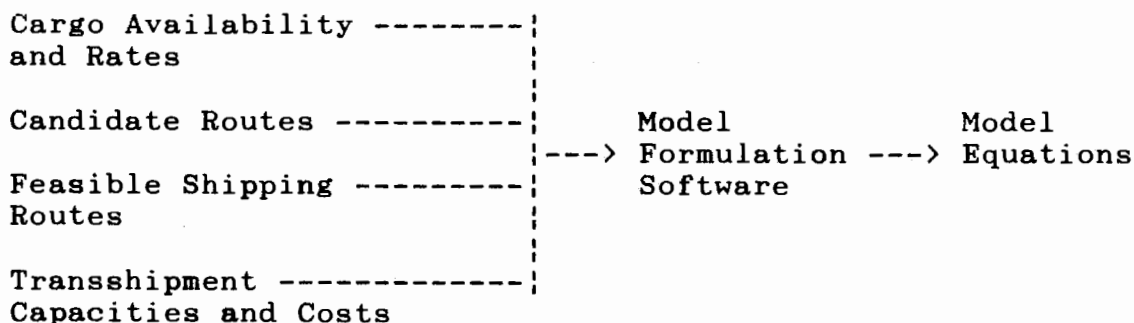
The sample problem considered only four demand points, two service levels, and seven routes. The model formulated from the sample problem included one hundred constraint equations. (This is the capacity of the "student version" software which was used to solve the sample problem.) In general, the number of equations for this model is approximately equal to:

$$3 * \text{Number of point pairs} * \text{Number of service levels} \\ + \text{Number of route legs} \\ + \text{Number of routes}$$

If the model were expanded to include transshipment equations, the number would be higher. The number of variables in the model is somewhat higher than the number of equations.

Since the number of point pairs doubles whenever a point is added, the nominal size of the problem grows by a factor of four whenever a point is added. If the model were expanded to include ninety demand points and four service levels, this would result in about 100,000 equations, not including transshipment equations.

The size of the model causes two potential problems. The first of these is the expense of formulating the model equations. The sample problem was formulated by hand. The formulation was time-consuming and error-prone. Formulating a large-scale model would require software to automate the formulation process. The software would transform a standard normalized database into the set of equations that are required by the solution software. This is illustrated below.



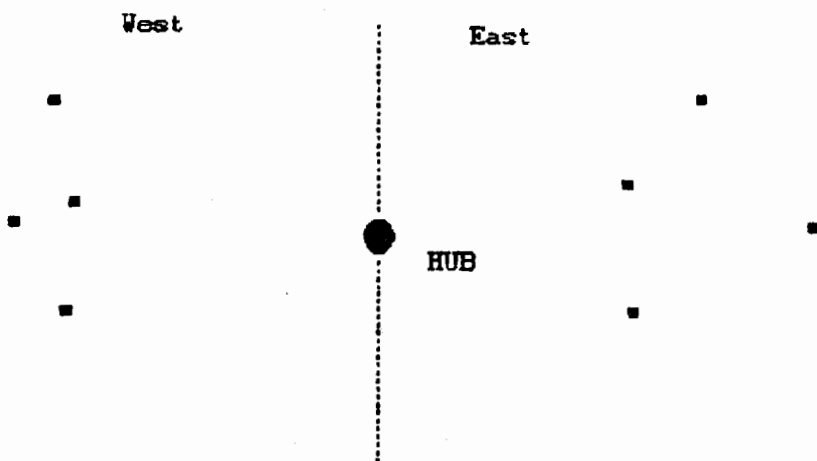
THE TRANSPORT OPERATIONS PROBLEM  
Extensions to the Simple Model

The second problem associated with large model size is the feasibility of solving the model. Large models require large amounts of storage and CPU time. In general, the CPU time required for solution grows more rapidly than the storage requirements. The large model contemplated above (90 demand points and 4 service levels) would nominally require a matrix of 9-10 billion elements. The matrix is very sparse, however, and sparse-array techniques could probably be used to reduce storage requirements to feasible levels. However, CPU time requirements would still be large, and probably large enough to severely limit the amount that such a model could be used.

One approach to reducing the CPU time requirements of a large model is to start the solution process with a known good solution. In this case, the known good solution would be Emery's current operating policies. The solution procedure can quickly eliminate many of the options which are not as good as the best known solution.

