



Title: Linear Programming Model Forklift Driving Schedule Optimization

Course:

Year: 1990

Author(s): M. Coleman, F. Forstner, K. Hsu, E. Pancoko and D. Yuwono

Report No: P90012

ETM OFFICE USE ONLY

Report No.: See Above

Type: Student Project

Note: This project is in the filing cabinet in the ETM department office.

Abstract: This report resulted from the need for a less time consuming method of handling driving assignments expressed by the management of a forklift test facility. With six drivers at various levels of efficiency in driving eight types of forklifts, the time that could be spent manipulating the schedule to match the driver efficiency to each type, while minimizing total expenses became prohibitive. A Linear Programming model was developed to optimize driver assignments in order to minimize cost.

LINEAR PROGRAMMING MODEL
FORKLIFT DRIVING SCHEDULE OPTIMIZATION

*Good project
You should also
compare your results with
current performance
measures.*

EMGT 543
Dr. Dunder F. Kocaoglu
Spring Term
1990

by

Mike Coleman
Fred Forstner
Kun Hsu
Edmundus Pancoko
Dharwin Yuwono

Engineering Management Program
Portland State University



**LINEAR PROGRAMMING MODEL
FORKLIFT DRIVING SCHEDULE OPTIMIZATION**

TABLE OF CONTENTS

	<u>PAGE</u>
I. EXECUTIVE SUMMARY	1
II. INTRODUCTION	2
III. PROBLEM FORMULATION	6
IV. SOLUTION	12
V. SENSITIVITY ANALYSIS	13
VI. DISCUSSION OF RESULTS AND CONCLUSIONS	24
VII. APPENDICES	27
VIII. CHARTS	99

I. EXECUTIVE SUMMARY

The need for a less time consuming method of handling driving assignments was expressed by the management of a forklift test facility. With six drivers having various levels of efficiency driving eight types of forklifts, the time that could be spent manipulating the schedule to match the driver efficiency to each truck while minimizing the total dollars spent became prohibitive.

In order to streamline the resource allocation problem a linear programming model was developed. The base model consists of 48 variables and 16 constraint equations. The linear programming software LINDO was used to optimize driver assignments. The objective function was to minimize the expense required for driving. The decision variables are the number of hours assigned to each driver for each truck. The contribution values are the wages and efficiencies of the drivers. The constraints are daily lap quotas, labor availability, and test track availability.

Feasible solutions were obtained and accepted as realistic after close scrutiny by the team members. The basic solution is presented, a sensitivity analysis was done, and a discussion of results and conclusions is included in this report.

II. Introduction

One of the primary uses for linear programming is in the allocation of finite resources to a problem with multiple channels of activity. In other words, the linear program seeks to optimize a parameter (often it is the minimization of cost) through the most efficient assignment of the available resources to the various parts of the problem. In some cases, a resource is totally committed to one part of the problem. In many cases, however, many if not most of the resources are divided up among two or more parts of the problem.

The problem chosen for examination in this project involves the kind of resource allocation described above, and pertains to the durability testing of prototype forklift trucks at Hyster Company's Technical Center in Troutdale, Oregon. Some of the information included in this report has been disguised for proprietary reasons.

Background

In the development of new forklift trucks, the testing of prototype machines is an important but expensive and time-consuming task. One phase of testing requires that the machines be durability tested by driving them for a period of time on a load-handling course designed to simulate actual

field conditions for load weight, travel surface, grade, and lift height. Usually, this course requires the operator to transport two test loads between three locations in a cyclical manner such that the machine is carrying a load approximately 50 percent of the time. The remainder of the time the machine travels empty. A typical course map with driver instructions is presented in Figure 1, page 99.

At Hyster Company, most durability driving tasks are handled by test drivers. The primary responsibility of these drivers is the durability testing of forklift trucks, but they also have certain facilities maintenance responsibilities, as well as responsibility for the monitoring of automated test stands. The drivers operate in two shifts, from 3:30 PM to 7:00 AM. Each shift is comprised of a foreman and two subordinates, each with a different hourly pay rate.

At certain times, there can be as many as six different machines undergoing durability testing, which means that at least three machines will be idle at any time during each shift.

