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Abstract: This report describes a linear programming (LP) model to minimize the parts placement cost of a printed circuit board by determining the optimal combination of component mounting techniques while operating within severe constraints. There are two primary methods of placement available to printed circuit board manufacturers today: surface mount (SMD) and through-hole (THD). To rationally compare these two methods, it is assumed that engineers designing the PCB may choose either SMD or THD technology. Both production methods yield identical function, although the physical characteristics of comparable boards differ.

PRINTED CIRCUIT BOARD PLACEMENT
COST OPTIMIZATION

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*Good, but the following
enhancements are
needed:*

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- Discussion of results
summarizing the
model results
and comparing
with sensitivity
analysis in the
integrating section*
- Comparison of
your results with
current performance
of the system*

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Executive Summary

This project describes a minimization, using linear programming techniques, of the cost of placing various types of components on a printed circuit board.

There are two types of placements available to choose from, through-hole design and surface mount design, as well as two different mounting techniques for each of the above, standard placement and unique placement. Each placed component has a specific cost for placement and a specific area associated.

The objective of this formulation will be to determine the optimal combination of component mounting techniques while operating within a given circuit board size and number of required components. These are fixed constraints in our formulation, but can be varied by the design engineer to alter this optimization program to his specific needs.

The data used for the model formulation was based on data from an actual circuit board design, which yielded the size of the board and the number and types of components required. Costs of the placement techniques were obtained from a local circuit board manufacturing company.

In our study we found that our model was not sensitive to the cost of through-hole placements. These are placed by hand and directly reflect labor rates. Our model was insensitive to labor rates due to the larger size of the through-hole components.

We found that the solution will pick the surface mount designs as long as they are mounted using standard automated placement. If a unique placement is required, the solution will favor the use of through-hole designs, which are less expensive, only if there is enough area on the circuit board to accommodate these larger parts. By either increasing the area of the circuit board or by developing a board which does not require these unique components, we have the possibility to reduce the placement cost by 47%. There is a great incentive then to limit the number of uniquely placed components in the design and function of the PCB.

2. INTRODUCTION

The purpose of this project is to use a linear programming (LP) model to minimize the parts placement cost of a printed circuit board. There are two primary methods of placement available to printed circuit board manufacturers today: surface mount and through-hole. For the remainder of this project, we refer to printed circuit boards as PCB, to surface mount devices as SMD, and to through-hole devices as THD. To facilitate rational comparison between the two methods, we have assumed that hardware engineers designing the PCB may choose either SMD or THD technology; either production method will yield identical function, although the physical characteristics of two comparable boards will differ. It is also possible to mix the two methods. To simplify the analysis and make comparison between THD and SMD more clear, we ignored the technical capability to mix production methods.

An engineer may normally choose either type of component, SMD or THD, before laying out the PCB for production. Once he has decided on the production method to be used and has finalized the board layout, he generally cannot change that decision. The choice will affect the total board size and the cost of production. In today's increasingly competitive and technologically demanding market, even a small "edge" in production cost can give one producer a large advantage over another. This fact makes the engineer's initial decision on whether to use THD or SMD methods all the more critical!

An engineer starts the decision process with a parts list, driven by the desired capabilities of the PCB under review. The end capability of the PCB, and the schematic describing it, are beyond the scope of this project.

In general terms, a PCB configured as we describe in the problem formulation and solutions phases of this analysis could serve a function similar to :

- act as a storage buffer between a computer keyboard and a memory unit.
- relay information between a cash register and a central pricing/ credit system.
- transmit or store information between larger components of a communications package.

When analyzing PCB manufacture, it is important to understand that parameters governing function and layout often drive the engineer to make different decisions for each PCB designed.

2.1 Assumptions

This group made eleven key assumptions and based its problem formulation and conclusions on them.

