ETM
ENGINEERING & TECHNOLOGY MANAGEMENT

Title: Optimum Production Levels for Rifle Scope Manufacturing

Course:
Year: 1989
Author(s): A. Oubari, R. Nayyar, H. Oberhelman and B. Yaghmaie

Report No: P89007

Abstract: This report reflects ways to minimize the annual production cost of optical devices within the boundaries of the sales forecast. A Linear Programming model was used to determine the production levels at regular wage and overtime, and the amounts of storage needed for each unit.

OPTIMUM RIFLE SCOPE PRODUCTION

A. Oubari, R. Nayyar, H. Oberhelman,
and B. Yaghmaie

EMP - P8907

24

OPTIMUM PRODUCTION LEVELS FOR RIFLESCOPE MANUFACTURE

Raman Nayyar

Ayman Obari

Henry Oberhelman

Bahram Yaghmaie

EAS 543

Table of Contents

1 *EXECUTIVE SUMMARY*

A manufacturing company designs and produces optical devices for the commercial sporting goods market. These products account for approximately 80% of all corporate revenue. Market demand for these products reached unexpectedly high levels in the last two years. Significant revenue has been lost because production has failed to meet demand in spite of greatly increased hiring and equipment additions. Management has requested assistance in evaluating the production planning process in order to avoid repeating this scenario.

The following issues are of major concern:

- Recruitment, training and placement of assembly workers.
- Inability to rapidly increase production levels to meet sales demand.
- High cost and long lead time of glass lenses.
- High cost associated with holding unsold inventory.
- Recognition of a need to reduce product line variety.

A linear programming cost minimization program was developed and used for the systems analysis that was requested. Major features of this model were:

Grouping the 49 product models into four major types.

Recognizing three major cost contributions to the objective function namely: cost of units produced during regular time, cost of units produced on overtime, and cost of stored units.

Defining the annual production cycle as two six month periods.

Constraining periodic labor hour fluctuation, periodic overtime limitation, profit, minimum and maximum inventory, maximum allowable production rate, availability of regular and overtime labor hours, regular testing hours (assuming no testing in overtime), machine hours, and raw material (lenses). In total, there were 52 constraints identified and applied.

Our conclusions identified the need to provide training for replacement of skilled workers, develop increased flexibility in reducing product variety, and utilize system slack to assist in implementing new technology for process improvement.

2 INTRODUCTION

A manufacturing company produces optical products for the commercial sporting goods market. These products, which include riflescopes, pistol scopes, spotting scopes, and binoculars, account for approximately 80% of corporate revenue. Experience in the marketplace indicates that:

- Product demand from the distribution network is highly seasonal. The maximum demand is slightly less than twice the lowest demand for the year.
- A four year cycle in market demand has been observed over the last 12 years.
- Long term product demand is stable to slightly declining.
- Competition from large and widely known manufacturers of optical goods is becoming much more intensive.
- Brand loyalty erosion resulting from inability to meet sales demand. In a specific instance this has led to the reentry into the marketplace of a former competitor.

In spite of sales forecast from past experience, product demand in the last two years surged to unexpectedly high levels. Production assembly failed to meet demand even with greatly increased employee and equipment additions, significant overtime and curtailment of lower volume product lines. As a result significant revenue was lost. The following specific issues were identified during these periods of high demand.

- Recruitment, training and placement of assembly workers is both more time consuming and costly than expected. Growth in the local economy has led to increased competition for qualified production workers, thus the available supply is both more limited and more unskilled.
- The production cycle time to respond to increased product demand is approximately 90 days. Although machine shop capacity is adequate, lead times for raw material such as aluminum bar stock and extrusions procurement are typically sixty to ninety days.
- Critical expensive components ,such as glass lenses, are contracted within specific volume requirements over lead times of 18 months. The ordering quantities and material characteristics are based, in selected cases, on firm contracts negotiated twelve months in advance of delivery.
- Inventory carryover is costly, both from the standpoint of carrying cost and the high risk of defective parts.

Sales for the current year are forecast at the same level of units as the previous year. Thus the firm is seeking to avoid repetition of the high cost and lost sales from previous years.

The current product mix consists of 49 models which are individually characterized by differences in surface appearance, magnification, field of view and power selection. The product mix has been grouped into the following distribution for this model:

Group	Model Designation	Number of Models
1	FP	22
2	VariXII	10
3	VariXIII	10
4	Spotting Scopes	7

Within each group the production process is standardized thus it is assumed that the highest volume product is suitable as a representative sample for the evaluation of process requirements.

The firm has stipulated that labor fluctuations shall be stabilized within 10% in order to minimize employee recruitment, crosstraining and relocation expense.

The objective of the company is then to minimize annual production cost within the bounds of meeting the minimum sales forecast, the contractual requirements for lens supplies and the limitation on labor supply fluctuation.

3 PROBLEM FORMULATION

A DECISION VARIABLES

X_{11} # of scopes type 1 produced and tested in regular time, in period 1.

X_{21} # of scopes type 2 produced and tested in regular time, in period 1.

X_{31} # of scopes type 3 produced and tested in regular time, in period 1.

X_{41} # of scopes type 4 produced and tested in regular time, in period 1.

Y_{11} # of scopes type 1 produced in overtime and tested in regular time, in period 1.

Y_{21} # of scopes type 2 produced in overtime and tested in regular time, in period 1.

Y_{31} # of scopes type 3 produced in overtime and tested in regular time, in period 1.

Y_{41} # of scopes type 4 produced in overtime and tested in regular time, in period 1.

W_{11} # of scopes type 1 in inventory at the end of period 1.

W_{21} # of scopes type 2 in inventory at the end of period 1.

W_{31} # of scopes type 3 in inventory at the end of period 1.

W_{41} # of scopes type 4 in inventory at the end of period 1.

LR_1 total regular labor hours available in period 1.

LO_1 total overtime labor hours available in period 1.

X_{12} # of scopes type 1 produced and tested in regular time, in period 2.

X_{22} # of scopes type 2 produced and tested in regular time, in period 2.

X_{32} # of scopes type 3 produced and tested in regular time, in period 2.

X_{42} # of scopes type 4 produced and tested in regular time, in period 2.

Y_{12} # of scopes type 1 produced in overtime and tested in regular time, in period 2.

Y_{22} # of scopes type 2 produced in overtime and tested in regular time, in period 2.

Y_{32} # of scopes type 3 produced in overtime and tested in regular time, in period 2.

Y_{42} # of scopes type 4 produced in overtime and tested in regular time, in period 2.

W_{12} # of scopes type 1 in inventory at the end of period 2.

W_{22} # of scopes type 2 in inventory at the end of period 2.

W_{32} # of scopes type 3 in inventory at the end of period 2.

W_{42} # of scopes type 4 in inventory at the end of period 2.

LR_2 total regular labor hours available in period 2.

LO_2 total overtime labor hours available in period 2.

B OBJECTIVE FUNCTION

MINIMIZE

$$
\begin{aligned}
&58.74\ X_{11} + 65.70\ X_{21} + 86.68\ X_{31} + 122.69\ X_{41} + \\
&61.73\ Y_{11} + 69.26\ Y_{21} + 90.01\ Y_{31} + 125.95\ Y_{41} + \\
&07.38\ W_{11} + 08.21\ W_{21} + 10.84\ W_{31} + 015.34\ W_{41} + \\
&58.96\ X_{12} + 65.89\ X_{22} + 87.26\ X_{32} + 124.33\ X_{42} + \\
&61.95\ Y_{12} + 69.34\ Y_{22} + 91.59\ Y_{32} + 127.59\ Y_{42} + \\
&07.37\ W_{12} + 08.24\ W_{22} + 11.45\ W_{32} + 015.95\ W_{42} + \\
&07.75\ LR_1 + 11.63\ LO_1 + 07.75\ LR_2 + 011.63\ LO_2
\end{aligned}
$$

C CONSTRAINTS

ROWS 1 TO 21 REFER TO OPERATIONAL CHARACTERISTICS FOR PERIOD 1.
ROWS 1-4 PRODUCTION, INVENTORY & SHIPMENT CONSTRAINTS.

