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Abstract: This paper emphasizes Transportation and Assignment Problems

TRANSPORTATION
AND
ASSIGNMENT PROBLEMS

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I. FORMULATION OF TRANSPORTATION AND ASSIGNMENT PROBLEMS

An important class of linear programming problems can be formulated by using a special kind of network model. Transportation problems refer, simply, a selection of routes to transfer commodities from a number of sources to a number of destinations with the objective function of minimizing total cost. The classical transportation problem has one more characteristic that is the total demand equals to total supply. Cost of each shipment is proportional to the amount shipped.

General formulation is:

minimize $z=\sum_i \sum_j c_{ij}x_{ij}$

subject to:

$$\sum_{i=1}^{n} x_{ij}=a_j, \quad j=1,2,...,m \qquad (1)$$

$$\sum_{j=1}^{m} x_{ij}=b_i, \quad i=1,2,...,n \qquad (2)$$

where;

a_j *indicate supply amounts*

b_i *indicate demand amounts*

c_{ij} *indicate the costs of carrying one unit of* x_{ij} *from* i *to* j

By changing some constraints or adding some constraints, it is possible to formulate a number of problems as a special type of transportation problem.

Constraints (1) or (2) could be relaxed by changing the equality sign to inequalities (i.e. less than or equal to type constraints)

Physically, this simply means that more units may be available at the origins than are required at the destinations (or, more units may be acceptable at the destinations than are available at the origins).

These inequalities can be handled easily by the addition of m (or n) slack variables. Then the constraints become:

$$\sum_{i=1}^{n} x_{ij} + x_{in} = a_i, \quad j=1,2,\ldots,m \qquad (3)$$

$$\sum_{j=1}^{m} x_{ij} \quad = b_j, \quad i=1,2,\ldots,n \qquad (4)$$

As cost coefficient to these new slack variables, we could assign zero and then problem turns out to be a classical transportation problem.

We might note that after a transportation problem is formulated in the classical form (constraint sets (1) to (3)), we can replace each cost coefficient, c_{ij} , by $c_{ij}+Y$ for any constant Y without changing the x_{ij} which give an optimal solution [11]. This can be done because this substitution for each c_{ij} changes the objective function value only by the constant

$$Y\sum_{i=1}^{m} a_{ij}$$

Generalized Transportation Problem's formulation is different from transportation problem. It's formulation is stated by Hadley [11] as follows:

$$\textit{minimize (or maximize)} \qquad z=\sum_i \sum_j c_{ij}x_{ij}$$

subject to:

$$\sum_{j=1}^{n} d_{ij}x_{ij} \mp x_{si} = a_i, \quad a_i \geq 0, \quad i=1,2,\ldots,m \qquad (5)$$

$$\sum_{i=1}^{m} x_{ij} \qquad = b_j, \quad b_j > 0, \quad j=1,2,\ldots,n \qquad (6)$$

(Note that x_{si} can be considered to be slack or surplus variables)

One particular example could be the machine assignments for the generalized transportation problem. Two basic differences between transportation and generalized transportation problems are:

1. The rank of the matrix of the coefficients of the x_{ij} in (5) and (6) is, in general, (m+n) rather than (m+n-1). It means that all constraints are independent [11].
2. The optimal basic solutions may involve noninteger values of the x_{ij}, even though a_i , b_j are integers.

Transshipment Problem

The original transportation problem deals with the selection of shipping routes so as to minimize the cost of shipping a uniform commodity from specified origins to specified destinations. The amounts to be sent from each origin, the amounts to be received by each destination, and the cost per unit shipped from any origin to any destination are specified. Transshipment is not considered. Transshipment is that any shipping or receiving point is also permitted to act as an intermediate point in seeking an optimum solution. This technique is used to find the shortest route from one point in a network to another [14].

For the formulation of problem, let us assume N points which are either shippers or receivers. g_i , (i=1,2,...,N) is the net amounts to be shipped by each point. If a_i=amount shipped by each point including transshipment and b_i=amount received by each point including transshipment; they have to satisfy the equality $g_i=a_i-b_i$, i=1,2,..,N. c_{ij} is the unit cost of shipment from i to j, and $c_{ii}=0$.