Another approach to reducing the CPU time requirements would also reduce storage requirements. This approach is based on a characteristic of the transportation operations problem and uses the technique of splitting the problem into multiple subproblems.

Consider the following diagram:



Hub-Dominated Network

## THE TRANSPORT OPERATIONS PROBLEM Extensions to the Simple Model

The diagram shows a centrally-located hub and a set of demand points. In the diagram, the demand points are partitioned into two groups, one to the east and the other to the west of the hub. This is, of course, an exaggeration of the true situation, but it illustrates a point. Suppose that east-west traffic travels through the hub. Then, if we consider just the points to the east of the hub, the transportation problem can be considered as involving just the hub and points east. Any cargo which actually originates in the West looks to the eastern points as if it originated at the hub. This characteristic of the problem provides a basis for splitting or 'decomposing' the problem into sub-problems.

Although the transportation network does not in fact split into a number of sub-problems which are strictly separated, it is possible to separate the problem into sub-problems which have limited inter-connectedness. This is so because of the same set of factors which favor the hub-and-spoke network. See Hall (1989). These factors include the preponderance of less-than-load demand between point pairs and the need to operate dedicated craft. Another factor which promotes separability is that the desirability of providing service to a point is strongly affected by the volume of demand (incoming and outgoing) at that point and is much less affected by whether or not any particular other point is served.

A number of problem decomposition approaches have been reported in the literature. The paper by Leung, Magnanti and Singhal (1990) which is described above is one example. In addition, there are more-generally-applicable decompositions available, such as the Dantzig-Wolfe and Bender decompositions.



## THE TRANSPORT OPERATIONS PROBLEM

### Alternative Approaches

The bulk of this paper has been concerned with one type of approach to the Emery planning problem, mathematical programming. There are numerous variations of the mathematical programming approach. They all consist of first formulating the problem as a set of mathematical equations which describe the possible strategies, then using a solution procedure which either identifies the best possible strategy or identifies "good" strategies.

These mathematical programming approaches have some limitations. Some of the limitations are discussed in the preceding section, along with some of the strategies which are available for overcoming them.

A basic limitation of these approaches is that it is necessary to simplify the problem sometimes drastically to formulate the problem in a way that can be solved. The simple model which was discussed in the preceding two sections was large enough that computer size and time to solve it would be problematic. Yet that model does not consider terminal pickup-and-delivery operations, and how those are related to transport operations. Nor does it consider that cargo availability is not fixed, but varies from day to day.

An alternative to mathematical programming-based models, which prescribe a best set of decisions given all the options, is a predictive/descriptive model. A very simple predictive/descriptive model would just evaluate what the revenue and costs would be, given a set of routes and a set of assignments of vehicles and cargo to them. At this level, the model would be like the simple mathematical programming model presented earlier, but with only one option.

This predictive/descriptive type of model has a couple of potential advantages over mathematical programming models. First, they are simple to construct, quick to execute, and do not suffer from the geometric growth patterns of the mathematical programming models. Second, it is possible to make them as detailed as needed, again without the geometric growth problems of the mathematical programming models.

The drawback of the predictive/descriptive models is the strength of the mathematical programming models. The predictive/descriptive models do not tell what the best set of decisions is. In terms of Emery's problem, however, this may not be a major drawback. As discussed above, decisions about

THE TRANSPORT OPERATIONS PROBLEM  
Alternative Approaches

service, vehicles and routing tend to have consequences that are largely local to the points and routes being considered. In addition, there are generally only a small number of options that directly affect a particular point. It may be possible to develop very good solutions without an optimizing model by making a systematic search of local options and evaluating the different strategies with a simple predictive/descriptive model.