1) $0.9\ X_{11} + 0.9\ Y_{11} - W_{11} \le 12730$
2) $0.9\ X_{21} + 0.9\ Y_{21} - W_{21} \le 15145$
3) $0.9\ X_{31} + 0.9\ Y_{31} - W_{31} \le 10944$
4) $0.9\ X_{41} + 0.9\ Y_{41} - W_{41} \le 02650$

LOWER LIMITS ON FOR EACH TYPE IN PERIOD 1:

5) $X_{11} + Y_{11} \ge 18000$
6) $X_{21} + Y_{21} \ge 28750$
7) $X_{31} + Y_{31} \ge 14950$
8) $X_{41} + Y_{41} \ge 03150$

MAXIMUM AND MINIMUM AVAILABLE ENDING INVENTORY FOR EACH TYPE IN PERIOD 1:

9) $W_{11} <= 05797$
10) $W_{11} >= 05270$
11) $W_{21} <= 14965$
12) $W_{21} >= 13645$
13) $W_{31} <= 04407$
14) $W_{31} >= 04006$
15) $W_{41} <= 00550$
16) $W_{41} >= 00500$

REGULAR LABOR HOUR CONSTRAINTS:

17) $0.45\ X_{11} + 0.35\ X_{21} + 0.68\ X_{31} + 0.85\ X_{41} - 1.00\ LR_1 <= 0$

OVERTIME LABOR HOUR CONSTRAINTS:

18) $0.45\ Y_{11} + 0.35\ Y_{21} + 0.68\ Y_{31} + 0.85\ Y_{41} - 1.00\ LO_1 <= 0$

REGULAR TEST HOUR AVAILABILITY CONSTRAINTS:

19) $0.01\ X_{11} + 0.01\ X_{21} + 0.01\ X_{31} + 0.01\ Y_{11} +$
$0.01\ Y_{21} + 0.01\ Y_{31} <= 1696$

MACHINE HOUR AVAILABILITY CONSTRAINTS:

20) $0.04\ X_{11} + 0.06\ X_{21} + 0.05\ X_{31} + 0.03\ X_{41} +$
$0.04\ Y_{11} + 0.06\ Y_{21} + 0.05\ Y_{31} + 0.03\ Y_{41} <= 3640$

RAW MATERIAL (LENSES) AVAILABILITY CONSTRAINTS:

21) $2\ X_{21} + 2\ Y_{21} <= 80000$
22) $X_{11} + X_{21} + X_{31} + Y_{11} + Y_{21} + Y_{31} <= 85000$

ROWS 10 & 11 ARE REGULAR LABOR HOUR FLUCTUATION CONSTRAINTS:

23) $LR_1 >= 49608$
24) $LR_1 <= 60632$

OVERTIME LIMIT AS A % OF REGULAR LABOR HOURS:

25) $5\ LO_1 - LR_1 <= 0$

PROFIT CONSIDERATION:

26) $85.69\ X_{11} + 125.25\ X_{21} + 139.56\ X_{31} + 73.03\ X_{41} + 83.91\ Y_{11} + 123.05\ Y_{21} + 136.70\ Y_{31} + 71.78\ Y_{41} >= 5000000$

Rows 27 - 52 REFER TO PERIOD 2

PRODUCTION, INVENTORY AND SHIPMENT CONSTRAINTS:

27) $0.9\ X_{12} + 0.9\ Y_{12} + W_{11} - W_{12} <= 21580$
28) $0.9\ X_{22} + 0.9\ Y_{22} + W_{21} - W_{22} <= 56600$
29) $0.9\ X_{32} + 0.9\ Y_{32} + W_{31} - W_{32} <= 18960$
30) $0.9\ X_{42} + 0.9\ Y_{42} + W_{41} - W_{42} <= 03940$

31) $X_{12} + Y_{12} >= 21650$
32) $X_{22} + Y_{22} >= 30950$
33) $X_{32} + Y_{32} >= 18850$
34) $X_{42} + Y_{42} >= 05050$

35) $W_{12} <= 5874$
36) $W_{12} >= 5340$
37) $W_{22} <= 5264$
38) $W_{22} >= 4785$
39) $W_{32} <= 4286$
40) $W_{32} >= 3896$
41) $W_{42} <= 1771$
42) $W_{42} >= 1610$

REGULAR LABOR HOUR CONSTRAINTS:

43) $0.45\ X_{12} + 0.35\ X_{22} + 0.68\ X_{32} + 0.85\ X_{42} - 1.00\ LR_2 <= 0$

OVERTIME LABOR HOUR CONSTRAINTS:

44) $0.45\ Y_{12} + 0.35\ Y_{22} + 0.68\ Y_{32} + 0.85\ Y_{42} - 1.00\ LO_2 <= 0$

REGULAR TEST HOURS AVAILABLE:

45) $0.01\ X_{12} + 0.01\ X_{22} + 0.01\ X_{32} + 0.01\ Y_{12} + 0.01\ Y_{22} + 0.01\ Y_{32} <= 1904$

MACHINE HOUR AVAILABILITY:

46) $0.04\ X_{12} + 0.06\ X_{22} + 0.05\ X_{32} + 0.03\ X_{42} + 0.04\ Y_{12} + 0.06\ Y_{22} + 0.05\ Y_{32} + 0.03\ Y_{42} <= 4000$

RAW MATERIAL AVAILABILITY:

47) $2\ X_{22} + 2\ Y_{22} <= 61900$
48) $1\ X_{12} + 1\ X_{22} + 1\ X_{32} + 1\ Y_{12} + 1\ Y_{22} + 1\ Y_{32} <= 71450$

LABOR HOUR FLUCTUATION:

49) $LR_2 - 0.9\ LR_1 >= 0$

50) $LR_2 - 1.1\ LR_1 <= 0$

OVERTIME LIMIT:

51) $LO_2 - 0.20\ LR_2 <= 0$

PROFIT CONSIDERATIONS:

52) $85.59\ X_{12} + 125.16\ X_{22} + 139.31\ X_{32} + 72.71\ X_{42} +$
$83.74\ Y_{12} + 123.00\ Y_{22} + 136.00\ Y_{32} + 71.00\ Y_{42} >= 5000000$

D PARAMETERS

The parameters for the formulation of the problems are as follows:

Cost coefficients:

C_1 thru C_{28} are the unit cost contributions of the decision variables X_{11} thru LO_2 to the objective function.

Right hand-side constraints:

B_1 thru B_{52} are the resource levels for each of the constraints.

Technological coefficients:

$A_{1,1}$ thru $A_{28,52}$ are the technological coefficients of the decision variables X_{11} thru LO_2 in the constraint equations.

Note: refer to the problem formulation table at the end of this section.

E COST COEFFICIENTS

Coefficient	Unit costs				
C_i	Unit labor cost *	material cost	vendor services ++	fixed labor overhead	total unit costs
C_1	5.97	27.44	3.41	21.92	58.74
C_2	7.12	31.32	3.88	23.38	65.70
C_3	8.67	41.19	4.53	32.29	86.68
C_4	6.53	79.39	4.38	32.38	122.69
C_5	8.96 +	27.44	3.41	21.92	61.73
C_6	10.68 +	31.32	3.88	23.38	61.73
C_7	13.00 +	41.19	4.53	32.29	91.01
C_8	9.80 +	79.39	4.38	32.38	125.95
C_9					7.34 #
C_{10}					8.21 #
C_{11}					10.84 #
C_{12}					15.34 #

Coefficient	Unit costs				
C_i	Unit labor cost *	material cost	vendor services ++	fixed labor overhead	total unit costs
C_{13}					7.75
C_{14}					11.63 **
C_{15}	5.97	27.66 ##	3.41	21.92	58.96
C_{16}	7.12	31.52 ##	3.88	23.38	65.90
C_{17}	8.67	41.77 ##	4.53	32.29	87.26
C_{18}	6.53	81.04 ##	4.38	32.38	124.33
C_{19}	8.96 +	27.66 ##	3.41	21.92	61.95
C_{20}	10.56 +	31.52 ##	3.88	23.38	69.34
C_{21}	13.00 +	41.77 ##	4.53	32.29	91.59
C_{22}	9.80 +	81.04 ##	4.38	32.38	127.60
C_{23}					7.37 #
C_{24}					8.24 #
C_{25}					11.45 #
C_{26}					15.95 #
C_{27}					7.75
C_{28}					11.63 **

* labor hours cost/unit consists of all costs other than regular hour cost for assembly and testing.