1. SMD components may be placed only by automated devices.

2. THD components may be placed only manually.
3. Non-standard (unique) SMD require a special loader to function with the automated placement machine.
4. The production facility has unlimited assembly capability available for one production run of a given PCB.
5. The engineer knows in advance the quantity of time it will take to assemble a PCB either manually or with AD.
6. The placement cost for assembly is fully burdened by manufacturing overhead (MOH), machine cost and personnel benefits such as insurance, retirement , and social security.
7. THD placement requires a special device to set up the builder's assembly line. It is called a parts placement machine (PPM). The PPM represents a large initial start-up cost.
8. Certain SMD components can be mounted only if a specialized loader is mounted on the PPM. Mounting these loaders incurs a unique setup charge for each SMD part.
9. Although it has utility to the manufacturer after our PCB production run is complete, we assume that the purchase cost is amortized exclusively during this run.
10. The production rate for machine assembly versus manual assembly is irrelevant for this analysis. Therefore, there is slack assembly resource.
11. If a manufacturer chooses automated assembly the firm will need to purchase the necessary equipment. This purchase will influence the per unit cost of parts placement built with automated techniques; as with the PPM, initial cost is very high. Once purchased, machine operation and maintenance costs are extremely low, approaching zero.

2.2 Applications

As the group progresses through its analysis of PCB manufacture, we found variables that impacted our production LP model in ways we had initially overlooked. The new LP model incorporating these variables is depicted in the problem formulation section. The model may be expanded to deal with other relevant manufacturing functions besides simple assembly, such as testing or packaging. Finally, it may be expanded to reflect multiple types of PCBs with various design functions.

The Lindo system proved itself indispensable in analyzing various PCB configurations and parameters. It pinpointed the optimal size of SMD and THD circuit boards quickly. More important, Lindo allowed us to make incremental changes in design parameters and constraints which would have taken an inordinate amount of time to compute manually or with hand-produced matrices.

Lindo's greatest contribution, however, comes in the area of sensitivity analysis. The sensitivity analysis is a particularly critical phase of this project for two reasons:

1. The sample PCB we studied is rather generic in design. It does not have an extensive production history pointing to an optimal size and production method. Without this known "track record", our PCB would be a new entity on the hardware designer's drafting table or screen.
2. The cost of placement for PCB components is very uniform and predictable. For example: resistors, transistors and integrated circuits for THD circuit boards all have placement costs of 12 cents each. The same components for a unique SMD circuit board cost 4 cents each. Unique SMD parts cost \$1.20 each to place. Analyzing the effects of incremental changes in these prices would be extremely tedious and would often yield trivial results. The Lindo system allows us to make bold adjustments and observe significant results immediately.

3. PROBLEM DEFINITION

Once the engineer designs electronic circuit and has captured the functional design in schematic form, his next decision is to determine the optimal packaging solution for his application. Components with identical functional performance are available in either a Through-Hole or Surface Mount technology. They differ only in the size and area required to place them on a printed circuit board, and in the method required for their placement. Generally the area and size of the SMD component is smaller than that of the THD, and the placement cost of THD is greater than that of SMD. Additionally, some of both types of components are not standard, but are unique, which will require special setups and equipment to place. The material costs for both types of components are typically the same and will not be considered in our formulation.

Generally the engineer has the flexibility to chose either type of component package, but once his decision is made it is very difficult to reverse that decision due to the major impact in would have on the cost and schedule of the project. The purpose of this formulation is to give the engineer some guidance and insight into what type of components to choose for the printed circuit board layout. As a result he may decide to change the design in order to meet established targets, whether it be cost or physical size of the board.

The objective is to determine the optimal combination of component packages that will result in the minimum cost for that board design while staying within specifications. Constraints consist mainly of available surface area for the PCB, and the quantity of parts for this PCB by part category.

3.1 Formulation Definitions

Following are the various device categories that can be in the design:

Category 1: Consists of two lead discrete devices such as resistors, capacitors, and diodes.

Category 2: Consists of three lead transistors.

Category 3: Consists of 8 pin integrated circuit.

Category 4: Consists of 14,16, and 18 pins integrated circuit.

Category 5: Consists of 20, and 24 pin integrated circuit.

Following are board area required, and the costs for placing each component by category:

Parts Category	THD			SMD		
	Standard Placement Cost (\$)	Unique Placement Cost (\$)	Area Required (inch ²)	Standard Placement Cost (\$)	Unique Placement Cost (\$)	Area Required (inch ²)
Category 1	0.12	0.12	0.08	0.04	1.2	0.02
Category 2	0.12	0.12	0.03	0.04	1.2	0.03
Category 3	0.12	0.12	0.20	0.04	1.2	0.04
Category 4	0.12	0.12	0.40	0.04	1.2	0.28
Category 5	0.12	0.12	0.80	0.04	1.2	0.65

The setup charge for using a standard and/or unique THD is \$7.00 regardless of the category.