## THE TRANSPORT OPERATIONS PROBLEM

### Conclusions

These major points have been raised in the body of this report:

- \* Emery's transportation operations problem is 'mission critical' and is very complex.
- \* No integrated model exists for solving it.
- \* Practical models reduce the complexity of the problem by several strategies. These strategies include looking at the problem at a higher level, simplifying the problem, or splitting up (decomposing) the problem into sub-problems.
- \* The literature includes much research into closely-related problems. This research reports on many techniques that might be applicable to Emery's problem with some modification.
- \* Emery's medium-range planning problem can be formulated into a fairly simple mixed integer program which can be solved with off-the-shelf software.
- \* This problem formulation, though simple, becomes a very large problem if it is extended to cover Emery's real-world transportation network. Special techniques, such as decomposition, may be needed to make the problem manageable.
- \* Predictive/descriptive models are an alternative to mathematical programming models. Although they do not identify the best solution, they are relatively easy to develop and quick to execute. It may be possible to use such models in concert with localized searches to develop good solutions to the Emery planning problem.

## THE TRANSPORT OPERATIONS PROBLEM

### References

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Leung J. M. Y., Magnanti T. L., Singhal V. (1990) Routing in Point-to-Point Delivery Systems: Formulations and Solution Heuristics. *Transportation Science*, Vol. 24, No. 4, 245-260.

Solomon M. M. and Desrosiers J. (1988) Time Window Constrained Routing and Scheduling Problems. *Transportation Science*, Vol. 22, No. 1, 1-11.

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

Appendix - Sample Problem

PROBLEM FORMULATION

```

MAX 1.10 WABAM + 0.80 WABPM          ! revenue
+ 1.84 WADAM + 1.34 WADPM
+ 2.39 WAEAM + 1.74 WAEPM
+ 1.10 WBAAM + 0.80 WBAPM
+ 2.56 WBDAM + 1.86 WBDPM
+ 3.16 WBEAM + 2.30 WBEPM
+ 1.84 WDAAM + 1.34 WDAPM
+ 2.56 WDBAM + 1.86 WDBPM
+ 0.60 WDEAM + 0.44 WDEPM
+ 2.39 WEAAM + 1.74 WEAPM
+ 3.16 WEBAM + 2.30 WEBPM
+ 0.60 WEDAM + 0.44 WEDPM
- 19000 USE1                          ! route fixed costs
- 15600 USE2
- 22400 USE3
- 18000 USE4
- 6600 USE5
- 15000 USE6
- 20400 USE7
- .23 SABAMR1 - .23 SABPMR1          ! route costs/pound
- .23 SADAMR1 - .23 SADPMR1
- .23 SAEAMR1 - .23 SAEPMR1
- .23 SBAAMR1 - .23 SBAPMR1
- .23 SDAAMR1 - .23 SDAPMR1
- .23 SEAAMR1 - .23 SEAPMR1
- .30 SBAAMR2 - .30 SBAPMR2
- .30 SBDAMR2 - .30 SBDPMR2
- .30 SBEAMR2 - .30 SBEPMR2
- .30 SABAMR2 - .30 SABPMR2
- .30 SDBAMR2 - .30 SDBPMR2
- .30 SEBAMR2 - .30 SEBPMR2
- .39 SDAAMR3A - .39 SDAPMR3A
- .39 SDBAMR3A - .39 SDBPMR3A
- .39 SEAAMR3A - .39 SEAAMR3A
- .39 SEBAMR3A - .39 SEBPMR3A
- .39 SADAMR3A - .39 SADPMR3A
- .39 SAEAMR3A - .39 SAEPMR3A
- .39 SBDAMR3A - .39 SBDPMR3A
- .39 SBEAMR3A - .39 SBEPMR3A
- .15 SAEAMR3B - .15 SAEPMR3B
- .15 SBEAMR3B - .15 SBEPMR3B
- .15 SDEAMR3B - .15 SDEPMR3B
- .15 SEBAMR3B - .15 SEBPMR3B

```

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

- .15 SEDAMR3B - .15 SEDPMR3B  
- .39 SDAAMR4 - .39 SDAPMR4  
- .39 SDBAMR4 - .39 SDBPMR4  
- .39 SEAAMR4 - .39 SEAPMR4  
- .39 SEBAMR4 - .39 SEBPMR4  
- .39 SADAMR4 - .39 SADPMR4  
- .39 SAEAMR4 - .39 SAEPMR4  
- .39 SBDAMR4 - .39 SBDPMR4  
- .39 SBEAMR4 - .39 SBEPMR4  
- .05 SAEAMR5 - .05 SAEPMR5  
- .05 SBEAMR5 - .05 SBEPMR5  
- .05 SDEAMR5 - .05 SDEPMR5  
- .05 SEBAMR5 - .05 SEBPMR5  
- .05 SEDAMR5 - .05 SEDPMR5  
- .28 SABAMR6 - .28 SABPMR6  
- .28 SBAAMR6 - .28 SBAPMR6  
- .47 SADAMR7 - .47 SADPMR7  
- .47 SAEAMR7 - .47 SAEPMR7  
- .47 SEAAMR7 - .47 SEAPMR7

SUBJECT TO

! product weight availability constraints  
!