** overtime hours cost/hour = 1.5 x regular hours cost/hour (assembly and testing).

+ labor hours cost/unit during overtime = 1.5 x regular time cost.

++ vendor cost is the cost related to anodization of aluminum purchased services.

inventory carrying cost = regular time unit cost x 25%/year x year/2periods e.g. 7.34 = 58.74 x .25 x 1/2.

raw material cost during period 2 = 104% of raw material cost during period 1.

F RIGHT HAND SIDE CONSTRAINTS

Constraints 1 to 4

b_i (units) planned shipment for period 1 - ending inventory from previous period

$b_1 = 18470 - 5740 = 12730$

$b_2 = 16830 - 1685 = 15145$

$b_3 = 11690 - 746 = 10944$

$b_4 = 2910 - 260 = 2650$

Constraints 5 to 8

b_i (units) minimum production plan for period 1

$b_5 = 18000$

$b_6 = 28750$

$b_7 = 14950$

$b_8 = 3150$

Constraints 9 to 16:

b_{10}, b_{12}, b_{14}, b_{16} = minimum planned inventory for period 1

b_9, b_{11}, b_{13}, b_{15} = maximum allowable inventory = 110% of the planned inventory

$b_9 = 5797$

$b_{10} = 5270$

$b_{11} = 14965$

$b_{12} = 13645$

$b_{13} = 4407$

$b_{14} = 4006$

$b_{15} = 550$

$b_{16} = 500$

Constraint 17

The regular labor hours used in period 1 must be <= available regular time labor hours

$b_{17} = 0$

Constraint 18

The overtime labor hours used in period 1 must be <= available overtime labor hours.

$b_{18} = 0$

Constraint 19

Assume all testing is performed during regular time, the available test hours are taken as 6% of the available regular hours.

b_{19} = 0 in period 1

Constraint 20

This constraint deals with the most critical machine in the manufacturing process line which has the capability of performing several processes.

b_{20} (hours) =1 machine x 26 weeks/period x 7 days/week x 2 shifts/day x 10 hours/shift = 3640

Constraint 21 & 22

Lens availability for the most critical type of lenses

b_{21} = 80000 lens type A, (units)

b_{22} = 85000 lens type B, (units)

Constraint 23 & 24

Regular labor hours in period 1 are constrained to fluctuate between +/- 10% of the regular labor hours in the previous period

regular labor hours for the period before period 1 = 55120

b_{23} = .9 x 55120 = 49608

b_{24} = 1.1 x 55120 = 60632

Constraint 25

Overtime hours i period 1 are limited to 20% of regular hours for the same period.

b_{25} = 0

Constraint 26

Desirable profitability is assumed as:

b_{26} = 5000000

RIGHT HAND SIDE CONSTRAINTS, PERIOD 2

Constraints 27 to 30

b_i (units) planned shipment for period 2

b_{27} = 21580

b_{28} = 56600

b_{29} = 18960

b_{30} = 3940

Constraints 31 to 34

b_i (units) minimum production plan for period 2

b_{31} = 21650

b_{32} = 30950

b_{33} = 18850

b_{34} = 5050

Constraints 35 to 42:

b_{36}, b_{38}, b_{40}, b_{42} = minimum planned inventory for period 2

b_{35}, b_{37}, b_{39}, b_{41} = maximum allowable inventory = 110% of the planned inventory

b_{35} = 5874

b_{36} = 5340

b_{37} = 5264

b_{38} = 4785

b_{39} = 4286

b_{40} = 3896

b_{41} = 1771

b_{42} = 1610

Constraint 43

The regular labor hours used in period 2 must be <= available regular time labor hours

b_{43} = 0

Constraint 44

The overtime labor hours used in period 2 must be <= available overtime labor hours.

b_{44} = 0

Constraint 45

Assume all testing is performed during regular time, the available test hours are taken as 6% of the available regular hours in period 2.

b_{45} = 0

Constraint 46

This constraint deals with the most critical machine in the manufacturing process line which has the capability of performing several processes.

b_{46} (hours) =1 machine x 26 weeks/period x 7 days/week x 2 shifts/day x 10.5 hours/shift = 3840

Constraint 47 & 48

Lens availability for the most critical type of lenses

b_{47} = 80000 lens type A, (units)

b_{48} = 85000 lens type B, (units)

Constraint 49 & 50

Regular labor hours in period 1 are constrained to fluctuate between +/- 10% of the regular labor hours in the previous period

$b_{49} = 0$

$b_{50} = 0$

Constraint 51

Overtime hours i period 2 are limited to 20% of regular hours for the same period.

$b_{51} = 0$

Constraint 52

Desirable profitability is assumed as:

$b_{52} = 5000000$

G TECHNOLOGICAL COEFFICIENTS

Constraints 1 to 4 (Period 1)

Ten per cent of the production for each type is assumed to be defective resulting in:

$a_{1,1} = a_{1,5} = a_{2,2} = a_{2,6} = a_{3,3} = a_{3,7} = a_{4,4} = a_{4,8} = 0.9$

The ending inventory is subtracted from the periods production to arrive at the shipment less the beginning inventory for period 1:

$a_{1,9} = a_{2,10} = a_{3,11} = a_{4,12} = -1$

Constraints 27 to 30, Period 2

Similar to the above except that the beginning inventories for period 2 (i.e. the ending inventory for period 1) enter the equations as new variables

$a_{27,15} = a_{27,19} = a_{28,16} = a_{28,20} = a_{29,17} = a_{29,21} = a_{30,18} = a_{30,22} = .9$

$a_{27,23} = a_{28,24} = a_{29,25} = a_{30,26} = -1$

$a_{27,09} = a_{28,10} = a_{29,11} = a_{30,12} = 1$

Constraint 5 to 8, period 1

Lower limits for production of each type in regular time plus overtime is a management decision.

$a_{5,1} = a_{5,6} = a_{6,2} = a_{6,6} = a_{7,3} = a_{7,7} = a_{8,4} = a_{8,8} = 1$

Constraints 31 to 34, period 2

Same as above for the second period

$a_{31,15} = a_{31,19} = a_{32,16} = a_{32,20} = a_{33,17} = a_{33,21} = a_{34,18} = a_{34,22} = 1$

Constraint 9 to 16, period 1

Lower and upper bounds for the inventory of each type is decided by management based upon sales forecast.

$a_{9,9} = a_{10,9} = a_{11,10} = a_{12,10} = a_{13,11} = a_{14,11} = a_{15,12} = a_{16,12} = 1$

Constraints 35 to 42, period 2

Same as above for period 2

$a_{35,23} = a_{36,23} = a_{37,24} = a_{38,24} = a_{39,25} = a_{40,25} = a_{41,26} = a_{42,26} = 1$

Constraints 17 and 18, period 1

With due account of the processing breakdown an average labor time required for each type of production was derived. In doing so, the best representative item was chosen from the group of production under each type. The final coefficients are shown in the following table:

Scope type	1	2	3	4
labor hours per unit	.45	.35	.68	.85

These required hours are the same, both for a unit produced during regular or overtime

$a_{17,1} = a_{18,5} = .45$
$a_{17,2} = a_{18,6} = .35$
$a_{17,3} = a_{18,7} = .68$
$a_{17,4} = a_{18,8} = .85$

Constraints 43 & 44, period 2

Same as above for the second period

$_{3,15} = a_{44,19} = .45$
$a_{43,16} = a_{44,20} = .35$
$a_{43,17} = a_{44,21} = .68$
$a_{43,18} = a_{44,22} = .85$

Constraints 19 & 45 period 1 and 2

Same approach as discussed for constraints 17 and 18, i.e. the investigation of processing and deriving at the representative testing hours per unit.

scope type	1	2	3	4
test hours required	0.01	0.01	0.01	0.0

The total labor hours is assumed to be 6% of the available regular time per period.

