ETM

ENGINEERING & TECHNOLOGY MANAGEMENT

Title: Excavation Rate and Work Force Planning

Course:

Year: 1989

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Report No: P89006

ETM OFFICE USE ONLY	
Report No.:	See Above
Type:	Student Project
Note:	This project is in the filing cabinet in the ETM department office.

Abstract: A contractor is dealing with the construction of two dams 515 miles apart. The objective of the firm is to minimize total excavation cost for the two projects, the transportation cost of machinery, and losses due to shut-down of machinery during transportation.

EXCAVATION RATE AND WORK FORCE PLANNING

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EMP - P8906

EXCAVATION RATE & WORK FORCE PLANNING FOR TISAN CONSTRUCTION INC. USING MIXED-INTEGER LINEAR PROGRAMMING

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June 5, 1989

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DEFINITION OF PROBLEM

DEFINITION OF THE PROBLEM

TISAN CONSTRUCTION Inc. is a contractor of mid-size dams in Turkey. Currently it is dealing with the construction of two dams which are located in Burdur-Yaprakli and Amasya-Sarayozu(characteristics are given in appendix D. & E) 2 construction sites are 860 km (515 miles) apart.

Although the firm has enough machinery to continue the constructions independent of each other, especially, due to increasing excavation cost during rainy seasons, it is more preferable to make excess excavation in dry season and hold it in inventory with a trade off by paying inventory holding cost. Of course this requires the transportation of heavy-duty machinery used in excavation which includes transportation cost and working day losses, back and forth between the 2 sites. Constructions are controlled by DSI (State Hydraulic Works) every three months to check if work is going on schedule. Firm is obliged to satisfy minimum excavation volumes per control time unit set by Master Plan which is issued by DSI just after bidding.(Master Plan is given in appendix G for two dams)

DEFINITIONS OF TERMS USED IN REPORT

DEFINITIONS

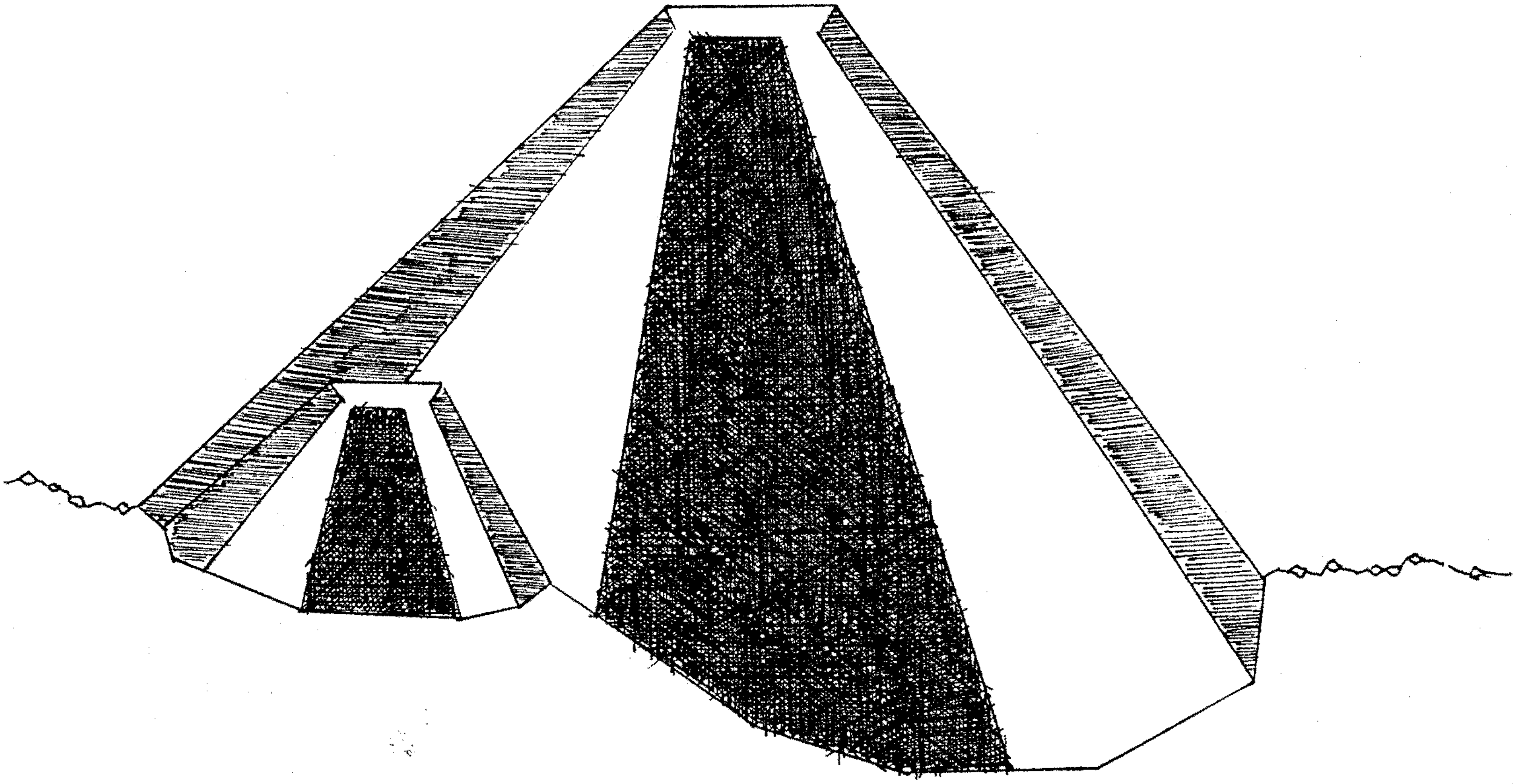
...<u>Filler type</u> : 2 dams which will be constructed by TISAN CONSTRUCTION Inc. have three basic ingredients (Rock, Clay, Filler #3). Two dams have a common structure as shown at the next page. Each filler type for each dam is associated with corresponding facility as shown below. (Filler types may be seen as different products produced by manufacturing firm.)