WABAM)           WABAM <= 3388  
WABPM)           WABPM <= 5288  
WADAM)           WADAM <= 4525  
WADPM)           WADPM <= 6654  
WAEAM)           WAEAM <= 2378  
WAEPM)           WAEPM <= 4391  
WBAAM)           WBAAM <= 3336  
WBAPM)           WBAPM <= 5347  
WBDAM)           WBDAM <= 5941  
WBDPM)           WBDPM <= 10521  
WBEAM)           WBEAM <= 4493  
WBEPM)           WBEPM <= 6674  
WDAAM)           WDAAM <= 4465  
WDAPM)           WDAPM <= 6763  
WDBAM)           WDBAM <= 6415  
WDBPM)           WDBPM <= 10302  
WDEAM)           WDEAM <= 5916  
WDEPM)           WDEPM <= 8832  
WEAAM)           WEAAM <= 3153  
WEAPM)           WEAPM <= 3905  
WEBAM)           WEBAM <= 4294  
WEBPM)           WEBPM <= 6723  
WEDAM)           WEDAM <= 3149  
WEDPM)           WEDPM <= 5101

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

: routing constraints

: A-B shipments can travel by routes R1-R2 or R6.

: WABAM - SABAMR1 - SABAMR6 = 0 ! balance at A  
: SABAMR1 - SABAMR2 = 0 ! balance at H

: WABPM - SABPMR1 - SABPMR6 = 0 ! balance at A  
: SABPMR1 - SABPMR2 = 0 ! balance at H

: A-D shipments can travel by routes R1-R3A, R1-R4, or  
: R7.

: WADAM - SADAMR1 - SADAMR7 = 0 ! balance at A  
: SADAMR1 - SADAMR3A - SADAMR4 = 0 ! balance at H

: WADPM - SADPMR1 - SADPMR7 = 0 ! balance at A  
: SADPMR1 - SADPMR3A - SADPMR4 = 0 ! balance at H

: A-E shipments can travel by routes R1-R3A-R3B,  
: R1-R3A-R5, R1-R4-R3B, R1-R4-R5, R7-R3B or R7-R5.

: WAEAM - SAEAMR1 - SAEAMR7 = 0 ! balance at A  
: SAEAMR1 - SAEAMR3A - SAEAMR4 = 0 ! balance at H  
: SAEAMR7 + SAEAMR3A + SAEAMR4 - SAEAMR3B - SAEAMR5 = 0  
: ! balance at D

: WAEPM - SAEPMR1 - SAEPMR7 = 0 ! balance at A  
: SAEPMR1 - SAEPMR3A - SAEPMR4 = 0 ! balance at H  
: SAEPMR7 + SAEPMR3A + SAEPMR4 - SAEPMR3B - SAEPMR5 = 0  
: ! balance at D

: B-A shipments can travel by routes R2-R1 or R6.

: WBAAM - SBAAMR2 - SBAAMR6 = 0  
: SBAAMR2 - SBAAMR1 = 0

: WBAPM - SBAPMR2 - SBAPMR6 = 0  
: SBAPMR2 - SBAPMR1 = 0

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

! B-D shipments can travel by routes R2-R3A or R2-R4.  
!

$$\begin{aligned} \text{WBDAM} - \text{SBDAMR2} &= 0 \\ \text{SBDAMR2} - \text{SBDAMR3A} - \text{SBDAMR4} &= 0 \end{aligned}$$

$$\begin{aligned} \text{WBDPM} - \text{SBDPMR2} &= 0 \\ \text{SBDPMR2} - \text{SBDPMR3A} - \text{SBDPMR4} &= 0 \end{aligned}$$

! B-E shipments can travel by routes R2-R3A-R3B,  
! R2-R3A-R5, R2-R4-R3B, or R2-R4-R5.  
!