$a_{19,5} = a_{19,6} = a_{19,7} = .01, \quad a_{19,8} = 0, \; a_{19,13} = -.06$

$a_{45,19} = a_{45,20} = a_{45,21} = .01, \; a_{45,22} = 0, \; a_{45,27} = -.06$

Constraint 20 and 46 period 1 and 2

Same approach as discussed for constraints 17 and 18 with regards to the machine hour requirements per unit.

scope type	1	2	3	4
machine hours required to produce each type	.04	.06	.05	.03

$a_{20,1} = a_{20,5} = a_{46,15} = a_{46,19} = .04$
$a_{20,2} = a_{20,6} = a_{46,16} = a_{46,20} = .06$
$a_{20,3} = a_{20,7} = a_{46,17} = a_{46,21} = .05$
$a_{20,4} = a_{20,8} = a_{46,18} = a_{46,22} = .03$

Constraints 21,22,47 and 48 period 1 and 2

The critical lenses are type A and B, the availability and cost considerations of these lenses are substantial. The table below, reflects the number of each type required per type of scopes:

Scope type	1	2	3	4
# of lens model A required per type	0	2	0	0
# of lens model B required per type	1	1	1	0

$a_{21,1} = a_{21,3} = a_{21,4} = a_{22,4} = a_{47,15} = a_{47,17} = a_{47,18} = a_{48,18} = 0$
$a_{21,2} = a_{47,16} = 2$
$a_{22,1} = a_{22,2} = a_{22,3} = a_{48,15} = a_{48,16} = a_{48,17} = 1$

Same relations for scopes produced on overtime, i.e.:

$a_{21,5} = a_{21,7} = a_{21,8} = a_{22,8} = a_{47,19} = a_{47,21} = a_{47,22} = a_{48,22} = 0$
$a_{21,6} = a_{47,20} = 2$
$a_{22,5} = a_{22,6} = a_{22,7} = a_{48,19} = a_{48,20} = a_{48,21} = 1$

Constraints 23,24,49, and 50 period 1 and 2
10% fluctuation regular hours in each period is allowed as compared to the previous period.

$a_{23,13}, a_{24,13}, a_{49,27}, a_{50,27} = 1$
$a_{49,13} = -0.90$
$a_{50,13} = -1.10$

Constraints 25 and 51 period 1 and 2

Overtime per each period is limited to 20% of regular time in that period:

$a_{25,13} = a_{51,27} = -0.2$
$a_{25,14} = a_{51,28} = 1.0$

Constraints 26 and 52 period 1 and 2

The following formulation is utilized to calculate the expected profitability per type, where profitability is:.

[sales price - (cost/unit + labor cost/unit)] x gross profit margin

product type coef.	sales price	cost per unit	labor hour per unit	labor cost per hour	gross profit margin	gross profit
$a_{26,1}$	260.70	58.74	.45	7.75	.43	85.35
$a_{26,2}$	340.70	65.70	.35	7.75	.46	125.26
$a_{26,3}$	415.85	86.68	.68	7.75	.43	139.28
$a_{26,4}$	513.60	122.69	.85	7.75	.19	73.02
$a_{26,5}$	260.70	61.73	.45	7.75 x 1.5	.43	83.31
$a_{26,6}$	340.70	69.14	.35	7.75 x 1.5	.46	123.04
$a_{26,7}$	415.85	91.01	.68	7.75 x 1.5	.43	136.28
$a_{26,8}$	513.60	125.95	.85	7.75 x 1.5	.19	71.78
$a_{52,15}$	260.70	58.96	.45	7.75	.43	85.25
$a_{52,16}$	340.70	65.9	.35	7.75	.46	125.16
$a_{52,17}$	415.85	67.26	.68	7.75	.43	139.02
$a_{52,18}$	513.60	124.33	.85	7.75	.19	72.71
$a_{52,19}$	260.70	61.95	.45	7.75 x 1.5	.43	83.21
$a_{52,20}$	340.70	69.34	.35	7.75 x 1.5	.46	122.95
$a_{52,21}$	415.85	91.59	.68	7.75 x 1.5	.43	136.03
$a_{52,22}$	513.60	127.60	.85	7.75 x 1.5	.19	71.46

H PROBLEM FORMULATION TABLE

See following page

4 SOLUTION

OBJECTIVE FUNCTION VALUE = $11298600

VARIABLE	VALUE
X_{11}	18000.000000
X_{21}	28750.000000
X_{31}	14950.000000
X_{41}	3150.000000
Y_{11}	.000000
Y_{21}	.000000
Y_{31}	.000000
Y_{41}	.000000
W_{11}	5270.000000
W_{21}	13645.000000
W_{31}	4006.000000
W_{41}	500.000000
X_{12}	21650.000000
X_{22}	30950.000000
X_{32}	18850.000000
X_{42}	5050.000000
Y_{12}	.000000
Y_{22}	.000000
Y_{32}	.000000
Y_{42}	.000000
W_{22}	5340.000000
W_{32}	4785.000000
W_{42}	3896.000000
LR_1	1610.000000
LO_1	49608.000000
LR_2	.000000
LO_2	44647.200000

5 SENSITIVITY ANALYSIS

A Sensitivity analysis-computer printout

NO. ITERATIONS= 43

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X_{11}	58.740000	2.989998	58.740000
X_{21}	65.700000	3.560005	65.700000
X_{31}	86.680000	4.330002	86.680000
X_{41}	122.690000	3.259995	122.690000
Y_{11}	61.730000	INFINITY	2.989998
Y_{21}	69.260000	INFINITY	3.560005
Y_{31}	91.010000	INFINITY	4.330002
Y_{41}	125.950000	INFINITY	3.259995
W_{11}	7.380000	INFINITY	7.380000
W_{21}	8.210000	INFINITY	8.210000
W_{31}	10.840000	INFINITY	10.840000
W_{41}	15.340000	INFINITY	15.340000
X_{12}	58.960000	2.990002	58.960000
X_{22}	65.890000	3.449997	65.890000
X_{32}	87.260000	4.329994	87.260000
X_{42}	124.330000	3.259995	124.330000
Y_{12}	61.950000	INFINITY	2.990002
Y_{22}	69.340000	INFINITY	3.449997
Y_{32}	91.590000	INFINITY	4.329994
Y_{42}	127.590000	INFINITY	3.259995
W_{12}	7.370000	INFINITY	7.370000
W_{22}	8.240000	INFINITY	8.240000
W_{32}	11.450000	INFINITY	11.450000
W_{42}	15.950000	INFINITY	15.950000
LR_1	7.750000	INFINITY	14.725000
LO_1	11.630000	INFINITY	11.630000
LR_2	7.750000	INFINITY	7.750000
LO_2	11.630000	INFINITY	11.630000

RIGHT HAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
1	12730.000000	INFINITY	1800.000000
2	15145.000000	INFINITY	2915.001000
3	10944.000000	INFINITY	1495.000000
4	2650.000000	INFINITY	315.000100
5	18000.000000	2000.001000	18000.000000
6	28750.000000	3238.890000	19557.510000
7	14950.000000	1661.112000	14950.000000
8	3150.000000	350.000100	3150.000000
9	5797.000000	INFINITY	527.000000
10	5270.000000	527.000000	1800.000000
11	14965.000000	INFINITY	1320.000000
12	13645.000000	1320.000000	2915.001000
13	4407.000000	INFINITY	401.000000
14	4006.000000	401.000000	1495.000000
15	550.000000	INFINITY	50.000000
16	500.000000	50.000000	315.000100
17	.000000	INFINITY	18602.000000
18	.000000	INFINITY	.000000
19	.000000	INFINITY	2359.480000
20	3640.000000	INFINITY	353.000000
21	80000.000000	INFINITY	22500.000000
22	85000.000000	INFINITY	23300.000000
23	49608.000000	11024.000000	7735.222000
24	60632.000000	INFINITY	11024.000000

RIGHT HAND SIDE RANGES (cont'd)

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
25	.000000	INFINITY	49608.000000
26	5000000.000000	2449774.000000	INFINITY
27	21580.000000	INFINITY	2165.000000
28	56600.000000	INFINITY	19885.000000
29	18960.000000	INFINITY	1885.000000
30	3940.000000	INFINITY	505.000100
31	21650.000000	575.001300	21650.000000
32	30950.000000	383.334200	29618.710000
33	18850.000000	460.001000	18850.000000
34	5050.000000	561.111300	5050.000000
35	5874.000000	INFINITY	534.000000
36	5340.000000	534.000000	2165.000000
37	5264.000000	INFINITY	479.000000
38	4785.000000	479.000000	4785.000000
39	4286.000000	INFINITY	390.000000
40	3896.000000	390.000000	1885.000000
41	1771.000000	INFINITY	161.000000
42	1610.000000	161.000000	505.000100
43	.000000	INFINITY	6961.699000
44	.000000	INFINITY	.000000
45	.000000	INFINITY	1964.332000
46	3840.000000	INFINITY	23.000050
47	80000.000000	INFINITY	18100.000000
48	85000.000000	INFINITY	13550.000000
49	.000000	9921.603000	6961.699000
50	.000000	INFINITY	9921.603000
51	.000000	INFINITY	8929.439000
52	5000000.000000	3707077.000000	INFINITY

B Sensitivity analysis-variable coefficients

Sensitivity analysis is also referred to as post optimality analysis. After we arrive at a feasible solution, we perform the sensitivity analysis. The parameters as specified in the model could change over a period of time . The Manager's apprehension and curiosity to know the effect of these changes on the day to day affairs could be taken care of by analyzing the model when subjected to different changes. We change the value of the constraints, costs of the variables and also the technological coefficients, and study the impact on the model.