Durability test goals are established according to the individual unit being tested, based on the type of machine and the components of the machine under scrutiny. For example, a large prototype truck with all-new components would have a different durability test goal than, say, a small truck in which only the transmission and brake systems were being evaluated. Usually the test goal is expressed in

terms of accumulated hours on a specific test course. The aforementioned large prototype truck might have a goal of 1000 hours, whereas the small truck may only be tested for 500 hours.

Different types of trucks operate on different designated test courses; these courses can usually be categorized as indoor warehouse simulation, outdoor paved surface, outdoor on/off road, or rough terrain. A number of different test courses are available, but in some cases there are not enough courses of a particular type to accommodate all the vehicles needing to operate on them. The best example of this is the indoor warehouse area, which under some conditions can accommodate at most two simultaneous test courses. Thus, if there are three or more trucks designated for testing on indoor warehouse courses, at least one of these machines will have to be idle at all times.

Each unit being tested has a corresponding account number. Corporate procedures dictate that each project's account be billed by each driver for the time that the driver operates that truck. For accounting purposes, it is desirable to minimize the total cost of testing that is billed to these accounts, even though the drivers are paid for a full shift no matter how much of the shift they spend driving.

The testing of forklift trucks was chosen by this team because it appeared that it would lend itself very well to the linear programming process, and by virtue of this process some improvement in operating efficiency could be effected.

The present-day system does not utilize all available information.

For example:

1. The current method does not take into account that each driver operates with a different efficiency, depending on experience and natural ability.
2. The current method does not account for the fact that certain drivers may prefer certain trucks and/or test courses, and that if possible, it would be desirable to allow them to decide among themselves which driver operates which truck. 3.

The current method does not consider the fact that not all drivers are paid the same.

4. The current method assumes that the severity of the test is independent of the driver. For example, it is assumed that 500 hours driven by a slow driver would put as much wear and tear on a machine as 500 hours driven by a fast driver. Common logic dictates otherwise. Therefore, it seems appropriate to use accumulated test cycles (this refers to the number of times the truck completes one circuit around the test course, as shown in Figure 1) rather than total hours as the durability goal.

The linear model derived for this problem is a kind of hybrid in that it uses both actual and hypothetical data. The truck designations, course descriptions and availability, and test cycle goals are based on fact. The operator efficiency and pay rates are hypothetical, but are reasonable based on the observations of two of the team members who are Hyster employees.

What follows is the documentation of the formulation of the linear model, the solution, and the sensitivity analysis of the model using the Lindo software package.

III. PROBLEM FORMULATION

A base linear model was developed for optimizing the forklift driving schedules. Included are the objective function, and constraints that are formulated to be used with the LINDO program.

The objective function describes the cost incurred for one day's operation (two consecutive shifts) given that the test drivers are not all paid the same, nor do they operate all machines with equal efficiency. The goal is to minimize the objective function, subject to a number of constraints.

The driver productivity chart on page 99 gives information on the six driver's hourly pay, their efficiencies in laps per hour on the eight trucks, and the laps per day goal for each truck. Drivers C and F are the second and third shift foremen.

The constraints of the base model include the following:

1. The daily lap quota. Each truck must be driven a certain number of laps per day, which is calculated at the beginning of each project by dividing the total number of laps desired for the truck by the number of working days allowed for the completion of the durability test.
e.g. $25,000 \text{ laps} / 100 \text{ days} = 250 \text{ laps/day}$ required

Some projects may also have a requirement that there be a minimum number of hourmeter hours per day accumulated

2. The labor supply constraints. The total hours driven by each driver cannot exceed seven. The drivers are paid for eight hours but actually work only seven.

In addition, the total daily cost cannot exceed the sum of the driver's wages multiplied by the total number of

hours worked in a regular shift. There is no overtime permitted.

3. Test track availability constraint. The number of test courses is limited. No more than one truck is permitted on each test course at any time. In some cases there will be a restriction on the total number of hours that can be accumulated per day on a particular classification of machine.
4. Non-zero constraints. Constraint equations must have non-zero and positive values.

If the cost per day is less than the maximum allowable, then the manager is free to either use driver free time for maintenance duties, or he can change the constraints to shorten the overall duration of some test projects until the free time is eliminated.

The problem formulation is as follows:

GIVEN: The numbers 1, 2, 3, 4, 5, 6, 7, and 8 refer to the forklift trucks that are to be tested.

a, b, c, d, e, and f are the six drivers that test drive the trucks seven hours per day. They are paid during their lunch break making a total of eight paid hours per day.