$$\begin{aligned} \text{WBEAM} - \text{SBEAMR2} &= 0 \\ \text{SBEAMR2} - \text{SBEAMR3A} - \text{SBEAMR4} &= 0 \\ \text{SBEAMR3A} + \text{SBEAMR4} - \text{SBEAMR3B} - \text{SBEAMR5} &= 0 \end{aligned}$$

$$\begin{aligned} \text{WBEPM} - \text{SBEPMR2} &= 0 \\ \text{SBEPMR2} - \text{SBEPMR3A} - \text{SBEPMR4} &= 0 \\ \text{SBEPMR3A} + \text{SBEPMR4} - \text{SBEPMR3B} - \text{SBEPMR5} &= 0 \end{aligned}$$

! D-A shipments can travel by routes R7, R3A-R1, or  
! R4-R1.  
!

$$\begin{aligned} \text{WDAAM} - \text{SDAAMR7} - \text{SDAAMR3A} - \text{SDAAMR4} &= 0 \\ \text{SDAAMR3A} + \text{SDAAMR4} - \text{SDAAMR1} &= 0 \end{aligned}$$

$$\begin{aligned} \text{WDAPM} - \text{SDAPMR7} - \text{SDAPMR3A} - \text{SDAPMR4} &= 0 \\ \text{SDAPMR3A} + \text{SDAPMR4} - \text{SDAPMR1} &= 0 \end{aligned}$$

! D-B shipments can travel by routes R3A-R2 or R4-R2.  
!

$$\begin{aligned} \text{WDBAM} - \text{SDBAMR3A} - \text{SDBAMR4} &= 0 \\ \text{SDBAMR3A} + \text{SDBAMR4} - \text{SDBAMR2} &= 0 \end{aligned}$$

$$\begin{aligned} \text{WDBPM} - \text{SDBPMR3A} - \text{SDBPMR4} &= 0 \\ \text{SDBPMR3A} + \text{SDBPMR4} - \text{SDBPMR2} &= 0 \end{aligned}$$

! D-E shipments can travel by routes R3B or R5.  
!

$$\text{WDEAM} - \text{SDEAMR3B} - \text{SDEAMR5} = 0$$

$$\text{WDEPM} - \text{SDEPMR3B} - \text{SDEPMR5} = 0$$



THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

```

!
!   E-A shipments can travel by routes R3B-R7,
!       R3B-R3A-R1, R3B-R4-R1, R5-R7, R5-R3A-R1, or
!       R5-R4-R1.
WEAAM - SEAAMR3B - SEAAMR5 = 0
SEAAMR3B + SEAAMR5 - SEAAMR7 - SEAAMR3A - SEAAMR4 = 0
SEAAMR3A + SEAAMR4 - SEAAMR1 = 0
!
!   WEAPM - SEAPMR3B - SEAPMR5 = 0
!   SEAPMR3B + SEAPMR5 - SEAPMR7 - SEAPMR3A - SEAPMR4 = 0
!   SEAPMR3A + SEAPMR4 - SEAPMR1 = 0
!
!   E-B shipments can travel by routes R3B-R3A-R2,
!       R3B-R4-R2, R5-R3A-R2, or R5-R4-R2.
WEBAM - SEBAMR3B - SEBAMR5 = 0
SEBAMR3B + SEBAMR5 - SEBAMR3A - SEBAMR4 = 0
SEBAMR3A + SEBAMR4 - SEBAMR2 = 0
!
!   WEBPM - SEBPMR3B - SEBPMR5 = 0
!   SEBPMR3B + SEBPMR5 - SEBPMR3A - SEBPMR4 = 0
!   SEBPMR3A + SEBPMR4 - SEBPMR2 = 0
!
!   E-D shipments can travel by routes R3B or R5.
!
!   WEDAM - SEDAMR3B - SEDAMR5 = 0
!
!   WEDPM - SEDPMR3B - SEDPMR5 = 0
!
!   route capacity constraints
!
!   A-H leg of R1
R1AH)   SABAMR1 + SABPMR1 +
        SADAMR1 + SADPMR1 +
        SAEAMR1 + SAEPMR1 <= 78000
!
!   H-A leg of R1
R1HA)   SBAAMR1 + SBAPMR1 +
        SDAAMR1 + SDAPMR1 +
        SEAAMR1 + SEAPMR1 <= 78000
!
!   B-H leg of R2
R2BH)   SBAAMR2 + SBAPMR2 +
        SBDAMR2 + SBDPMR2 +
        SBEAMR2 + SBEPMR2 <= 62000