The sensitivity analysis gives an idea about the flexibility in the system, i.e. we can determine the amount by which each parameter could be changed without altering optimality.

In our analysis, we would study the effect of changes in the values of right hand side, and the coefficients of the decision variables in the objective function. We would also like to venture outside the range as specified by the sensitivity analysis and see the effect on the model. The idea is to prepare the Manager for the worst case situation which has a very remote chance of occurrence, but still cannot be ignored. We feel that would end the chapter of post optimality analysis in a complete way.

I) Effect of change in the values of C_{ij} on the system.

The model is very sensitive to changes in the coefficients. It is also very rigid and, hence, we don't see any radical changes after we change the coefficients value.

a) Effect of Coefficient of X_{11}:

The current value is $58.74 and it can be increased by $2.99 and decreased by $58.74 so as to retain optimality. If the value is increased beyond $2.99, we would have Y_{11} as a candidate for the basis. But, from an economic viewpoint, we know that it is not possible because it implies that the cost of production during overtime is less than that during regular time. It's value can be decreased by $58.74 as the objective is to minimize cost and the decrease will not effect any other variable.

A possibility could be that the cost of producing X_{11} equals that of X_{21}. This means, we are going beyond the range as specified by the analysis. But we have very rigid constraints on the system. The constraint demands that a minimum amount of X_{11} has to be produced. Thus, even if cost of X_{11} increases beyond the cost of X_{21} or others, we would still be producing it.

We removed the " >= " constraint from the production constraints (actually we deleted rows 5 to 8). The system responded by reducing the production of X_{11} to 0 and produced only a particular type (X_{21}) and (X_{31}). This was the response even without changing the cost coefficient. But, "Leupold and Stevens" has to maintain market presence for certain types even though it may not be very profitable to produce it. This is a part of their long term strategy.

b) Effect of coefficient of X_{21}:

The current value is $65.70 and it can be increased by $3.56 and decreased by $65.70. An increase beyond this range would make Y_{21} a candidate for the basis. Y_{21} is the number of type 2 scopes produced in first quarter. An increase in the cost during regular hours would also increase the cost during overtime hours. Thus, the possibility of Y_{21} entering the system is remote due to an increase in the cost of production of X_{21}. The cost can be reduced to zero, since this is a cost minimization function.

c) Effect of coefficient of X_{31}:

The effect of change is same as discussed above for X_{11} and X_{21}.

The allowable range is $-\$86.68 <= \Delta C_3 <= \$4.33$. In case the cost is increased beyond range specified, Y_{31} would be a candidate for the basis.

d) Effect of coefficient of X_{41}:

The effect is same as that for X_{11}, X_{21} or X_{31}.

The range in which the value of coefficient can be changed is:

$$-\$122.69 <= \Delta C_4 <= \$3.26.$$

The upper limit corresponds to the value at which Y_{41} would be a candidate for the basis. The lower limit specifies a value of 0 for the coefficient so as to minimize the cost.

e) Effect of coefficient of Y_{11}:

The range as specified by the sensitivity analysis is:

$$\$2.99 <= \Delta C_5 <= \text{Infinity}$$

If the value is reduced more than $2.99, then we have a new candidate for the basis, i.e. X_{11}. It means that the overtime hours are cheaper to produce Type 1 scopes than regular hours. We know that it is not logical and a situation of this type would also be accompanied by a corresponding decrease in the value of coefficient of X_{11}. Hence, even though the system predicts that Y_{11} would enter the solution, this would not happen in practice.

We can increase the cost of production to infinity without changing the solution. It means that there is no restriction on the increase in the cost of overtime production and it doesn't affect any other variable or constraint. Y_{11} would enter the optimum solution only when the production demands force production during overtime.

f) Effect of coefficient of Y_{21}:

The range over which the coefficient value can be changed while ensuring optimality is:

$$-\$3.56 <= \Delta C_6 <= \text{Infinity}$$

If we reduce below $3.56 then we have a candidate for the basis and that is X_{21}. We can increase it to any value without upsetting optimality. But, as explained above, value of coefficient of X_{21} would decrease corresponding to a decrease in the coefficient of Y_{21}. Hence, practically speaking, X_{21} would not leave the basis.

g) Effect of coefficient of Y_{31}:

The range over which we can change this is:

$$-\$4.33 <= \Delta C_7 <= \text{Infinity}$$

The explanation is the same as that for Y_{11} and Y_{21}.

h) Effect of coefficient of Y_{41}:

The safe range of operation is:

$$-\$3.26 <= \Delta C_8 <= \text{Infinity}$$

Same explanation holds good for Y_{41} as that for Y_{11} and Y_{21}.

i) Effect of coefficient of W_{11}:

The values of this coefficient can be changed over the range:

$$-\ \$7.38 <= \Delta C_9 <= \text{Infinity}$$

Since, we have a cost minimization objective, we would like to have a cost of $0 for the inventory. The cost can be raised to infinity but it would still not change the solution. W_{11} is one of those variables that would always be in the basis.

This is understandable, because we ought to be carrying a minimum inventory under any condition and thus we should be ready to pay whatever the price. Note that, since it is a cost minimization problem, the system has the minimum amount possible of finished goods in the inventory.

We changed the inventory constraints and studied the effect on the solution. We were really changing the model itself at this stage but wanted to see the effect. If just given a " <= " in the model formulation and no upper bound is specified (we deleted rows 10, 12, 14 and 16) the system tries to take a value of "0" for W_{11}. This is understandable, since we are increasing the cost of operation from Lindo's point of view and hence it tries to drop any variable that adds to the cost. We were only easing the constraints a bit.

However, in practice, the system experiences a step increase in demand during the offset of the hunting season and the inventory is in fact a veritable life saver in meeting a sudden rise in demand.

j) Effect of coefficient of W_{21}:

The range for this coefficient is:

$$-\$8.21 <= \Delta C_{10} <= \text{Infinity.}$$

The variable W_{21} like W_{11} can never be removed from the basis and hence, we see that we can reduce the cost to zero or increase the cost to infinity.

Looking at it from an economic point of view, it is a carrying cost which is always built into the system and can never be dropped.

k) Effect of coefficient of W_{31}:

The safe range of operation is:

$$-\$10.84 <= \Delta C_{11} <= \infty$$

The explanation is the same as that for W_{11} and W_{21}. This variable would always be in the basis.

Ok

l) Effect of coefficient of W_{41}:

The value of the coefficient can be reduced by \$15.34 and increased by infinity. W_{41} like other inventory variables would always be in the basis.

m) Effect of coefficient of X_{12}:

The cost for manufacturing is very slightly higher than the cost during first period. This might seem strange, but the data received from the company substantiates the fact. The increase is due to changes in the material cost.