The setup charge for using a standard SMD is \$0.0

The setup charge for using a unique SMD is \$1.25 per category used.

Following are specifications for the design being formulated:

- Available board area: 8.75 (inch²)
- Number of standard components of category 1: 282
- Number of standard components of category 2: 50
- Number of standard components of category 3: 4
- Number of standard components of category 4: 2
- Number of standard components of category 5: 0
- Number of unique components of category 1: 10
- Number of unique components of category 2: 4
- Number of unique components of category 3: 0
- Number of unique components of category 4: 2
- Number of unique components of category 5: 0

4. MODEL FORMULATION

4.1 Decision Variables

X's: Represent Through-Hole Mounted Devices(THD).

Z's: Represent Surface Mounted Devices(SMD).

X1s: Represents the number of category 1 devices, which are standard and Through-Hole mounted.

X2s: Represents the number of category 2 devices, which are standard and Through-Hole mounted.

X3s: Represents the number of category 3 devices, which are standard and Through-Hole mounted.

X4s: Represents the number of category 4 devices, which are standard and Through-Hole mounted.

X5s: Represents the number of category 5 devices, which are standard and Through-Hole mounted.

X1u: Represents the number of category 1 devices, which are unique and Through-Hole mounted.

X2u: Represents the number of category 2 devices, which are unique and Through-Hole mounted.

X3u: Represents the number of category 3 devices, which are unique and Through-Hole mounted.

X4u: Represents the number of category 4 devices, which are unique and Through-Hole mounted.

X5u: Represents the number of category 5 devices, which are unique and Through-Hole mounted.

Sx: Indicates the selection of a Through-Hole mounted device.

Z1s: Represents the number of category 1 devices, which are standard and surface mounted.

Z2s: Represents the number of category 2 devices, which are standard and surface mounted.

Z3s: Represents the number of category 3 devices, which are standard and surface

mounted.

Z4s: Represents the number of category 4 devices, which are standard and surface mounted.

Z5s: Represents the number of category 5 devices, which are standard and surface mounted.

Z1u: Represents the number of category 1 devices, which are unique and surface mounted.

Z2u: Represents the number of category 2 devices, which are unique and surface mounted.

Z3u: Represents the number of category 3 devices, which are unique and surface mounted.

Z4u: Represents the number of category 4 devices, which are unique and surface mounted.

Z5u: Represents the number of category 5 devices, which are unique and surface mounted.

Sz1u: Indicates the selection of Z1u devices.

Sz2u: Indicates the selection of Z2u devices.

Sz3u: Indicates the selection of Z3u devices.

Sz4u: Indicates the selection of Z4u devices.

Sz5u: Indicates the selection of Z5u devices.

4.2 Objective Function

Minimize:

$$\begin{aligned}
 &0.12X_{1s} + 0.12X_{2s} + 0.12X_{3s} + 0.12X_{4s} + 0.12X_{5s} + \\
 &0.12X_{1u} + 0.12X_{2u} + 0.12X_{3u} + 0.12X_{4u} + 0.12X_{5u} + \\
 &7.00S_x + \\
 &0.04Z_{1s} + 0.04Z_{2s} + 0.04Z_{3s} + 0.04Z_{4s} + 0.04Z_{5s} + \\
 &1.20Z_{1u} + 1.20Z_{2u} + 1.20Z_{3u} + 1.20Z_{4u} + 1.20Z_{5u} + \\
 &1.25S_{z1u} + 1.25S_{z2u} + 1.25S_{z3u} + 1.25S_{z4u} + 1.25S_{z5u}.
 \end{aligned}$$

Where the coefficients of X^{**} and Z^{**} are the costs of parts placement. The coefficients of S_x and S_z^{**} are the setup charges to use these parts.