```

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

```

!
!   H-B leg of R2
R2HB)   SABAMR2 + SABPMR2 +
        SDBAMR2 + SDBPMR2 +
        SEBAMR2 + SEBPMR2 <= 62000
!
!   D-H leg of R3A
R3ADH)  SDAAMR3A + SDAPMR3A +
        SDBAMR3A + SDBPMR3A +
        SEAAMR3A + SEAPMR3A +
        SEBAMR3A + SEBPMR3A <= 62000
!
!   H-D leg of R3A
R3AHD)  SADAMR3A + SADPMR3A +
        SAEAMR3A + SAEPMR3A +
        SBDAMR3A + SBDPMR3A +
        SBEAMR3A + SBEPMR3A <= 62000
!
!   D-H leg of R4
R4DH)   SDAAMR4 + SDAPMR4 +
        SDBAMR4 + SDBPMR4 +
        SEAAMR4 + SEAPMR4 +
        SEBAMR4 + SEBPMR4 <= 62000
!
!   H-D leg of R4
R4HD)   SADAMR4 + SADPMR4 +
        SAEAMR4 + SAEPMR4 +
        SBDAMR4 + SBDPMR4 +
        SBEAMR4 + SBEPMR4 <= 62000
!
!   D-E leg of R3B
R3BDE)  SAEAMR3B + SAEPMR3B +
        SBEAMR3B + SBEPMR3B +
        SDEAMR3B + SDEPMR3B <= 62000
!
!   E-D leg of R3B
R3BED)  SEBAMR3B + SEBPMR3B +
        SEDAMR3B + SEDPMR3B <= 62000
!
!   D-E leg of R5
R5DE)   SAEAMR5 + SAEPMR5 +
        SBEAMR5 + SBEPMR5 +
        SDEAMR5 + SDEPMR5 <= 45000
!
!   E-D leg of R5
R5ED)   SEBAMR5 + SEBPMR5 +
        SEDAMR5 + SEDPMR5 <= 45000

```

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

```

!
R6AB)      A-B leg of R6
           SABAMR6 + SABPMR6 <= 62000
!
R6BA)      B-A leg of R6
           SBAAMR6 + SBAPMR6 <= 62000
!
R7AD)      A-D leg of R7
           SADAMR7 + SADPMR7 +
           SAEAMR7 + SAEPMR7 <= 62000
!
R7DA)      D-A leg of R7
           SDAAMR7 + SDAPMR7 +
           SEAAMR7 + SEAPMR7 <= 62000
!
           ! route use indicator constraints
!
USE1)      SABAMR1 + SABPMR1 +
           SADAMR1 + SADPMR1 +
           SAEAMR1 + SAEPMR1 +
           SBAAMR1 + SBAPMR1 +
           SDAAMR1 + SDAPMR1 +
           SEAAMR1 + SEAPMR1 - 156000 USE1 <= 0
!
USE2)      SBAAMR2 + SBAPMR2 +
           SBDAMR2 + SBDPMR2 +
           SBEAMR2 + SBEPMR2 +
           SABAMR2 + SABPMR2 +
           SDBAMR2 + SDBPMR2 +
           SEBAMR2 + SEBPMR2 - 124000 USE2 <= 0
!
USE3)      SDAAMR3A + SDAPMR3A +
           SDBAMR3A + SDBPMR3A +
           SEAAMR3A + SEAPMR3A +
           SEBAMR3A + SEBPMR3A +
           SADAMR3A + SADPMR3A +
           SAEAMR3A + SAEPMR3A +
           SBDAMR3A + SBDPMR3A +
           SBEAMR3A + SBEPMR3A +
           SAEAMR3B + SAEPMR3B +
           SBEAMR3B + SBEPMR3B +
           SDEAMR3B + SDEPMR3B +
           SEBAMR3B + SEBPMR3B +
           SEDAMR3B + SEDPMR3B - 248000 USE3 <= 0