$h_a, h_b, h_c, h_d, h_e, h_f$ refer to the hourly pay of each driver

Each truck is to be driven a specific number of laps around a testing course. The minimum number of laps required for each truck are identified as $L_1, L_2, L_3, L_4, L_5, L_6, L_7,$ and L_8 .

Each drivers average productivity varies with each truck. Productivity is measured in laps per hour and is presented in the following chart.

<u>trucks</u>							
1	2	3	4	5	6	7	8

a P_{a1} P_{a2} P_{a3} P_{a4} P_{a5} P_{a6} P_{a7} P_{a8}

b P_{b1} P_{b2} P_{b3} P_{b4} P_{b5} P_{b6} P_{b7} P_{b8}

c P_{c1} P_{c2} P_{c3} P_{c4} P_{c5} P_{c6} P_{c7} P_{c8}

drivers

d P_{d1} P_{d2} P_{d3} P_{d4} P_{d5} P_{d6} P_{d7} P_{d8}

e P_{e1} P_{e2} P_{e3} P_{e4} P_{e5} P_{e6} P_{e7} P_{e8}

f P_{f1} P_{f2} P_{f3} P_{f4} P_{f5} P_{f6} P_{f7} P_{f8}

Note: productivity and hourly pay are not necessarily related.

B is the amount of money budgeted to test the eight vehicles.

FIND: The number of hours each driver spends on each truck in order to satisfy the given constraints and minimize the testing cost.

SOLUTION

THE MODEL

Let $X_{a1}, X_{a2}, X_{a3}, \dots, X_{a7}, X_{a8}$ equal the hours spent by each driver on each truck.

The objective function is :

$$\text{min. } Z = h_a \left(\sum_{i=1}^8 X_{ai} \right) + h_b \left(\sum_{i=1}^8 X_{bi} \right) + \dots + h_f \left(\sum_{i=1}^8 X_{fi} \right) - \sum_{i=1}^8 (h_i * \text{hrs})$$

The constraints ~~equations~~

Min. number of test laps (8 eqn, 1 per truck),

$$P_{a1}X_{a1} + P_{b1}X_{b1} + P_{c1}X_{c1} + P_{d1}X_{d1} + P_{e1}X_{e1} + P_{f1}X_{f1} = L_1$$

$$P_{a2}X_{a2} + P_{b2}X_{b2} + P_{c2}X_{c2} + P_{d2}X_{d2} + P_{e2}X_{e2} + P_{f2}X_{f2} = L_2$$

$$P_{a3}X_{a3} + P_{b3}X_{b3} + P_{c3}X_{c3} + P_{d3}X_{d3} + P_{e3}X_{e3} + P_{f3}X_{f3} = L_3$$

$$P_{a4}X_{a4} + P_{b4}X_{b4} + P_{c4}X_{c4} + P_{d4}X_{d4} + P_{e4}X_{e4} + P_{f4}X_{f4} = L_4$$

$$P_{a5}X_{a5} + P_{b5}X_{b5} + P_{c5}X_{c5} + P_{d5}X_{d5} + P_{e5}X_{e5} + P_{f5}X_{f5} = L_5$$

$$P_{a6}X_{a6} + P_{b6}X_{b6} + P_{c6}X_{c6} + P_{d6}X_{d6} + P_{e6}X_{e6} + P_{f6}X_{f6} = L_6$$

$$P_{a7}X_{a7} + P_{b7}X_{b7} + P_{c7}X_{c7} + P_{d7}X_{d7} + P_{e7}X_{e7} + P_{f7}X_{f7} = L_7$$

$$P_{a8}X_{a8} + P_{b8}X_{b8} + P_{c8}X_{c8} + P_{d8}X_{d8} + P_{e8}X_{e8} + P_{f8}X_{f8} = L_8$$