Filler type	facility #
Burdur - Rock	1
,, - Clay	2
,, - #3	3
Amasya - Rock	4
,, - Clay	5
,, - #3	6

...<u>Facility</u> : is the common name for the group of mines which are excavated for a particular type of filler in a particular dam.

There are currently 6 facilities :

1. Burdur-Yaprakli dam ; Rock mines
2. ,, ,, ,, ; Clay mines



Cross section of the dams which are undertaken by Tisan Construction Inc.(Not drawn to scale)

- Rock
- Filler #3
- Clay

3. Burdur-Yaprakli dam ; Filler #3 mines

4. Amasya-Sarayozu dam ; Rock mines

5. ,, ,, ,, ; Clay mines

6. ,, ,, ,, ; Filler #3 mines

For example although there are 2 clay mines for Amasya-Sarayozu dam, all are considered to be at the same location and are altogether called as facility 5. Also there are 3 filler #3 mines in Amasya,again they are considered to be at the same location(although not) and altogether called as facility 6, etc.

With the introduction of the 'facility' description, we are making the assumption that, there are no significant transportation cost difference between different mines of same filler type. For example for Amasya-Sarayozu dam , we are considering that unit transportation cost of clay from two different mines to the dam construction site are same.

...Complementary facility : is defined as the facility at the other dam site which is used for the excavation of same filler type.

For example complementary facilities of Burdur-Clay mines are the clay mines in Amasya-Sarayozu. Therefore, we can summarize the facilities as below:

Facility #	Complementary Facility #
1	4
2	5
3	6
4	1
5	2
6	3

...<u>Class $(i+i_c)$ Machine</u> : Firm has different heavy duty machinery with different daily excavation capacities. The ones which are devoted to the body filling(of dam) related excavation activities are shown in Appendix F with specs.

Also these machines are subdivided within themselves depending on which facility they are working. Among those, 2 units of TEREX C550 wheel loaders are especially designed to work in rock mines. 5 units of CAT 988B wheel loaders are grouped for working in clay mines. Remaining 4 units of CAT 992C wheel loaders and 2 units of CAT 245BH Backhoe excavators are grouped for working in Filler #3 mines. These groupings are based on 3 reasons:

1. Machines in the same group have approximately same periodic maintenance plans.

2. Machines in the same group have approximately(although shown as equal) same excavation rates per

working hour.

3. And machines in the same group are more suitable for in that facility due to working environment, and due to machine specifications.

Therefore; 2 units of TEREX C550 wheelloaders..any rock mine
5 ,, of CAT 988B ,, ..any clay ,,
4 ,, of CAT 992C ,, ..any filler #3 ''
2 ,, of CAT 245BH Excavators . ,, ,, ,, ,,

So we come up with the following table :

	facility #					
	1	2	3	4	5	6
TEREX C550	X			X		
CAT 988B		X			X	
CAT 992C			X			X
CAT 245BH			X			X

(X shows the facility that a specified machine can work)

If i denotes a facility and i_c denotes its complimentary facility (defined above) Then class($i+i_c$) machine is a machine which can work in facility i and i_c. For example a class(1+4) machine will indicate a TEREX C550, a class(3+6) machine will indicate either a CAT 992C or CAT 245BH.