Next step is to form the problem as a transportation problem with N destinations and N origins.

A transshipment problem is actually a transportation problem and by this formulation, we can conclude that this transportation problem can have 2(m+n)-1 variables different from zero [11]. However, (m+n) of these variables represent the stockpiles (shipping from origin i to destination i), there are no more then (m+n-1) variables of interest which are different from zero.

The Capacitated Transportation Problem

With the available solution techniques for transportation problems, it is easier to solve these problems treating with bounded variables. This is called as capacitated transportation problem. General formulation is same as the classical transportation problem except that there are additional upper-bound constraints on each variable:

$$0 \leq x_{ij} \leq d_{ij}, \quad for \quad \forall\ i, j$$

Assignment Problems

The term "assignment" describes the problem as finding the optimal way to assign n persons to n jobs. Assumption is that the individuals have various suitability index for a particular job [Dantzig, page-316]. This is a combinatorial problem. For example, if there are n individuals and n jobs for assignment, there are n! different possibilities for the assignment. Since the number grows rapidly, it is being seeked for more practical solution methods rather than checking every permutation combination.

Formulation of an assignment problem shows slight differences from transportation problem;

$$\text{minimize} \quad z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to:

$$\sum_{i=1}^{n} x_{ij}=1, \quad \text{for} \quad \forall j \qquad (7)$$

$$\sum_{j=1}^{n} x_{ij}=1, \quad \text{for} \quad \forall i \qquad (8)$$

$$x_{ij}=\begin{cases} 1, & \text{if the } i^{th} \text{ person is assigned to the } j^{th} \text{ job} \\ 0, & \text{otherwise} \end{cases}$$

$$c_{ij}\geq 0, \quad \text{for} \quad \forall x_{ij}$$

Generalized Assignment Problem

This problem, a generalization of the classical assignment problem. A better formulation would allow the assignment of several tasks to a single individual, provided these tasks do not require more of resource then is available to the individual. It can be seen as a specialized transportation problem in which the amount demanded at each destination must be supplied by a single origin if the resource required by individual i to do task j is constant for each individual i [Ross, Soland pp:91-93].

General formulation is as follows:

$$\text{minimize} \quad z=\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}$$

subject to:

$$\sum_{j\in J} r_{ij}x_{ij} \leq b_i, \quad \text{for } \forall\ i\in I \qquad (9)$$

$$\sum_{i\in I} x_{ij} =1, \quad \text{for } \forall\ j\in J \qquad (10)$$

$$x_{ij}=0 \text{ or } 1$$

In this formulation, I={1,2,...,m}, is a set of individuals, J={1,2,...,n} is a set of tasks. c_{ij}=cost if i assigned to task j, r_{ij}=the resource required by agent i to do task j, $b_i>0$ is the amount of resources available to i [Nauss, page-48].

There are many potential application areas for generalized assignment problems such as, assigning software development tasks to programmers [Ross, page-92].

Quadratic Assignment Problem

This is a more generalized formulation and has more difficulties to solve. Now, our extra conditions are;

- each individual must be assigned to exactly one member of tasks,
- each task must have exactly one member of individual assigned to it.

Typical applications include problems of facilities location, space allocation, scheduling and routing. These problems different from the classical linear assignment problem in that the members to be assigned are treated as a set of interconnected rather than independent objects [Ligget, page-442].

II. SOLUTION TECHNIQUES

2.1.1. Primal-Dual Algorithm

The primal-dual algorithm has been developed by Dantzig, Ford and Fulkerson, eliminates the problems coming from introducing artificial variables in two-phase method and Big-M method. Since the criterion used in two-phase and Big-M methods to select the variable to enter the basis is concerned with driving the artificial variables to zero, there is no guarantee that it works for optimality [Hadley, page-258].

Primal-dual algorithm introduces the artificial variables into the primal problem. But, the dual problem is used to determine which variables can enter the primal basis.

In other words, solution of the dual problem gives the entering vector of primal problem.

A new solution for the dual can be found which gives new entering vectors for primal basis maintained. Each solution to the dual gives the decrease in the dual objective [Hadley, p-258].