Subject to:

The board area constraint:

$$\begin{aligned}
 &0.08X_{1s} + 0.03X_{2s} + 0.20X_{3s} + 0.40X_{4s} + 0.80X_{5s} + \\
 &0.08X_{1u} + 0.03X_{2u} + 0.20X_{3u} + 0.40X_{4u} + 0.80X_{5u} + \\
 &0.02Z_{1s} + 0.03Z_{2s} + 0.04Z_{3s} + 0.28Z_{4s} + 0.65Z_{5s} + \\
 &0.02Z_{1u} + 0.03Z_{2u} + 0.04Z_{3u} + 0.28Z_{4u} + 0.65Z_{5u} \leq 8.75
 \end{aligned}$$

Where the coefficients are the board areas required by the individual components.

The total number of standard category 1 devices constraint:

$$X_{1s} + Z_{1s} = 282$$

The total number of standard category 2 devices constraint:

$$X_{2s} + Z_{2s} = 50$$

The total number of standard category 3 devices constraint:

$$X_{3s} + Z_{3s} = 4$$

The total number of standard category 4 devices constraint:

$$X_{4s} + Z_{4s} = 2$$

The total number of standard category 5 devices constraint:

$$X_{5s} + Z_{5s} = 0$$

The total number of unique category 1 devices constraint:

$$X_{1u} + Z_{1u} = 10$$

The total number of unique category 2 devices constraint:

$$X_{2u} + Z_{2u} = 4$$

The total number of unique category 3 devices constraint:

$$X_{3u} + Z_{3u} = 0$$

The total number of unique category 4 devices constraint:

$$X_{4u} + Z_{4u} = 2$$

The total number of unique category 5 devices constraint:

$$X_{5u} + Z_{5u} = 0$$

Through-Hole devices setup charge constraint:

$$X_{1s} + X_{2s} + X_{3s} + X_{4s} + X_{5s} + X_{1u} + X_{2u} + X_{3u} + X_{4u} + X_{5u} - 10000S_x \leq 0$$

Z_{1u} setup charge constraint:

$$Z_{1u} - 10000S_{z1u} \leq 0$$

Z_{2u} setup charge constraint:

$$Z_{2u} - 10000S_{z2u} \leq 0$$

Z_{3u} setup charge constraint:

$$Z_{3u} - 10000S_{z3u} \leq 0$$

Z_{4u} setup charge constraint:

$$Z_{4u} - 10000S_{z4u} \leq 0$$

Z_{5u} setup charge constraint:

$$Z_{5u} - 10000S_{z5u} \leq 0$$

Our linear program model sets up a tension between the choice of a particular part type and the setup charge in the objective function for that part style. The program will attempt to remove the setup charge by reducing the number of parts in either SMD or THD placement to zero unless otherwise constrained to use a part style.

5. DISCUSSION

This formulation is meant to be a tool used by the engineer in determining parts choice. A Linear Programming formulation of this problem is flexible in that it can be made as specific as desired, by adding parts categories, to as fine a level of detail as desired. Parts quantities in specific categories left at zero do not impair the solution. In this formulation, problem size can be accommodated by solving a large problem in sections with some part categories compared in this section and the remaining part categories in another section.

Lindo has an advantage over a spreadsheet in this case because Lindo is designed to shift decisions in response to costs as constrained by limits of the situation. This branching capability would be tedious to program with a spreadsheet.

The sensitivity analysis in Linear programming can be viewed the same as studying partial differentials of a differential equation. That is, several coefficients can remain constant while another is varied. A spreadsheet program is valuable to this study because it can output a text file appropriate for Lindo input. Changing numerical values in the spreadsheet is quite easy since many of the coefficients can be changed at once by a constant ratio. Thus groups of coefficients are changed with little effort.

5.1 Examination of the Area Constraint

Considering each solution of this problem to be equivalent to solving partial differential equations, the values are first set at the cost coefficients in the Objective Function to the present costs incurred in circuit board assembly. Second; let us set the individual part areas to those required by the components for the present circuit board layout technology. By varying the Right Hand Side (RHS) value of the circuit board area constraint, the smallest possible size of the PCB can be found by noting the area at which the solution just becomes feasible as the board area is increased in size from zero. Note also the cost of the Objective Function at this size. Next increase the area of the board until the Objective Function no longer decreases with further increases in size. Note the value of the slack variable in the area constraint. Subtracting this value from the present area gives the most economical large area. The difference in the two values of the Objective Function is the cost differential for the size difference.