7 hours/day of driving per driver,

$$X_{a1} + X_{a2} + X_{a3} + X_{a4} + X_{a5} + X_{a6} + X_{a7} + X_{a8} \leq 7$$

$$X_{b1} + X_{b2} + X_{b3} + X_{b4} + X_{b5} + X_{b6} + X_{b7} + X_{b8} \leq 7$$

$$X_{c1} + X_{c2} + X_{c3} + X_{c4} + X_{c5} + X_{c6} + X_{c7} + X_{c8} \leq 7$$

$$X_{d1} + X_{d2} + X_{d3} + X_{d4} + X_{d5} + X_{d6} + X_{d7} + X_{d8} \leq 7$$

$$X_{e1} + X_{e2} + X_{e3} + X_{e4} + X_{e5} + X_{e6} + X_{e7} + X_{e8} \leq 7$$

$$X_{f1} + X_{f2} + X_{f3} + X_{f4} + X_{f5} + X_{f6} + X_{f7} + X_{f8} \leq 7$$

A matrix showing the problem formulation is shown on pages 101-104. Putting the model in the matrix form helped to define the decision variables, the minimization equation, and the constraint equations. The base problem formulation is a cost minimization model having 48 decision variables, and 16 constraint equations.

IV. SOLUTION

The chart on page 105 shows the optimum driving time assignments for each driver-forklift combination. The driving time assignments satisfy all the model's constraints without requiring two of the drivers to drive a full seven hours per day. The minimum daily salary cost for completing the necessary testing was found to be \$256.20.

The optimum assignment of drivers to the various forklifts

does not offer the drivers much variety. Four of the eight forklifts were assigned a single driver and only one forklift was assigned as many as three drivers.

An intuitive review of the decision variables having non-zero values tends to support the computed results. For example Driver A is fairly productive and is paid a low wage. It is not surprising that he has been assigned a full seven hours of work on the two forklifts he drives most productively.

V. SENSITIVITY ANALYSIS

Using the basic run we identified many changes that could be made to the constraints in order to get to the optimal solution. The ones that we focused on were as follows. 1) Require that all drivers drive exactly seven hours. 2) Add an additional driver 3) Eliminate each driver individually and eliminate multiple drivers 4) Revise the driver payscale. 5) Change the measures of effectiveness of the drivers of each truck by plus or minus 10%; or change the laps/hour required per driver 6) Put restrictions on the type of driving, indoor or outdoor, that a particular driver can do.

PART 1 SENSITIVITY ANALYSIS Basic Run (Appendix A)

The basic model comes up with the values of decision

variables as follows:

Total driving hour of driver A : 7 hours
Total driving hour of driver B : 7 hours
Total driving hour of driver C : 1.68 hours
Total driving hour of driver D : 7 hours
Total driving hour of driver E : 3.8 hours
Total driving hour of driver F : 6.7 hours

The optimal cost is \$ 256.199900.

PART 2
SENSITIVITY ANALYSIS
Relaxation of Required Lap Constraints
(Appendix B)

This is the basic run with relaxation of required laps per truck; the equalities are changed to inequalities (greater than or equal to).

This run produced the same decision variables as the basic run.

The optimal cost is also \$ 256.199900. This shows that the model has no sensitivity to the relaxation of the equality of the required lap constraint.

PART 2, CON'T
SENSITIVITY ANALYSIS
Fixed Driving hours and Relaxation of Required Lap
Constraints
(Appendix B)

In this instance, we assume that each driver is required to drive a total of seven hours per shift. The objective function in this case becomes rather trivial, and is simply the sum of the drivers' pay rates times the total number of

hours worked.

This solution clearly sets the sum of all decision variables for each driver equal to seven.

The results of this analysis are as follows:

Driver C: 316.67% increase
Driver E: 84.21% increase
Driver F: 4.48% increase

All other drivers have no change in workload.

The workload of driver C is the most sensitive in this case, while driver E's workload is moderately sensitive, and driver F's workload shows very little sensitivity.

After examination of this analysis, it becomes incredibly obvious that the LINDO program is not required to determine the additional workload of each driver. This additional workload is simply the difference between the sum of the decision variables for each driver in the basic run and the seven-hour constraint.

PART 2, CON'T
SENSITIVITY ANALYSIS
Substituting Foreman C
(Appendix B)

In this instance, we eliminate the second-shift foreman (driver C) and replace him with a new driver (driver G). Driver G is a new, minimum wage employee who is paid the same as (and has the same productivity as), driver A.

In this case, the optimal cost decreases by .09%, to \$255.97.