Here we see that we can not define if a class (3+6) machine is CAT 992C or CAT 245BH. But however we do not need to discriminate among them, since based on the assumptions for which the groupings of machines are made there will be no significant operating cost differences and no significant excavation volume differences; if the working machine is CAT 992C or CAT245BH.

OBJECTIVE

Objective of the firm is to minimize total excavation cost (which are due to the operating cost of heavy-duty excavation machinery) for two dams, the transportation cost of machinery between two sites and losses which occur due to not working of machinery during transportation (which generally takes 24 to 28 hrs for a machine) over the planning horizon. In other words:

$$\min: \begin{bmatrix} \text{Total} \\ \text{Excavation} \\ \text{Cost} \end{bmatrix} + \begin{bmatrix} \text{Transportation} \\ \\ \text{Cost} \end{bmatrix} + \begin{bmatrix} \text{Losses due to} \\ \text{not working} \\ \text{of Machinery} \end{bmatrix}$$

MODEL

i. LP Formulation

ii. Decision Variables & Parameters

iii. Constraints

MODEL:

min: $$z = \sum_{t=1}^{T} \sum_{i=1}^{n} \left[c_{it} H_{it} + \lambda_{it} T_{it} + \lambda'_{it} L_{it} \right]$$

subject to:

$$D_{i1} \le \alpha_i \cdot H_{i1}$$

$$\sum_{t=1}^{k} \alpha_i H_{it} \ge \sum_{t=1}^{k} D_{it} \qquad \forall i \ \forall k \quad k = 2, 3, \ldots T-1$$

$$\sum_{t=1}^{T} \alpha_i H_{it} \ge \sum_{t=1}^{T} D_{it}$$

$$\sum_{t=1}^{T} \alpha_i H_{it} \le (1.04) \sum_{t=1}^{T} D_{it}$$

(2)

$$H_{it} \le M_{it} W_{it} \quad ; \quad \forall i, \forall t$$ (3)

$$W_{it} = W_{i,t-1} + T_{it} - L_{it} \; ; \; \forall i, \forall t$$ (4)

$$W_{it} + W_{i_c t} \le M_{i i_c, t} \quad ; \forall i, \forall i_c, \forall t$$ (5)

$$(0.995)(\alpha_{i+1}) H_{i+1,t} \le (G_{i,i+1})(\alpha_i) H_{it} \le (1.02)(\alpha_{i+1}) H_{i+1,t} \quad ; \forall i, \forall t$$ (6)

$$(0.995)(\alpha_{i+2}) H_{i+2,t} \le (G_{i,i+2})(\alpha_i) H_{it} \le (1.02)(\alpha_{i+2}) H_{i+2,t} \quad ; \forall i, \forall t$$ (7)

$$H_{it} \ge 0$$

$$W_{it}, L_{it}, T_{it} = 0, 1, 2, 3, \ldots$$

DECISION VARIABLES AND PARAMETERS:

t:each time unit is 3-month in length. In order to coincide with the control times of DSI. Therefore planning horizon is 5 time units for Burdur and 6 time units for Amasya.

c_{it}: Hourly operating cost of class $(i+i_c)$ machine during the t^{th} 3-month period.

H_{it}:number of total machine hours which will be performed in facility i during t^{th} 3-month period.(continuous variable)

W_{it} : number of total machines present in facility i during t^{th} 3-month period (integer variable)

T_{it}: number of machines transported to facility i from its complimentary facility i_c at the beginning of t^{th} 3-month period(integer variable)

L_{it}:number of machines transported from facility i to complimentary facility i_c at the beginning of t^{th} 3-month period.(integer variable)

λ_{it}: cost of bringing a class $(i+i_c)$ machine to facility from its complimentary facility i_c at the beginning of t^{th} 3-month perid.

λ'_{it}: cost of sending a class $(i+i_c)$ machine from facility i to i_c at the beginning of t^{th} 3-month period(This cost is a result of revenue losses which incurs during the transportation of machinery due to not working of machine for at least amount of hours during transportation.

D_{it}:Minimum amount of excavation that must be performed at facility i in order to meet Master Plan requirements in the t^{th} 3-month period.