Individual areas cannot be smaller than the physical size of the parts, except that SMD parts may be placed on both sides of the circuit board without significant increase in cost. In the formulation, the areas of SMD parts may be changed to $1/2$ of the values for single sided boards to accommodate a double sided policy. THD components may not be placed on both sides of the board without significant increase in cost.

Board area required is also a function of the technological capability of the layout process for the circuit board. As the technology increases, the area per component decreases. However, as board area increases, the number of circuit board layers may be reduced, and this decreases costs of the manufacture of the bare circuit board. This cost comparison is beyond the scope of this problem.

Board areas required by the individual components can be changed to represent various circuit board parts densities. Some layout methods will accommodate higher densities than others. This program can be used to solve the problem of permissible board size and the cost differential of the two different board sizes from the two values of the Objective Function mentioned above.

5.2 Cost Coefficients Of The Objective Function

The cost coefficients of the objective function can change due to production level and policy decisions. The designer needs to consider the cost of his parts choice decision in the

light of possible changes during the life of the design.

To study the effects of the cost coefficients, the circuit board area would be set large enough so that Lindo can choose between THD and SMD parts without the solution being determined by the area constraint.

The cause of the change of the above cost coefficients is as follows. $C(X1s)$ to $C(X5u)$ are largely dependent on the cost of labor. The cost of labor is presently rising only slightly with time, and is not sensitive to the number of circuit boards that are assembled. That is, it is insensitive to the number of boards produced because present policy is to lay off labor if it is not needed.

However; $C(Z1s)$ to $C(Z5s)$ are largely dependent on machine depreciation that is spread over the number of placements made per machine payment. As a result these C 's are almost directly proportional to the number of parts that are placed. A policy is to change these C 's about once per year based on present and projected quantities of circuit boards to be produced. Historically the quantity of circuit boards may change 2x in one year in either direction. Thus business level could cause a large change in the total parts placement cost.

$C(Z1u)$ to $C(Z5u)$ are determined by the cost of the loader for these unique parts (\$1000) spread over the expected usage for 1 year. This is a policy decision, as well as being dependent on the quantity of boards assembled per year. Policies of amortizing the loader over 1 to 3 years should be examined by appropriately reducing these costs.

In addition, the loader could be expensed by engineering before production starts so that $C(Z1u)$ to $C(Z5u)$ could become the same as $C(Z1s)$ to $C(Z5s)$.

Regarding policy decisions, the values of the C 's may be adjusted to force new designs preferential to SMD parts to keep the price of SMD placement down. Conversely, designs preferential to THD design where circuit board area is available can be instituted to retain the labor force. The individual designer should appraise the viability of these policies over the life of the project. Normally it is not possible to lay out the circuit board for alternate technology components during the life of the product. The wrong choice could force the product out of production if the assembly price rises drastically.

The coefficients of S_x , and S_{z1u} through S_{z5u} , in the Objective Function are incurred once per machine setup. Thus as the number of boards that are assembled in one lot is increased, this cost is decreased per board. When this cost is relatively small with respect to the total board cost, advantages of running small lots of boards should be considered.

The coefficients of S_x and S_{z1u} through S_{z5u} in the Setup Charge constraint are set to a large number to make the integer switch that includes the setup charge function. This number is set larger than the sum of the parts in each of these constraints. It is recognized that if the sum of the parts were to exceed this coefficient that the problem would become infeasible. These slack amounts have no other importance.

The solutions presented here are typical of what would be found with specific circuit designs. These solutions are presented as a guide for those who would use this formulation to consider the affects of part type choices on the cost of circuit board assembly.

6. SENSITIVITY ANALYSIS

6.1 Labor

Both Standard and Unique Through Hole components are then placed manually, their placement costs are then a direct function of labor rates.

Sensitivity of our objective function to labor rates can then be determined by examining changes caused by changes in Through Hole Placement Cost.

In our original problem formulation, our board size constraint dictates that none of the larger Through Hole (THD) components be used - only the smaller, machine-placed surface mount (SMD) components can be used.