The sums of the decision variables for each driver are as follows:

Total driving hour of driver A : 7 hours
Total driving hour of driver B : 7 hours
Total driving hour of driver G : 5.38 hours
Total driving hour of driver D : 4 hours
Total driving hour of driver E : 6.33 hours
Total driving hour of driver F : 5.14 hours

Driver G's total workload is 5.38 hours, which is a 220% increase over driver C's workload in the basic run. This seems to suggest that the pay differential between drivers C and G has more influence on the objective function than the difference in productivity between these two drivers.

In addition, other workload adjustments are as follows:

Driver D : 42.86% decrease
Driver E : 66.58% increase
Driver F : 23.28% decrease

This indicates moderate sensitivity for the workload of these three drivers.

PART 2, CON'T
SENSITIVITY ANALYSIS
Substituting Foreman F
(Appendix B)

In this instance, we eliminate the third-shift foreman (driver F) and replace him with a new driver (driver H). Driver G is a new, minimum wage employee who is paid the same as (and has the same productivity as), driver E.

In this case, the optimal cost increases by 1.81%, to \$260.85. The sums of the decision variables for each driver are

as follows:

Total driving hour of driver A : 7 hours
Total driving hour of driver B : 7 hours
Total driving hour of driver C : 2.55 hours
Total driving hour of driver D : 7 hours
Total driving hour of driver E : 6.09 hours
Total driving hour of driver H : 6.86 hours

Driver H's total workload is 6.86 hours, which is a 2.39% increase over driver F's workload in the basic run. This suggests that the pay differential between drivers F and H has approximately the same influence on the objective function as the difference in productivity between these two drivers.

In addition, other workload adjustments are as follows:

Driver C : 51.79% increase
Driver E : 60.26% increase

PART 2, CON'T
SENSITIVITY ANALYSIS
Substituting Foreman C and F
(Appendix B)

This is a combination of the two previous runs.

Interestingly, the result parallels the combined results of the two previous runs. The objective function increases by 1.5%, G's workload is a 220% increase over C's workload, and H's workload is a slight (7.61%) decrease from F's workload.

The sums of the decision variables for each driver are as

follows:

Total driving hour of driver A : 7 hours
Total driving hour of driver B : 7 hours

Total driving hour of driver G : 5.38 hours
 Total driving hour of driver D : 5 hours
 Total driving hour of driver E : 7 hours
 Total driving hour of driver H : 6.19 hours

In addition, other workloads are as follows:

Driver D : 28.57% decrease
 Driver E : 84.21% increase

PART 3
SENSITIVITY ANALYSIS
Elimination of selected drivers.
(Appendix C)

There is a possibility that each driver can not perform fully in the job, for example she or he is ill, or possibly a foreman (driver C or F) is required to do another job. In this case we set constraints 10 to 15 equal zero, one at a time. The analysis of the effect of the elimination of each driver is given below .

1. Basic run
 Objective function = 256.20
 Total driving hours of driver A = 7
 Total driving hours of driver B = 7
 Total driving hours of driver C = 1.68
 Total driving hours of driver D = 7
 Total driving hours of driver E = 3.8
 Total driving hours of driver F = 6.7
2. Elimination of driver A,
 Objective function = 267.25 4.3% increase
 Total driving hours of driver A = 0
 Total driving hours of driver B = 7 0% change
 Total driving hours of driver C = 7 316% increase
 Total driving hours of driver D = 7 0% change
 Total driving hours of driver E = 5.3 39% increase
 Total driving hours of driver F = 7 4% increase
3. Elimination of driver B
 Objective function = 264.86 3.4% increase
 Total driving hours of driver A = 7 0% change
 Total driving hours of driver B = 0