The value of the coefficient can be decreased by the value of the current coefficient itself. This is understandable as the system is trying its best to reduce the manufacturing cost. But, in case the value is increased beyond \$2.99 we have a new variable entering the basis, i.e. Y_{12}. this is also expected. But, at this point X_{12} doesn't leave the system and we still get an optimal solution instead of an infeasible solution.

n) Effect of coefficient of X_{22}:

The range as specified by Lindo is:

$$-\$65.89 <= X_{22} <= \$3.45$$

We can reduce the cost to '0' but we cannot increase the cost beyond \$3.45 If we go beyond this range, then Y_{22} would be a candidate for the basis. However, as explained earlier, this cannot happen in practice.

o) Effect of coefficient of X_{32}:

The range for this coefficient is:

$$-\$87.26 <= \Delta C_{17} <= \$ 4.33$$

The cost can be reduced till '0' but we would change the basis, if we go outside the upper limit. The new candidate: Y_{32} of course. Same explanation holds good as that for X_{12} and X_{22}.

p) Effect of coefficient of X_{42}:

The range for this coefficient is:

$$-\$124.33 <= \Delta C_{18} <= \$ 3.26$$

The cost should not be increased beyond approximately $13.15, otherwise Y_{42} would become a candidate for the basis. There is no problem even if we reduce the cost to zero.

q) Effect of coefficient of Y_{12}:

The allowable range is:

$$\$2.99 <= \Delta C_{19} = \text{Infinity}$$

If the cost is decreased beyond approximately $8.44, then Y_{12} would be a candidate for the basis. A similar observation is made here, if we venture outside the lower limit, Y_{12} enters the solution but X_{12} is still in the basis.

There is no limit to the increase in the cost of manufacture during overtime, i.e. there is no effect on other variables or constraints.

r) Effect of coefficient of Y_{22}:

The range allowed to change is:

$$-\$3.45 <= \Delta C_{20} <= \text{Infinity}$$

The same explanation as above holds good for Y_{22}.

s) Effect of coefficient of Y_{32}:

The allowable range to change is:

$$-\$ 4.33 <= \Delta C_{21} <= \text{Infinity}$$

The same explanation holds good as that for Y_{12}.

t) Effect of coefficient of Y_{42}:

The allowable range to change is:

$$-\$ 3.26 <= \Delta C_{22} = \text{Infinity}$$

The value of the objective function in this range does not change, since Y_{42}= 0. The explanation is the same as that for Y_{12}.

u) Effect of coefficient of W_{12}:

The range for this coefficient is:

$$-\$7.37 <= \Delta C_{23} <= \text{Infinity}$$

As explained earlier, the change in coefficients can not force out W_{12} from the basis. W_{12} has to be in the basis as required by the constraints. Even though, the cost were to be Infinity, we would still be forced to carry the inventory and come up with an optimal solution but at a substantially higher cost in the objective function's value.

v) Effect of coefficient of W_{22}:

The range in this case is:

$$-\$8.24 <= \Delta C_{24} <= \text{Infinity}$$

The same explanation as above holds good in this case too.

w) Effect of coefficient of W_{32}:

The range for the coefficient of W_{32} is:

$$-\$11.45 <= \Delta C_{25} <= \text{Infinity}$$

Same explanation as that for W_{12} holds good in this case.

x) Effect of coefficient of W_{42}:

The range for W_{42}'s coefficient is:

$$-\$15.95 <= \Delta C_{26} <= \text{Infinity}$$

Same explanation as that for W_{12} holds good in this case too.

y) Effect of coefficient of LR_1:

The range over which this coefficient can change is:

$$-\ \$14.73 <= \Delta C_{13} <= \text{Infinity}$$

This variable would always be in the basis, come what may. The range over which the coefficient values can change gives us an idea about this characteristic. Whatever would be the cost (it would be obviously lower than the overtime cost), we would have to live with it, be it inflation or deflation.

z) Effect of coefficient of LO_1:

The range for the coefficient of this variable is:

$$-\$11.63 <= \Delta C_{14} <= \text{Infinity}$$

This means that the change in coefficient values have no impact on the solution. LO_1 could enter the basis, but only when there is sufficient demand that forces the system to produce scopes in overtime hours. A change in the coefficient's value cannot bring LO_1 into the solution.

z1) Effect of coefficient of LR_2:

The range for the coefficient of this variable is:

$$-\$7.75 <= \Delta C_{27} <= \text{Infinity}$$

The same explanation as given for LR_1 applies to LR_2 also. The system would be happy to give a better value for the objective function, if the cost of LR_2 goes down, but even if the cost goes up, it has to live with it.

z2) Effect of coefficient of LO_2:

The range for this coefficient is:

$$-\$11.63 <= \Delta C_{28} <= \text{Infinity}$$

The explanation for LO_1 holds good here also.

C Sensitivity Analysis-right-hand side values

Row 1:

This constraint refers to the operational characteristic for period 1, optical device type 1. The number of devices produced on regular time, plus the number of devices produced on overtime, minus the number of devices in the inventory, should be more than 12730 units. There is a slack variable associated with this constraint which is not in the optimal solution. This means that 1800 units can be removed from the system (B = 12730) without making the problem infeasible.

$$-1800 \;=<\; \Delta B \;<=\; \text{infinity}$$
$$10930 \;=<\; b^{new} \;<=\; \text{infinity}$$

Row 2:

This constraint refers to the number of optical devices produced on regular time, plus the number of devices produced on overtime, minus the number of devices in the inventory, in period 1 for type 2 device and it should be more than 15145 units, there is a slack variable associated with this constraint which is not in the optimal solution. This means there is an excess amount of 2915 units can be removed from the system (b = 15145) without making the problem infeasible.

$$-2915 \;=<\; \Delta b \;<=\; \text{infinity}$$
$$12230 \;=<\; b^{new} \;<=\; \text{infinity}$$

Row 3:

This constraint refers to the number of optical devices produced on regular time, plus the number of devices produce on overtime, minus the number of devices in the inventory, in period 1, for type 3 optical device and it should be more than 10944 units. There is a slack variable associated with this constraint which is not in the optimal solution. This means there is an excess amount of 1495 units can be removed from the system (b =10944) without making the problem infeasible.

$$-1495 \;=<\; \Delta b \;<=\; \text{infinity}$$
$$9449 \;=<\; b^{new} \;<=\; \text{infinity}$$

Row 4:

This constraint refers to the number of devices produced on regular time, plus the number of devices produced during overtime, minus the number of devices in the inventory, for period 1, for type 4 optical device, and it should not be more than 2650 units. There is a slack variable associated with this constraint which is not in the optimal solution. This means there is an excess amount of 315 units that can be removed from the system (b = 2650) without making the problem infeasible.

$$-315 = < \Delta b < = \text{infinity}$$
$$2335 = < b^{new} < = \text{infinity}$$

Row 5:

This constraint represents the amount of type 1 device for period 1, produced on regular time, plus the number of devices in the inventory should not be more than 18000 units. There is a surplus variable associated with this constraint which is not in the optimal solution. This means the resources represented by constraint (18000 units) is fully utilized. The optimal solution remains feasible as long as the new values of the basic variable are still non-negative.

$$-18000 < = \Delta b < = 2000$$
$$0 < = b^{new} < = 20000$$

And the optimal value of Z is:

$$Z^{new} = Z^{old} + Z \times b$$
$$= 11298600 + (-58.74)(2000)$$
$$= 11181120$$

and $Z^{new} = 11298600 + (-58.74)(-18000)$

$$= 12355920$$

thus the range for z is:

$$11181120 = < Z^{new} = < 12355920$$

Row 6:

This constraint represents the amount of type 2 optical device produced on regular time, plus the amount of devices from the inventory, should not be more than 28750 units. There is a surplus variable associated with this constraint which is not in the optimal solution. This means the resources represented by constraint (28750 units) is fully utilized. The optimal solution remain feasible as long as the new values of the basic variable are still non-negative.

$$-19639 =< \Delta b =< 3238$$

$$9193 =< b^{new} =< 31988$$

$$\begin{aligned} Z^{new} &= Z^{old} + Z \times b \\ &= 11298600 + (-65.7)(3238) \\ &= 11085863 \\ Z^{new} &= 11298600 + (-65.7)(-19639) \\ &= 1258882 \end{aligned}$$

thus the range for z is:

$$11085863 =< Z^{new} =< 1258882$$

Row 7:

This constraint represents the amount of type 3 device in period 1, produced on regular time, plus the amount of device from the inventory, should not be more than 14950 units. There is a surplus variable associated with this constraint which is not in the optimal solution. This means the resources represented by constraint (14950 units) is fully utilized. The optimal solution remain feasible as long as the new values of the basic variable are still non-negative.