2.1.2. Primal Method for Transportation Problem

The well-known Hungarian Method is a dual method. Balinski and Gomory [1], describe a dual algorithm of Hungarian Method, so they called their technique as "Primal" technique. The Hungarian Method provides at each intermediate computational step a dual feasible vector (U,V) and a corresponding infeasible vector X orthogonal to first vector. Balinski-Gomory technique provides at each step a feasible X vector (a solution of transportation or assignment problem) and a corresponding orthogonal infeasible (U,V) vector. As a transportation problem, let's consider;

$$\textit{minimize} \quad \alpha(X)=\sum_{i,j} a_{ij}x_{ij}$$

subject to:

$$\sum_{j} x_{ij}=b_i$$

$$\sum_{i} x_{ij}=c_j$$

$$x_{ij}\geq 0$$

The dual problem is:

$$\textit{maximize} \quad \beta(U,V)=\sum_{i} b_i u_i+\sum_{j} c_j v_j$$

subject to:

$$u_i+v_j \leq a_{ij}, \qquad \forall\, i\,,j$$

If $u_i+v_j \leq a_{ij}$, condition for all (i,j), then X and U, V constitute optimal solutions. Otherwise, there exists some entry *(k,l)* such that $u_k+v_k > a_{kl}$.

The following conditions show a new optimal pair, X' and U', V' with X' feasible, for the computational stage l (column l).

$$u_i' + v_j' \leq a_{ij} \quad \text{if } u_i + v_j \leq a_{ij}$$

$$u_i' + v_j' \leq a_{il} \quad \text{for all } i, \text{ column } l$$

This new pair satisfies $\alpha(X') \leq \alpha(X)$ [1, page-586].

Each new derived transportation problem is considered identical to the first problem except that column l is replaced by two columns l_1 and l_2 having nonnegative integer demands. At every step, there are a derived transportation problem and its dual problem.

2.2. Transshipment Problem

The main idea of this problem is to find the shortest route from one point in a network to another. This is an extension of original transportation (T_0) problem [Orden, page-277].

Dantzig's simplex technique provides a satisfactory computation for transshipment problem (T_1) [Orden, page-278].

The general procedure of the solution is as follows : T_1 is to be converted to the form of T_0 by treating each point as shipping and receiving points. The unit cost of shipment from a point to the same point is equal to zero. Orden presents the procedure as follows:

1. Let the total amount of shipped equal to be to the total amount of received. c_{ij} is the specific cost of shipment from point i to j. $c_{ii}=0$ and gi is the specified net amounts of shipping (note that some g_i are positive and some are negative). If a_i=amount shipped by each point including transshipment and b_i=amount received by each point including each point then $g_i=a_i-b_i$ has to hold for i=1,2,...,M.

2. Set up the transshipment problem in the form of a transportation problem in which there are M origins and M destinations. The amounts to be shipped , a_i, and the amounts received, b_i, must be greater then zero.

$$\text{If } g_i > 0, \text{ set } a_i^0 = g_i, \; b_i^0 = 0$$

$$\text{If } g_i < 0, \text{ set } a_i^0 = 0, \; b_i = |g_i|$$

Let $a_i' = a_i^0 + s$ *and* $b_i' = b_i^0 + s$ *(A)*

where s is a positive constant (a stock pile)

3. Compute the minimum cost solution to the T_0 problem. Let C' be the total cost and x_{ij}' be shipments of the minimum cost solution.

4. Discard x_{ii}' , because redundant, items. Since each x_{ii}' is contained in both the amount shipped a_i' and the amount received b_i', may be deducted from both

$$a_i'' = a_i' - x_{ii}'$$

$$b_i'' = b_i' - x_{ii}' \qquad \text{(B)}$$

A feasible solution to the T_0:

$$x_{ij}'' = x_{ij}'$$

$$x_{ii}'' = 0 \qquad \text{(C)}$$

5. Convert the results in the form (B) and (C) to the solution to the transshipment problem. The min cost solution involves (M-1) point-to-point paths for which $x_{ij} \neq 0$.