```

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Formulation

USE4) !  
SDAAMR4 + SDAPMR4 +  
SDBAMR4 + SDBPMR4 +  
SEAAMR4 + SEAPMR4 +  
SEBAMR4 + SEBPMR4 +  
SADAMR4 + SADPMR4 +  
SAEAMR4 + SAEPMR4 +  
SBDAMR4 + SBDPMR4 +  
SBEAMR4 + SBEPMR4 - 124000 USE4 <= 0

USE5) !  
SAEAMR5 + SAEPMR5 +  
SBEAMR5 + SBEPMR5 +  
SDEAMR5 + SDEPMR5 +  
SEBAMR5 + SEBPMR5 +  
SEDAMR5 + SEDPMR5 - 90000 USE5 <= 0

USE6) !  
SABAMR6 + SABPMR6 +  
SBAAMR6 + SBAPMR6 - 124000 USE6 <= 0

USE7) !  
SADAMR7 + SADPMR7 +  
SAEAMR7 + SAEPMR7 +  
SDAAMR7 + SDAPMR7 +  
SEAAMR7 + SEAPMR7 - 124000 USE7 <= 0

END  
INTEGER USE1  
INTEGER USE2  
INTEGER USE3  
INTEGER USE4  
INTEGER USE5  
INTEGER USE6  
INTEGER USE7

LEAVE

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Solution

PROBLEM SOLUTION

OBJECTIVE FUNCTION VALUE

1) 89535.350

VARIABLE	VALUE	REDUCED COST
USE1	.000000	-69920.000000
USE2	1.000000	15600.000000
USE3	.000000	22400.000000
USE4	1.000000	18000.000000
USE5	1.000000	6600.000000
USE6	.000000	-86680.010000
USE7	1.000000	20400.000000
WABAM	.000000	.000000
WABPM	.000000	.300000
WADAM	4525.000000	.000000
WADPM	6654.000000	.000000
WAEAM	2378.000000	.000000
WAEPM	4391.000000	.000000
WBAAM	.000000	.000000
WBAPM	.000000	.300000
WBDAM	5941.000000	.000000
WBDPM	10521.000000	.000000
WBEAM	4493.000000	.000000
WBEPM	6674.000000	.000000
WDAAM	4465.000000	.000000
WDAPM	6763.000000	.000000
WDBAM	6415.000000	.000000
WDBPM	10302.000000	.000000
WDEAM	5916.000000	.000000
WDEPM	8832.000000	.000000
WEAAM	3153.000000	.000000
WEAPM	3905.000000	.000000
WEBAM	4294.000000	.000000
WEBPM	6723.000000	.000000
WEDAM	3149.000000	.000000
WEDPM	5101.000000	.000000
SABAMR1	.000000	.000000
SABPMR1	.000000	.000000
SADAMR1	.000000	.000000
SADPMR1	.000000	.000000
SAEAMR1	.000000	.000000
SAEPMR1	.000000	.720000
SBAAMR1	.000000	.000000
SBAPMR1	.000000	.000000

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Solution

SDAAMR1	.000000	.000000
SDAPMR1	.000000	.000000
SEAAMR1	.000000	.000000
SEAPMR1	.000000	.000000
SBAAMR2	.000000	.000000
SBAPMR2	.000000	.000000
SBDAMR2	5941.000000	.000000
SBDPMR2	10521.000000	.000000
SBEAMR2	4493.000000	.000000
SBEPMR2	6674.000000	.000000
SABAMR2	.000000	.000000
SABPMR2	.000000	.000000
SDBAMR2	6415.000000	.000000
SDBPMR2	10302.000000	.000000
SEBAMR2	4294.000000	.000000
SEBPMR2	6723.000000	.000000
SDAAMR3A	.000000	1.190000
SDAPMR3A	.000000	1.190000
SDBAMR3A	.000000	.000000
SDBPMR3A	.000000	.000000
SEAAMR3A	.000000	1.110000
SEBAMR3A	.000000	.000000
SEBPMR3A	.000000	.000000
SADAMR3A	.000000	.720000
SADPMR3A	.000000	.720000
SAEAMR3A	.000000	.720000
SAEPMR3A	.000000	.000000
SBDAMR3A	.000000	.000000
SBDPMR3A	.000000	.000000
SBEAMR3A	.000000	.000000
SBEPMR3A	.000000	.000000
SAEAMR3B	.000000	.100000
SAEPMR3B	.000000	.100000
SBEAMR3B	.000000	.100000
SBEPMR3B	.000000	.100000
SDEAMR3B	.000000	.100000
SDEPMR3B	.000000	.100000
SEBAMR3B	.000000	.100000
SEBPMR3B	.000000	.100000
SEDAMR3B	.000000	.100000
SEDPMR3B	.000000	.100000
SDAAMR4	.000000	1.190000
SDAPMR4	.000000	1.190000
SDBAMR4	6415.000000	.000000
SDBPMR4	10302.000000	.000000
SEAAMR4	.000000	.720000
SEAPMR4	.000000	.720000
SEBAMR4	4294.000000	.000000
SEBPMR4	6723.000000	.000000