Therefore, our original problem is insensitive to labor rates. *ok*

6.2 SMD (Automated) Component Placement Costs

Our original problem dictates the exclusive use of the automatically placed SMD components. These placement costs could increase as a result of decreased machine amortization periods, or changes in costing procedures. Affects of these increases are shown below:

Change in Component Placement Cost	New Placement Cost	Change in Objective Function	Objective Function	Component Mix
+ 25%	0.05	39.85	+ 9.3%	All SMD
+ 50%	0.06	43.23	+ 18.5%	All SMD
+ 100%	0.08	49.99	+ 37%	All SMD
+ 200%	0.12	63.51	+ 74%	All SMD
+ 210%	0.12	63.51	+ 74%	All SMD
+ 225%	0.13	66.89	+ 83%	All SMD
+ 250%	0.14	70.27	+ 93%	All SMD
+ 275%	0.15	73.58	+ 100%	Some THD
+ 300%	0.16	76.46	+ 110%	Some THD
+ 350%	0.18	82.22	+ 125%	Some THD
+ 500%	0.24	99.50	+ 173%	Some THD

From the information above, we can learn:

The component placement cost of the board will increase about 0.035% for every 1% increase in SMD placement costs.

As SMD placement cost increases approach +275%, the use of through hole components for parts category 2 will result in the least costly board configuration. The model allows the use of through hole components from category 2 because, in this category, both through hole and SMD components are the same size, and their substitution will not violate the maximum board size constraint.

6.3 Unique SMD Component Placement Cost

The unique components must be placed by specialized loading machines. Costs of using these loading machines can vary with utilization rates, and operating procedures. Affects of changes in these costs are shown below:

Change in Placement Cost	New Placement Cost	Objective Function	Change in Objective Function	Component Mix
*	0.12	19.19	- 47%	All SMD
-	0.25	21.27	- 42%	All SMD
-	0.50	25.27	- 31%	All SMD
-50%	0.60	26.87	- 26%	All SMD
-25%	0.90	31.67	- 13%	All SMD
+25%	1.50	41.27	+ 13%	All SMD
+50%	1.80	45.10	+ 24%	Some THD
+100%	2.40	52.30	+ 43%	Some THD
+200%	3.60	66.70	+ 83%	Some THD
+ 300%	4.80	81.10	+ 122%	Some THD

* This placement cost (\$0.12) represents unique placement cost reduced to the equivalent of automatic placement cost.

From this, then, we learn that:

If provisions can be made to convert the 3 part categories requiring unique placement (16 individual parts out of total of 354 parts required per board), we can reduce the total cost of parts placement for the board by 47%.

If the costs of operating the specialized loading machines increases by 50%, it will be necessary to substitute unique through hole components for parts category 2 in order to minimize the total cost of parts placement. Again, through hole components from category 2 are allowed, without violating the maximum board size constraint, because they are the same size as the like surface mount components.

6.4 Surface Mount Component Area

With the use of more technologically advanced board layout techniques, it may become possible to reduce the size of the board area that must be allocated for each surface mount device. In an effort to determine the incentive for pursuing these more advanced (and certainly more costly) techniques, the following data is presented:

Reduction In SMD Placement Area	Objective Function	Change in Objective	Component Mix
- 50%	22.44	+38%	Std.-SMD, Unique-THD
- 40%	22.44	+38%	Std.-SMD, Unique-THD
- 20%	24.71	+32%	Std.-SMD, Unique-THD
- 10%	25.35	+30%	Std.-SMD, Unique-mix
- 5%	25.62	+30%	Std.-SMD, Unique-mix
- 1%	36.47	No Change	All SMD
+ 1%	36.47	No Change	All SMD
+ 5%	Infeasible		

From this we learn that if we can reduce the board area required by the standard surface mount components by 5%, across all SMD categories, enough extra space will remain on the board that the unique components can be installed as through hole devices.

The use of through hole unique components improves our objective function by eliminating the costly set-up charges incurred when board size constraint required the use of surface mount unique devices.

Initially, all part quantities were specified as General Integer Numbers. This formulation was not solvable for some values of the area constraint that should have worked. This is interpreted as an infeasible solution for either integer value. Changing the part quantities to real numbers (continuous variables) produced feasible solutions with fractional part quantities. Practically, the designer would round the part quantity value to the nearest whole number.