2.3. Assignment Problem Solution Techniques

2.3.1. The Alternating Basis Algorithm for Assignment Problems

When solving assignment problems by simplex method we face with the unnecessary inspection of alternative basis representations of the extreme points. Barr, Glover and Klingman's [3] alternating basis algorithm for assignment problems reduces the convergence time for degenerate problems compared to simplex method. They show that if an assignment problem has a feasible solution, optimal solution can be found by considering only bases of this type [3,page-2]. In this method, problem solver does not have to consider all feasible bases to be candidates for processing to an optimal basis. It has been shown that if assignment problem has an optimal solution then it also has an optimal solution with the unique basis tree structure, completed with the Alternating Path (AP) structure. So, it is not necessary to check all points in a "feasible spanning tree" like in the case of simplex method. Barr, Glover and Klingman state that "AB algorithm is a procedure to exploit the properties of the AP basis structure in a manner that substantially reduces the impact of degeneracy" [3]. Giving some definitions makes easy to understand the algorithm. They define "Alternating Path" (AP) basis with the help of following conditions:

1. The root node is an origin node.
2. All 1-links are Origin-Destination links.
3. All 0-links are Destination-Origin links.

If a rooted basis tree for an assignment problem has above specifications, it becomes AP basis. They represent the assignment problem as a graph, consisting of a set of origin nodes with unit supplies and a set of destination nodes with unit demands. Directed arcs from origin nodes to destination nodes accommodate the flow and cost involved if the flow exists. Their algorithm basically;

1. Find an initial any feasible AP basis for the assignment problem. An artificially feasible initial AP basis can be constructed for an nxn assignment problem by assuming that arcs exist from each origin node to all destination nodes where the artificial arcs have a "Big-M" cost.
2. Successively apply the simplex pivot procedure keeping the root fixed and picking the link to leave. When the simplex method is applied to AP, the pivot can be carried out give a new AP basis for any entering non-basic arc by dropping the unique link in the basis equivalent path attached to the origin node of the entering arc.

Before the criteria are satisfied, the procedure does not stop. Authors of this algorithm have also proved that Alternating Path algorithm is finitely converging without using any external disturbance.

2.3.2.Bertsekas' Algorithm for Assignment Problem

Bertsekas' algorithm [4], is using some basic definitions which are common in Hungarian Method. Both algorithms involve flow augmentations along with alternating paths, changes in the dual variables. Difference is coming from the roles of these definitions.

In Bertsekas' method, augmentation is being used when augmentation is no more possible to continue the process of increasing prices of assigned sinks without violating the complementary slackness constraint.

This algorithm is also using the dual and primal variables together. In his paper, Bertsekas explains his algorithm with a maximization problem for which the dual problem is:

$$\textit{minimize} \quad z=\sum_{i=1}^{N} m_i+\sum_{j=1}^{N} p_j$$

subject to:

$$m_i+p_j \geq a_{ij}\ , \quad \forall\ (\ i\ ,\ j\ )\in L.$$

where p_j=prices and m_i=profit margins, and L=set of directed links.

His algorithm can be stated as below [4, pp:155-157]

- Initialize the problem

$$m_i^{k+1} + p_j^{k+1} = a_{ij} \qquad \textit{if (i,j) is an assigned pair}$$

$$m_i^{k+1} + p_n^{k+1} \geq a_{in} \qquad \textit{for every (i,n) unassigned pair}$$

- (k+1)[st] iteration of the algorithm

Choose a source i' which is unassigned.

Compute the maximum profit margin

$$m' = \max \{a_{ij}' - p_j^k \mid (i', j) \in \textit{unassigned}\ \}$$

$$m' = a_{ij} - p_j^k$$

Compute also the "second maximum", m", profit margin.

Then according to equalities or inequalities of m' and m", Case.1 and Case.2 procedures came. After the k[th] iteration of the algorithm some of the items have been assigned to persons that have prices p_j^k.

If Case.1 holds at the (k+1)[st] iteration, m' > m" or m'=m" and sink j' is an unassigned one, the unassigned i' selects this item j' that offers maximum profit margin.

Case.2 is that the m'=m" and for some i, was assigned to any item before, we have (i,j') pair. In this case, algorithm is trying to find an augmenting path not containing j' from source i" to an unassigned sink. There are two possibilities: Either an augmented path will be found, or a change in the dual variables will be effected.

For the first case, the old link is retained. For the second case, the old link (i,j') is replaced by (i',j') in X_{k+1} step and no new sink is assigned. In this case, the dual variables change. After this step, augmentation and change of variables come.