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Solution

SADAMR4	.000000	.720000
SADPMR4	.000000	.720000
SAEAMR4	.000000	.720000
SAEPMR4	.000000	.000000
SBDAMR4	5941.000000	.000000
SBDPMR4	10521.000000	.000000
SBEAMR4	4493.000000	.000000
SBEPMR4	6674.000000	.000000
SAEAMR5	2378.000000	.000000
SAEPMR5	4391.000000	.000000
SBEAMR5	4493.000000	.000000
SBEPMR5	6674.000000	.000000
SDEAMR5	5916.000000	.000000
SDEPMR5	8832.000000	.000000
SEBAMR5	4294.000000	.000000
SEBPMR5	6723.000000	.000000
SEDAMR5	3149.000000	.000000
SEDPMR5	5101.000000	.000000
SABAMR6	.000000	.000000
SABPMR6	.000000	.000000
SBAAMR6	.000000	.000000
SBAPMR6	.000000	.000000
SADAMR7	4525.000000	.000000
SADPMR7	6654.000000	.000000
SAEAMR7	2378.000000	.000000
SAEPMR7	4391.000000	.000000
SEAAMR7	3153.000000	.000000
SEAPMR7	3905.000000	.000000
SDAAMR7	4465.000000	.000000
SDAPMR7	6763.000000	.000000
SEAAMR3B	.000000	.000000
SEAAMR5	3153.000000	.000000
SEAPMR3B	.000000	.000000
SEAPMR5	3905.000000	.000000
SEAPMR3A	.000000	.330000

THE TRANSPORT OPERATIONS PROBLEM  
Appendix - Problem Solution

ROW	SLACK OR SURPLUS	DUAL PRICES
WABAM)	3388.000000	.000000
WABPM)	5288.000000	.000000
WADAM)	.000000	1.370000
WADPM)	.000000	.870000
WAEAM)	.000000	1.870000
WAEPM)	.000000	1.220000
WBAAM)	3336.000000	.000000
WBAPM)	5347.000000	.000000
WBDAM)	.000000	1.870000
WBDPM)	.000000	1.170000
WBEAM)	.000000	2.420000
WBEPM)	.000000	1.560000
WDAAM)	.000000	1.840000
WDAPM)	.000000	1.340000
WDBAM)	.000000	1.870000
WDBPM)	.000000	1.170000
WDEAM)	.000000	.550000
WDEPM)	.000000	.390000
WEAAM)	.000000	1.920000
WEAPM)	.000000	1.270000
WEBAM)	.000000	2.420000
WEBPM)	.000000	1.560000
WEDAM)	.000000	.550000
WEDPM)	.000000	.390000
26)	.000000	1.100000
27)	.000000	.300000
28)	.000000	1.100000
29)	.000000	.300000
30)	.000000	.470000
31)	.000000	-.330000
32)	.000000	.470000
33)	.000000	-.330000
34)	.000000	.520000
35)	.000000	-.280000
36)	.000000	.050000
37)	.000000	.520000
38)	.000000	.440000
39)	.000000	.050000
40)	.000000	1.100000
41)	.000000	.800000
42)	.000000	1.100000
43)	.000000	.800000
44)	.000000	.690000
45)	.000000	.390000
46)	.000000	.690000
47)	.000000	.390000