$$-14950 =< \Delta b =< 1661.112$$
$$0 =< b^{new} =< 16611.112$$

$$\begin{aligned} Z^{new} &= Z^{old} + Z . b \\ &= 11298600 + (-86.68)(-14950) \\ &= 12594466 \\ \text{and } Z^{new} &= 11298600 + (-86.68)(1661.112) \\ &= 11154615 \end{aligned}$$

thus the range for z is:

$$11154615 =< Z^{new} =< 12594466$$

Row 8:

This constraints represents the amount of type 4 device in period 1, produced on regular time plus the amount of devices from the inventory should not be more than 3050 units. There is a surplus variable associated with this constraint which is not in the optimal solution. This means the resources represented by constraint (3150 units) is fully utilized. The optimal solution remain feasible as long as the new values of the basic variable are still non-negative.

$$-3110 =< \Delta b =< 350$$
$$0 =< b^{new} =< 3500$$

and the new optimal value of Z is:

$$Z^{new} = Z^{old} + Z \cdot b$$
$$= 11298600 + (-122.69)(-3150)$$
$$= 11685074$$

and $Z^{new} = 11298600 + (-122.69)(350)$
$$= 11255659$$

thus the range for z is:

$$11255659 =< Z^{new} =< 11685074$$

Row 9:

This constraint represents the maximum allowable inventory of type 1 product in the first period. It should not be more than 5797 units. There is a slack associated with this constraint which is not in the optimal solution. This means there is an excess amount of 527 units in the system (b = 5797) without making the problem infeasible.

$$-527 =< \Delta b <= \text{infinity}$$
$$5270 =< b^{new} <= \text{infinity}$$

Row 10:

This constraint represents the minimum allowable inventory of type 1 product in the first period. It should be more than 5270 units. There is a surplus variable associated with this constraint. This means the resources represented by constraint (5270 units) is fully utilized, the optimal solution remains feasible as long as the new values of the basic variable are still non-negative.

$$-1800 =< \Delta b =< 527$$
$$3470 =< b^{new} =< 5797$$

and the new value of the optimal solution Z is:

$$Z^{new} = Z^{old} + Z \cdot b$$
$$= 11298600 + (-7.38)(-1800)$$
$$= 11311884$$

and $Z^{new} = 11298600 + (-7.38)(527)$
$$= 11294711$$

thus the range for z is:

$$11294711 =< Z^{new} =< 11311884$$

Row 11:

This constraint represents the maximum available inventory for product type 2, in the end of period 1. It should not be more than 14965 units. There is a slack variable associated with this constraint which is in the solution. This means there is an excess amount of (1320 units) which can be removed form the system (b = 14965 units) without making the problem infeasible.

$$-1320 =< \Delta b <= \text{infinity}$$
$$13645 =< b^{new} <= \text{infinity}$$

Row 12:

This constraint represents the minimum available inventory for product type 2 in the end of period 1. It should be more than 13645 units. There is a surplus variable associated with this constraint. This means the resources represented by constraint (13645 units) is fully utilized, the optimal solution should remain feasible as long as the new values of the basic variable are still non negative.

$$-2915 =< \Delta b =< 1320$$
$$10730 =< b^{new} =< 14965$$

and the optimal value of Z is:

$$Z^{new} = Z^{old} + Z \cdot b$$
$$= 11298600 + (-8.21)(-2915)$$
$$= 11322532$$

and

$$Z^{new} = 11298600 + (-8.21)(1320)$$
$$= 11287763$$

thus the range for z is:

$$11287763 =< Z^{new} =< 11322532$$

Row 13:

This constraint represents the maximum available inventory for product type 3 in the end of period 1. It should not be more than 4407 units. There is a slack variable associated with this constraint which is in the solution. This means there is an excess amount of (401 units) which can be removed from the system (b = 4407 units) without making the problem infeasible.

$$-401 =< \Delta b <= \text{infinity}$$
$$4006 =< b^{new} <= \text{infinity}$$

Row 14:

This constraint represents the minimum available inventory for product type 3 in the end of period 1. It should be more than 4006 units. There is a surplus variable associated with this constraint. This means the resources represented by constraint (4006 units) is fully utilized, the optimal solution should remain feasible as long as the new value of the basic variables are still non-negative.

$$-1495 =< \Delta b =< 401$$
$$2511 =< b^{new} =< 4407$$

The new optimal value of Z is:

$$Z^{new} = Z^{old} + Z \,.\, b$$
$$= 11298600 + (-10.48)(-1495)$$
$$= 11314806$$

and
$$Z^{new} = 11298600 + (-10.48)(401)$$
$$= 11294253$$

thus the range for z is:

$$11294253 =< Z^{new} =< 11314806$$

Row 15:

This constraint represents the maximum available inventory for product type 4 in the end of period 1. It should not be more than 550 units. There a slack variable associated with this constraint which is in the solution. This means there is an excess amount of (50 units) which can be removed from the system (b = 550 units) without making the problem infeasible.

$$-50 =< \Delta b =< \text{infinity}$$
$$500 =< b^{new} =< \text{infinity}$$

Row 16:

This constraint represents the minimum available inventory for product type 4 in the end of period 1. It should be more than 500 units. There is a surplus associated with constraint. This means the resources represented by constraint (500 units) is fully utilized, the optimal solution should remain feasible as long as the new values of the basic variable are still non-negative.

$$185 =< \Delta b =< 550$$

The new optimal value of Z is:

$Z^{new} = Z^{old} + Z . b$
$= 11298600 + (-15.34)(-315)$
$= 11303432$

and $Z^{new} = 11298600 + (-15.34)(50)$
$= 11297833$

thus the range for z is:

$$11297833 =< Z^{new} =< 11303432$$

Row 17:

This constraint represents the difference between regular labor hours for product 1, 2, 3, and 4 for the first period. and the regular labor hours available in the first period, should be equal to zero. There is a slack variable associated with this constraint which is in the solution.
This means there is an excess amount of 18687 hours which can be removed from the system without making the problem infeasible.

$$-18602 =< b^{new} =< \text{infinity}$$

Row 18:

This constraint refer to the overtime labor hours for each type, minus the required overtime available. There is a slack variable equal to zero, also the dual price is equal to zero. Because the labor hours available is not included in the objective function, that will set the value of the slack to be zero. This means that this constraint have no influence in the objective value. This constraint will be needed when the production will increase to the point where the it force the overtime to be used.

Row 19:

This constraint refer to the amount of regular testing hours for all types of optical devices produced on regular time and overtime. This amount should not be more than 1696 hours. There a slack variable associated with this constraint which is in the solution. This means there is an excess amount of (1079 hours) which can be removed from the system (b = 1696 hours) without making the problem infeasible.

$$-2359 =< \Delta b =< \text{infinity}$$
$$2359 =< b^{new} =< \text{infinity}$$

Row 20:

This constraint refer to the machine hours available for all type of product on regular time and overtime for period 1, and it should not reduced to any value below 3640 hours. This means there is an excess amount of 353 machine hours which can be removed from the system (b = 3640 hours) without making the problem infeasible.

$$-353 =< \Delta b =< \text{infinity}$$
$$3287 =< b^{new} =< \text{infinity}$$

Row 21:

This constraint refer to total lenses needed for product 1 only, in the first period, if the product produced in regular time and overtime. The total amount of lenses should be 8000 lenses or less. There is a slack variable associated with this constraint which is in the solution. This means there is an excess amount of 22500 lenses for this period. Which can be removed from the system (b = 80000 lenses) without making the problem infeasible.

$$-22500 =< \Delta b =< \text{infinity}$$
$$57500 =< b^{new} =< \text{infinity}$$

Row 22:

This constraint represents the total number of lenses available for 1,2,and 3 optical devices in the first period, which is 85000 lenses. There is a slack variable associated with this constraint which is in the solution. This means there is an excess amount of 2300 lenses for this period, which can be removed from the system (b = 85000 lenses) without making the problem infeasible.

$$-23300 =< \Delta b =< \text{infinity}$$
$$61700 =< b^{new} =< \text{infinity}$$

Row 23:

This constraint represents regular hours available in the first period, it should no be less than 49608 hours. There is a surplus variable associated with this constraint. This means the resources represented by constraint (49608 hours) is fully utilized, the optimal solution should remain feasible as long as the new value of the basic variable are still non-negative.