Termination comes in this algorithm with a change in the dual variables and a new iteration is started with a new unassigned sink. The iteration can terminate with a flow augmentation, and a change in dual variables.

2.3.3. Signature Method for the Assignment Problem

This method is a dual simplex method and developed by Balinski [2]. In this solution technique, each step goes from one dual feasible basis to a neighboring one. A difficulty comes from ignoring the primal problem. This difficulty is that the method may encounter a dual feasible basis that already gives an optimal assignment, prior to its termination [Balinski, p:527].

Signature method searches among dual feasible bases one that has signature.

The "signature" of a tree T is the vector of its row node degrees $a=(a_1, a_2,\ldots,a_n)$

$$\sum_i a_i = 2n-1$$

$$a_i \geq 1$$

Balinski also defines the "signature" of a dual feasible basis of the assignment problem as : "n-vector whose i^{th} component is the number of nonbasic activities of type (i,j)". His theorem is :

If T(u,v) is a tree with some one row node i* of degree 1 and the remaining rows of degree 2, then the permutation k defined as follows solves the assignment problem:

$$k(i^*)=j \qquad \text{for } (i^*,j) \in T(u,v)$$

$$k(i)=j \qquad i=i^* \ \text{ for } (i,j) \in T(u,v)$$

This theorem, and the signature methods, proposes a solution which is looking for a tree whose signature contains exactly one 1 and otherwise 2,s. This method, iterates from one tree T to another T'. T' is obtained by pivoting.

2.3.4. Branch and Bound Algorithm for the General AP

Ross and Soland [15], propose an algorithm for general AP. In Ross and Soland's algorithm, the bound is calculated in part by solving binary knapsack problems rather than using linear programming. Relaxed problem gives the lower bound. The lower bound is increased, in this algorithm, by the sum of the values of the objective functions obtained from solving for each binary knapsack problem. The solution of this knapsack problem shows the tasks which must be reassigned from agent i to another agent in order to satisfy the resource restriction on agent i. The optimal solution of these knapsack problems indicates those reassignments that lead to a minimal increase for the value of z.

By using binary-knapsack approach it is possible to assign new values to lower bound. Additionally, the solutions of these problems indicate new assignments to agents that could be a feasible solution.

2.3.5. Overview of Quadratic Assignment Problem Algorithms

In most of the cases, it is hard to solve quadratic assignment problems computationally. So, heuristic techniques are being employed to get a reasonable solution in real problems. Typical applications of quadratic assignment problems are : facilities location, space allocation, scheduling and routing. Ligget [12], examined the effective heuristic methods and compared them. Quadratic assignment problem solving methods has been classified into two groups : constructive initial placement techniques and iterative improvement techniques [Ligget, p:442].

a. Constructive Techniques

This method was developed by Graves and Whinston [10]. It combines the enumerative procedure with probability theory to construct an implicit enumeration algorithm. Ligget states these techniques as "n-stage decision process for intelligently building a solution from scratch". It has been added that constructive methods can be considered to take either "local" or "global" orientation to problem solving. Whinston-Graves method gives a global constructive procedure.

b. Improvement Techniques

A solution found by an improvement method, highly depends on the initial starting solution. Usually these starting solutions are generated randomly [Ligget, pp:442-443]. After finding an initial solution, method tries to improve the solution. Improvement methods differ according to the exchange selection process.

III. APPLICATIONS-DISCUSSION

3.1. Possible Applications in Industry

There is a wide range of applications for which transportation method of LP modelling is well suited. Some of the more typical applications may be summarized as follows:

1. Product Distribution

The objective in this application is to minimize the cost of serving multiple destinations from multiple sources of supply. The typical case involves to determine which particular factories should send specific destinations (markets or warehouses).

The simple version of this problem allocating the production of suppliers to particular markets by the objective function of minimizing total transportation cost. In this case it is assumed that cost is directly proportional to the amount shipped.

Other case could be to find the combined costs of production and distribution by the objective function of minimizing total cost. If the sales prices are a function of the supplier, we can enlarge the problem by the objective function of maximizing the total profit of serving the aggregate market. This involves explicit consideration of production costs, transportation costs, and market revenue [Siemens, page-124].