- | | |
|--|---------------|
| Total driving hours of driver C = 5.55 | 230% increase |
| Total driving hours of driver D = 7 | 0% change |
| Total driving hours of driver E = 7 | 84% increase |
| Total driving hours of driver F = 7 | 4% increase |
4. Elimination of driver C
- | | |
|---------------------------------------|----------------|
| Objective function = 257.37 | 0.46% increase |
| Total driving hours of driver A = 7 | 0% change |
| Total driving hours of driver B = 7 | 0% change |
| Total driving hours of driver C = 0 | |
| Total driving hours of driver D = 7 | 0% change |
| Total driving hours of driver E = 6.3 | 66% increase |
| Total driving hours of driver F = 6.7 | 0% change |
5. Elimination of driver D
- | | |
|---------------------------------------|---------------|
| Objective function = 267.41 | 4.4% increase |
| Total driving hours of driver A = 7 | 0% change |
| Total driving hours of driver B = 7 | 0% change |
| Total driving hours of driver C = 6.2 | 269% increase |
| Total driving hours of driver D = 0 | |
| Total driving hours of driver E = 7 | 84% increase |
| Total driving hours of driver F = 7 | 4% increase |
6. Elimination of driver E
- | | |
|-------------------------------------|---------------|
| Objective function = 262.39 | 2.4% increase |
| Total driving hours of driver A = 7 | 0% change |
| Total driving hours of driver B = 7 | 0% change |
| Total driving hours of driver C = 5 | 198% increase |
| Total driving hours of driver D = 7 | 0% change |
| Total driving hours of driver E = 0 | |
| Total driving hours of driver F = 7 | 4% change |
7. Elimination of driver F.
This is an infeasible solution, it means that everyone will work seven hours but the constraints still will not be met. All dual prices with the exception of those for lines 16 and 17 are non-zero. This says that none of the constraints in rows 2 through 15 are satisfied.
8. Elimination of driver C and F.
This is an infeasible solution, see above.

As we can see, the elimination of each driver affects the objective function, which is to be expected.

In addition, we see that driver C in most cases is very sensitive to this situation and is required to pick up most

of the additional load.

PART 4
SENSITIVITY ANALYSIS
PAY SCALE ADJUSTMENT
(APPENDIX D)

In this step, the sensitivity of the decision variables to changes in pay scale among the drivers was explored. The changes involved drivers "E" and "F", since these two drivers differ considerably in both pay and productivity. The background of the two drivers is as follows: Driver "E" is an entry-level employee who has only recently learned the basics of forklift driving. As a result, his productivity is low, but he is also paid the least of all the drivers, with a current hourly pay of \$6.55 per hour. Driver "F" is the foreman on the third (graveyard) shift. By virtue of his years of experience, Driver "F" is very productive, and he is at the top of the pay scale, at \$9.45 per hour.

In the first run, the Driver "E" was given a 26% pay raise, to \$8.25 per hour. As a result, his total assigned driving time decreased by 75%. The assigned driving time for Drivers "A", "B", and "D" was completely insensitive to this change (although the time originally scheduled for Driver "A" to drive Truck 3 was reassigned to Truck 1 for the same driver). The total driving time for the second (swing) shift foreman (Driver "C") increased by 149%, to a little over 4 hours. The objective function value increased by about 2.5%.

In the second run, Driver "F", the third shift foreman, was given a 26% pay raise, to \$11.91 per hour. As a result, his

own driving load was reduced by 98%. Amazingly, the assigned time to drivers "A", "B", and "D" was again completely insensitive to this change. The big loser in this scenario was Driver "C", the second shift foreman. As a result of Driver "F"'s raise in pay, Driver "C"'s total workload increased 318%.

A summary of the changes in driver workload as a result of the two scenarios is given below.

PERCENT CHANGE IN EACH DRIVER'S TOTAL DRIVING LOAD

	SCENARIO 1	SCENARIO 2
DRIVER "A"	0%	0%
DRIVER "B"	0%	0%
DRIVER "C"	149%	318%
DRIVER "D"	0%	0%
DRIVER "E"	-75%	84%
DRIVER "F"	0%	-98%

SCENARIO 1: GIVE DRIVER "E" A RAISE TO \$8.25 PER HOUR

SCENARIO 2: GIVE DRIVER "F" A RAISE TO \$11.91 PER HOUR

In summary, it was found that the decision variables for Drivers "A", "B", and "D" were completely insensitive to changes in the pay of Drivers "E" and "F". The driver that was mainly affected by these changes was the second-shift

foreman, Driver "C". This is probably because of the high productivity that Driver "C" provides, along with his correspondingly high pay rate.

PART 5

SENSITIVITY ANALYSIS

The Effect of Variations in the Drivers' Productivity Rates

The focus of this analysis was to determine how a driver's productivity influences which forklift he is assigned to. The effects of driver productivity on the actual amount of driving time per vehicle was not explored because the inverse relationship between driver productivity and driving time is obvious.

The productivity coefficient assigned to each non-zero solution variable was analyzed to determine how much the coefficients could change before a new non-zero variable appeared in the solution. These limits were identified by trial and error.