$$-7735 =< \Delta b =< 11024$$
$$41873 =< b^{new} =< 60632$$

$$Z^{new} = Z^{old} + Z . b$$
$$= 11298600 +(-14.725)(-7735)$$
$$= 11335148$$

and $Z^{new} = 11298600 +(-14.725)(11024)$
$$= 11136272$$

thus the range for z is:

$$11136272 =< Z^{new} =< 11335148$$

row 24:

This constraint represents the maximum available labor hours for period 1, and should not be more than 60632 hours. There is a slack variable associated with this constraint which is in the solution. This means there is an excess amount of (11024 hours) which can be removed from the system (b = 60632 hours) without making the problem infeasible:

$$-11024 =< \Delta b =< \text{infinity}$$
$$49608 =< b^{new} =< \text{infinity}$$

Row 25:

This constraint represents the relationship between the regular time and the overtime, where overtime limitation hours should be 20% of the regular labor hour. There is a slack variable associated with this constraint which is in the solution, This means there is an excess amount of 49608 hours which can be removed from the system without making the problem infeasible.

$$-49608 \le \Delta b \le \text{infinity}$$
$$49608 \le b^{new} \le \text{infinity}$$

Row 26:

This constraint represents the profit expected from all the products type in the first period. It should be more than $5000000. There is a slack variable associated with this constraint. The optimal solution over satisfies the profit by $2449774. Thus the optimal solution will remain optimal even if the values Of $5000000 increased by the amount:

$$\text{infinity} \le \Delta b \le 24497748$$
$$\text{infinity} \le b^{new} \le 74497748$$

6 DISCUSSION

The model is very rigid and it gives results as expected. The results can be known at the outset. To minimize the cost of operation, we had specified a minimum level of production of each type of scope, minimum amount of inventory, and labor hours. We were not surprised to see that the system picked up the lowest values from these constraints in the final solution.

For example, please refer to rows 5 to 8 in the problem formulation. They give us a lower bound on production for each model in the first period. The final values in the solution corresponding to the production volumes values are the lowest that the system could take, i.e. X_{11} = 18000, X_{21} = 28750, X_{31} = 14950 and X_{41} = 3150.

The total slack in the regular labor hours for both the periods is: 18602 and 6961 respectively, i.e. 25563 hrs. The cost saving is 25563 * 7.75 = $198113.25. This figure does not include the overtime hours but still is a sizeable amount.

Had it been a profit maximization problem, we would have seen all the slack resources being used up and the system trying to achieve as high a production target as possible.

The system is well covered to take care of any sudden rise in the demand. We have sufficient numbers of finished goods in the inventory and also there are sufficient lenses(raw material) in the inventory.

Test hours: Test hours is no problem and the manager need not worry about this aspect.

Machine hours: The model is extremely sensitive to changes in the machine hour availability. The system indicates that the machine hour availability is one of the most important factors to be taken care of to ensure a normal production flow. The system does has a slack in the first quarter but in the second quarter, there are just 23 hours as slack, which means that if anything happens to the machines then the entire production could come to a halt.
POTENTIAL HOT SPOT!! PLEASE WATCH OUT!!

Raw Materials: The raw material refers to the lenses which the company has to procure from outside and is totally dependent on the outside sources for sourcing them. Besides that, there is a huge lead time associated with the lenses. Row 21 says that model 'type 2' takes two lenses per unit and that fixes the limit to 40000 lenses. Row 22 says that the total availability of critical lenses is less than or equal to 85000. Please note that the lenses for model 'type 4' do not appear in the picture as there is no problem in sourcing them and these 85000 lenses are to be shared among three model types. Hence, the slack associated with them can be justified.

Profit considerations: The final solution suggests that the company can really be making more profit than what it is achieving now. This should be a please surprise for the top manager's in the company. The production could be raised in the factory by improving the productivity of the work force and proper management of resources. This would help in bringing the cost of production down and getting a better return on the dollar.

Ok

Moreover, when we eased the minimum constraints of production on the different models, we got a totally different result, i.e. the system suggests that the company could be doing a better job of bringing the costs down if manufacturing certain types of scopes only. This may not be practical but is worth looking into from a strategic point of view. By changing the production mix the company may end up with a better cost and hence profit.

For the first period the system suggests that we should produce X21 = 33455 , and X31 = 5811. for the second period, we should produce only X12 = 39948. The system is very forcefully trying to say that producing the scond model type would bring the costs down and also not violate any of the constraints. This is worth looking into.

We observed that there was no production during overtime. This means that we either dont have enough demand for production right now or the workers are highly productive. This contradicts the real life situation as the company does produce scopes during overtime. We investigated this disparity a little deeper and changed the technological coefficients(we increased them) in rows 17 and 43. The coefficients represent the time taken to produce a particular model of scope.

This forced the system to go for production into overtime. This was a very significant observation and also a very sensitive one. What it means is that the workers are taking more time to produce than what was planned for by the Production manager. The company can save a lot of money, approximately $200k by restricting the production to the time specified by the solution.

If we assume that the workers were indeed producing the scopes within the specified time limits, then why do we need so much of extra labor hours. The reason is that in case people leave the company, the cost of finding a suitable person, training him to the desired level of expertise takes a lot of time, effort and expense. The production also goes down as the smooth flow of operation is disrupted. The management after lot of analysis concluded that it was safer to have some slack in labor hours rather than to go looking for people whenever some extra labor hours were required. The figures for fluctuation rate control(<= 1.1 times or >= 0.9 times the regular hours in the previous period) in rows 23,24 and 49,50 are thus critical. Inventory figures: The system as specified earlier picks up the minimum values for the inventory. The point is whether we need to be so rigid to force inventory figures into the solution. The answer is yes, because, the demand shoots up during certain parts of the year and the inventory helps the management to meet the demand without overloading the production lines. However, if the constraint is removed, the system drops the inventory totally as it can get a better value for the objective function.

7 CONCLUSION

The results are understandable and there is close agreement between the model and the actual data taken from 1987 production records. Any disparities between the numbers in the model and the real life situation can be accounted for and explained.

LABOR HOURS & OVERTIME:

The management should look into the reason why the workers are taking more time to produce than expected. What is the reason? Is it due to lack of training or it is a motivation problem. The answer to this problem could save the company thousands of valuable dollars. We know from data collected during earlier years that the assembly times are achieveable. However, there has been a high percentage of turnover, terminations and retirements, during the last three years. The loss of these trained and experienced workers could help explain why the current labor performance is running about 50% over standard. This emphasizes the need to focus on training.

The company definitely does not need to produce during overtime. Sometimes, the workers want to push production to overtime as they get better benefits. The shift supervisors would have to be on their toes and find out what is really wrong.

MACHINE HOURS:

The Machine hours are very limited and this is another top priority issue which should be taken care of immediately. We recommend that there be a slack of about 200 hours for this constraint. In fact, we later found that the company had, early in 1988, purchased another machine tool to relieve this constraint.

IDLE RESOURCES:

Please refer to the following Table. This gives us an idea about the idle resources expressed as a % of the available resources. These resources could either be slacks or surpluses in the system. The management should look into these and checkup whether these numbers could be reduced as they would have a haevy impact on cutting down the costs. This raises the interesting possibility of trading off product cost for implementation of new technology, i.e., perhaps the manager could out-source parts manufacture and free internal resources for new process development.

Catagory	Slack(surplus/resource	
Available labor hours	18602/49608 = 37.5 6960/44647 = 15.6	Period 1 Period 2
Test hours	2359/(.06)(49608) = 79.3 1960/(.06)(44647) = 73	Period 1 Period 2
Machine hours	353/3640 = 28.1	
Raw material-lenses	22500/80000 = 28.1 18100/80000 = 22.6	Period 1 Period 2
Raw material-lenses	23300/85000 = 27.4 13550/85000 = 16	Period 1 Period 2

PRODUCT MIX SELECTION:

The management can try to change their focus of attention on a few particular models and see if it helps them to net better performance figures. Application of Pareto's rule to the product mix for several years would likely provide fruitful results.

The system is very tightly controlled. The reason could be that the company has been in this business for a long while (since 1906) and the product design is mature and stable. In addition the production process is well known and experience has likely exposed the management and supervision to almost all of the perturbations in the manufacturing system. It does seem inevitable that competitive pressure from outside will force the need to sharpen the mangement of the manufacturing process.