2. Production Planning and Scheduling

Our objective in this case is to develop a minimum cost production plan to serve the anticipated aggregate demand during some specified planning period. Again we could have different special cases related to production planning problem.

Production costs may be different for different plants, transportation costs are variable, and inventory costs must be considered when producing for inventory. The optimal production schedule involves determining which plants should be utilized during each production period, for regular or overtime.

For example, if a plant's products demand fluctuates over a period of time, there are three different ways of adjusting the production of the output to meet the demand :

a. Change the level of the regular production

b. Use overtime production at the necessary period

c. Store the present excess to cover future shortages [Chvatal, pages-322,323].

For the products which are not usable over a time period these options change. Different type of formulations are possible.

The problem of scheduling regular and overtime production can be solved by the transportation method. The problem is to schedule production so that storage (inventory) costs are balanced against overtime costs by minimizing the total cost.

General formulation can be in the form:

$$\textit{minimize} \quad z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_i x_{ij}+\sum_{i=1}^{n}\sum_{j=1}^{n} d_i y_{ij}+\sum_{j=2}^{n}\sum_{i=1}^{j-1}\left\{\sum_{k=i}^{j-1} f_k\right\}(x_{ij}+y_{ij})$$

subject to:

$$\sum_{i=1}^{j} x_{ij} + \sum_{i=1}^{j} y_{ij} = b_j , \quad j=1,2,\dots,n \qquad (D)$$

$$\sum_{j=1}^{n} x_{ij} \le a_i , \quad i=1,2,\dots,n \qquad (P_1)$$

$$\sum_{j=1}^{n} y_{ij} \le a_i' \quad i=1,2,\dots,n \qquad (P_2)$$

$$y_{ij} , x_{ij} \ge 0 \quad \forall\, i , j$$

[*Hadley*, *pages*:440–441]

Where (P_1) and (P_2) are the production constraints and (D) is the demand constraint.

x_{ij}-number of units of the product produced on regular time in period i for sale in period j

y_{ij}-number of units produced on overtime in period i for sale in period j

a_i-number of units produced in regular time, period i

a_i'amount of product produced in overtime and period i

b_j-number of units demanded in time period j

f_k-storage charge

3. Allocation of Production Facilities (machine Assignment Problem)

When a variety of jobs (or products) can be performed on a number of different machines which have different output capacity and cost, it is necessary to allocate the various jobs to the several machines by minimizing cost, time, etc. Formulation of this type of problem gives us the generalized transportation problem.

4. Personnel Assignment

In this case, objective function could be to minimize cost by assigning proper individuals to proper jobs. Another case is to maximize utilization of personnel. Each particular personnel has different preference for each job. Assignment problem formulation handles these kind of applications.

5. Assembly Line Balancing

The objective of assembly line balancing is to allocate work elements to various lines. Purpose of this study is to make the work load uniform. Another objective can be to minimize the total idle time [Siemens, page-125].

3.2. Plant Location Problems

There is a close relationship between transportation problem formulation and plant location problem. Efroymson and Roy [9], state the location as "a transportation problem with no constraint on the amount shipped from any source". If the plant is "closed", there is no cost associated to this route. If the plant is "open", the cost is positive and independent from the amount shipped. We can't treat this problem as a linear problem, because the fixed charge associated with each plant does not vary linearly with the amount shipped from the plant [9,page-361].

Efroymson and Roy have given a new formulation to this problem. They use;

N_k : the set of plants that can supply customer k

P_i : the set of those customers that can be supplied from Plant i

$$\textit{minimize} \quad z=\sum_i \sum_j c_{ij}x_{ij}+\sum_i f_i y_i$$

$$\textit{subject to:}$$

$$\sum_{i\in N_j} x_{ij} = 1 , \qquad j=1,2,...,n$$

$$0\le \sum_{j\in P_i} x_{ij} \le n_i y_i , \qquad i=1,2,...,m$$

$$y_i \textit{ is } 0 \textit{ or } 1 , \quad \textit{for} \quad i=1,2,...,m$$

Then they conclude that the optimal solution is;