The procedure could also be applied to the driver-forklift combinations that were assigned zero hours. In this case the sensitivity analysis would determine how a driver must improve before he is assigned time on a specific vehicle. This analysis could be valuable to a manager but does not contribute unique information about the model's overall sensitivity to changes in productivity rates. Therefore this sensitivity analysis was limited to the non-zero decision variables in the optimum solution.

The drivers' assignments were found to be very sensitive to the efficiency rates assigned to each driver-forklift combination. There were eleven non-zero decision variables in the optimum solution. None of their associated productivity rates could be reduced by more than two laps-per-hour without generating new non-zero decision variables. In six cases the coefficients could not be increased by more than one lap-per-hour before a new non-zero decision variable appeared.

Appendix E summarizes the specific results of this sensitivity analysis.

It can be concluded from this analysis that the applicability of the model depends primarily on the reliability of the driver productivity data.

PART 6
SENSITIVITY ANALYSIS RESTRICTIONS ON TYPE OF DRIVING DONE
(APPENDIX F)

First run:

When the constraints of not being able to drive outdoors (they can only drive indoor test courses) are imposed on

drivers C and D it changes the value of the objective function and the hours assigned to each driver. The table below shows the percent change of total assigned hours for each driver:

drivers	<u>new hours</u>		<u>basic hours</u>		%chg
	truck	hours	truck	hours	
A	5	1.6	2	4.2	0
	6	5.4	3	2.8	
B	7	4.1	6	2.0	+13
	8	2.9	7	4.1	
C	1	3.5	3	1.7	+411
	2	1.4			
	3	2.1			
D	2	3.2	5	4.0	0
	3	3.8	6	3.0	
E	4	1.7	1	3.8	-223
F	4	3.3	4	4.4	-8
	5	2.9	8	2.3	

The example run of this problem can be found in Appendix F. The summary table above shows that drivers C and D were assigned only hours for the indoor trucks 1, 2, & 3. The new solution reassigns the drivers and hours per truck and minimizes the cost based on the additional constraints. Driver C's workload has increased from 1.7 hours to 7.0 hours, and represents the largest change in workload. Driver E's assigned hours have been reduced by over half. The changes to the other driver's workloads did not significantly change. The value of the objective function has increased from \$256 to \$294. Adding the "indoor only" constraints has

increased the cost of driving.

Second run:

When additional driver E is added to the drivers that do not drive outdoors the solution becomes infeasible. The non-zero dual prices at rows 16, 18, 19, and 20 show that these are the rows contributing to infeasibility. The constraints of rows 18, 19, and 20 are not satisfied because there are not enough indoor driving hours to give to drivers C, D, and E. Also the row 16 constraint for test track availability cannot be satisfied due to "overbooking" the indoor test tracks.

VI. DISCUSSION OF RESULTS AND CONCLUSIONS

From the sensitivity analysis and the discussion of sections V the solutions obtained are feasible and realistic.

The user of this model has the flexibility to change parameters to suit changing conditions and needs. The feasible solution of the original model is the starting point which may be altered to find new solutions that are optimal given revised or additional constraints.

From the sensitivity analysis it can be shown that the

most sensitive area is that of the assigned laps per hour for each driver. The assignment of the efficiency levels of a particular driver for certain trucks must be accurately determined. A dilemma is generated by this fact. To get accurate efficiency data, every driver must be observed driving every vehicle. But the model proves that this is extremely inefficient.

Perhaps the most realistic application of this model would be during times of limited manpower (vacations, sickness, etc.) when an ongoing base of productivity data could be used to determine the optimal driving assignments.

The data obtained from a specific run can be used to determine the amount of time each driver spends on a particular truck. With this information in hand the supervisor or manager is able to assign driving time in the most efficient manner. Cost minimization will be achieved when this tool is used.

The program can be altered to assure that priority trucks are driven the required number of hours per day. We envision that the model could be updated with fresh data on a weekly or even daily basis to account for the beginning and/or ending of major projects, changes in staffing, and

other anomalies.

The model could also be used as a motivational device. If a driver requests a raise or more driving hours, the model can be modified to determine how the driver's productivity must improve. The model would also show how the other drivers would be impacted by the driver's increase in productivity and/or pay.