$$x_{ij}=\begin{cases} 1, & \textit{if} \quad c_{ij}+\dfrac{g_i}{n} = \min_{k\in K_1\cup K_2} [c_{kj}+\dfrac{g_n}{n_k}] \\ 0, & \textit{otherwise} \end{cases}$$

$$y_i=\left(\frac{1}{n}\right)\sum_{j\in P_j} x_{ij}, \qquad \textit{for } \forall \; i\in K_2$$

$$\textit{where } g_k$$

$$g_k=\begin{cases} f_k, & k\in K_2 \\ 0, & k\in K_1 \end{cases}$$

K_1,K_0 are the sets of y's that are fixed at 1, 0 respectively and

K_2 is the set of indices of remaining y's

One disadvantage of their formulation is the use of small number of plant k. On the other hand, they also extended their studies to handle plant location problems which has a variable plant cost as well as a fixed cost [9, pp:364,365].

Cooper [7], dealt with the transportation location problems. In his paper, he described an enumeration algorithm and two heuristic algorithms for the problem. In enumeration algorithm, after setting the transportation tableau, he generates the connected graph of all basic feasible solutions. Then for each solution, solves the set of location problems.

Cooper's first heuristic is "alternating transportation location". Basic idea in this heuristic is to locate sources alternatively, given a pattern of allocations and to determine an allocation given a set of source locations [7, page-104]. The drawback of this method is coming from convergence problem. There is no guarantee for the convergence to the global maximum.

3.3 Comparison of the Methods

It has been summarized several solution techniques for transportation and assignment problems. Each one has different specifications, but the purpose of these studies is same; to find a more efficient algorithm.

Balinski-Gomory's primal method for the assignment and transportation problems gives a chance to bound the number of steps required to solve these problems. The best bound is $n(n+1)/2$ labeling for the $n*n$ assignment problem. This bound is the same as the best known bound for the Hungarian Method [1,page:578]. But their algorithm does not require the use of a basic solution which is necessary for the simplex method and which makes the problem more difficult to solve (due to degeneracy).

Major differences are involved in the Alternating Basis Algorithm [3,page:2]. These basic differences between this algorithm and previous primal extreme point methods are;

1.the rules of the algorithm automatically gives the special structure basis,

2.the algorithm finitely convergent without using external techniques,

3.in some problems degenerate basis exchanges.

Bertsekas gives some comparison results between his technique and the Hungarian method. Bertsekas' algorithm can converge ten or more times faster than the Hungarian method with the $N > 100$ where N is the number of assigned people.

Signature method described by Balinski[2] solves the assignment problem at most $(n-1)(n-2)/2$ basis changes. This number is better than the known $n(n+1)/2$ steps.

As discussed before, for the quadratic assignment problem, there is no well established algorithm. Some heuristics usually gives a good feasible solution, by the consideration of cost.

In general, LP problems have a common difficulty: Data availability. To come up with a realistic formulation, problem solver needs accurate data. Same difficulty exists for transportation and assignment problems. In the simplest form of transportation problem, we need predetermined supply and demand numbers. In assignment problems usually it is difficult to quantify utility involves for each individual. In a well documented environment determination of the cost associated with each route in the transportation problems could not be a big problem. But, most of the time setting these data is a tedious job.

Additionally, for the real, large scale problems we may need some manipulation on formulation of problem to get a transportation or assignment problem. This kind of relaxation saves time to solve big, complex problems.

IV. EXAMPLE PROBLEM

For the illustration of an assignment problem Carpaneto, Martello, and Toth's computer code was used [5]. Their algorithm solves the assignment problem to give minimum sum of cost.

Linear minimum sum assignment problem defined as "given a square matrix of order n, assign each row to one column, and vice versa, so as to minimize the cost sum of the row-column assignment" [5,page:193]. Their algorithm is based on primal-dual algorithms.

Their code is using APC subroutine to find the minimum cost assignments. This subroutine needs INCR, INIT, and PATH subroutines. In the second case, subroutine APS gives the solution of the minimum sum assignment problem for the sparse matrix. With the given data set program was run. Output and the code of algorithm are added to the paper.

Meaning of output parameters:

z= cost of the optimal assignment= .....

F(I)= column assigned to row I

Meaning of input parameters:

N= number of rows and columns of the cost matrix.

A(I,J)= cost of the assignment of row I to column J.

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