α_i :Hourly excavation rate (m3/hr) of a class $(i+i_c)$ machine

M_{it}: number of available working hours for a class $(i+i_c)$ machine during t^{th} 3-month period.

$M_{ii_c,t}$: total number of class $(i+i_c)$ machines that may be available at facility i & facility i_c during the t^{th} 3-month period.

$G_{i,i+1}$: Volumetric conversion factor showing excavation volume that must be performed in facility i+1 if 1 m3 of excavation is done in facility i.

$G_{i,i+2}$: Volumetric conversion factor showing excavation volume that must be performed in facility i+2 if 1 m3 of excavation is done in facility i.

Excavation Prices for Facilities(Average)

		ROCK TL/m3	CLAY TL/m3	No.3 Tl/m3
Burdur	:	1300	345	400
Amasya	:	1450	330	380
% Cost (Avg.)	:	60%	80%	75%

Avg. Increase in cost due to Rain

Burdur	:	5%	10%	10%
(in Dec.,Jan.,Feb.,March,Apr. and May)				
Amasya	:	7%	12%	12%
(in Dec.,Jan.,Feb.,March,Apr. and May)				

Following table shows c_{it} :(x1000 TL)

			TIME					
			1	2	3	4	5	6
FACILITY	1	:	75	75	82	82	75	
	2	:	30	30	33	33	30	
	3	:	40	40	45	45	40	
	4	:	85	87	94	94	85	87
	5	:	29	30	33	33	29	30
	6	:	38	40	44	44	38	40

Tranportation Cost : 340 TL/km

note.1: Since tranportation of a TEREX C550 requires tranportation of CAT D9, CAT D8 or CAT D7 Bulldozer (in order to be able to work in rock mines) cost of transporting a TEREX C550 contains also the cost of transporting a Bulldozer.

note.2:Since the firm had 2 accidents in the past during winter while transporting, we assumed to inflate transportation cost by 10% in December, January and February.

Following table shows tranportation costs by class:

for t=1,2,4,5,6	λ_{it}
class(1+4) machines..........	585000.00 TL
class(2+5) machines..........	290000.00 TL
class(3+6) machines..........	295000.00 TL

for t=3	λ_{it}
class(1+4) machines..........	645000.00 TL
class(2+5) machines..........	320000.00 TL
class(3+6) machines..........	325000.00 TL

Following table for $\dot{\lambda}_{it}$ is found depending on the following fact: (x1000 TL)

'Allowable working hours for any class machine in a day is 20 hrs.Remaining 4 hrs is periodic maintenance time'

λ'_{it}

FACILITY		TIME 1	2	3	4	5	6
	1 :	700	700	615	615	700	
	2 :	105	105	55	55	105	
	3 :	187	187	112	112	187	
	4 :	780	740	685	685	780	740
	5 :	100	80	40	40	100	80
	6 :	180	150	95	95	180	150

CONSTRAINTS

Number-2 group of constraints satisfy to meet minimum excavation volumes & to to make excess if feasible at every facility over the planning horizon. These constraints help us to find # of hours required.

Number-3 group of constraints let us not to exceed total available machine working hours which is available thru the number of available machines at every facility during the t^{th} 3-month period. Note that conversion from continuous variables to integer variables are achieved by these constraints.

Number-4 group of constraints which are are called time phased balance constraints let us to decide number of machines at each facility for each 3-month working period with the previous one.Note that T_{it} and L_{it} are linearly dependent so only one of them may be at the basis at the same time or they may together be at zero level.

Number-5 group of constraints let us not to exceed available number of class ($i+i_c$) machines for each class which are working at facility i & i_c during the t^{th} 3-month period.

Number-6 and Number-7 group of constraints let us to perform scaled volumes of excavation at each facility at each period. Forexample realizing 50000 m3 of rock excavation at Burdur at any t and not doing any excavation for clay and filler #3 would be irrational and would end up with a non-scaled excavation. Rock excavation (being the most time consuming and most costly) has been chosen as basis and number-6 & 7 group of constraints are built upon volumetric conversion factors from rock to clay and from rock to filler #3. Also it is assumed that performing 0.5% less or 2% extra can easily be compensated in the following period. That is why coefficients 0.995 and 1.02 appear in these constraints.

SOLUTION & COMMENTS

Solutions are shown at the following pages for the amount of excavation volumes performed vs. required and for the time-phased machine flow between the facilities. Here we would like to make some comments on the overall problem model.

.....I.We took each period as 3-month in length because of 2 reasons.

1.It coincides with the cotrol time unit of DSI

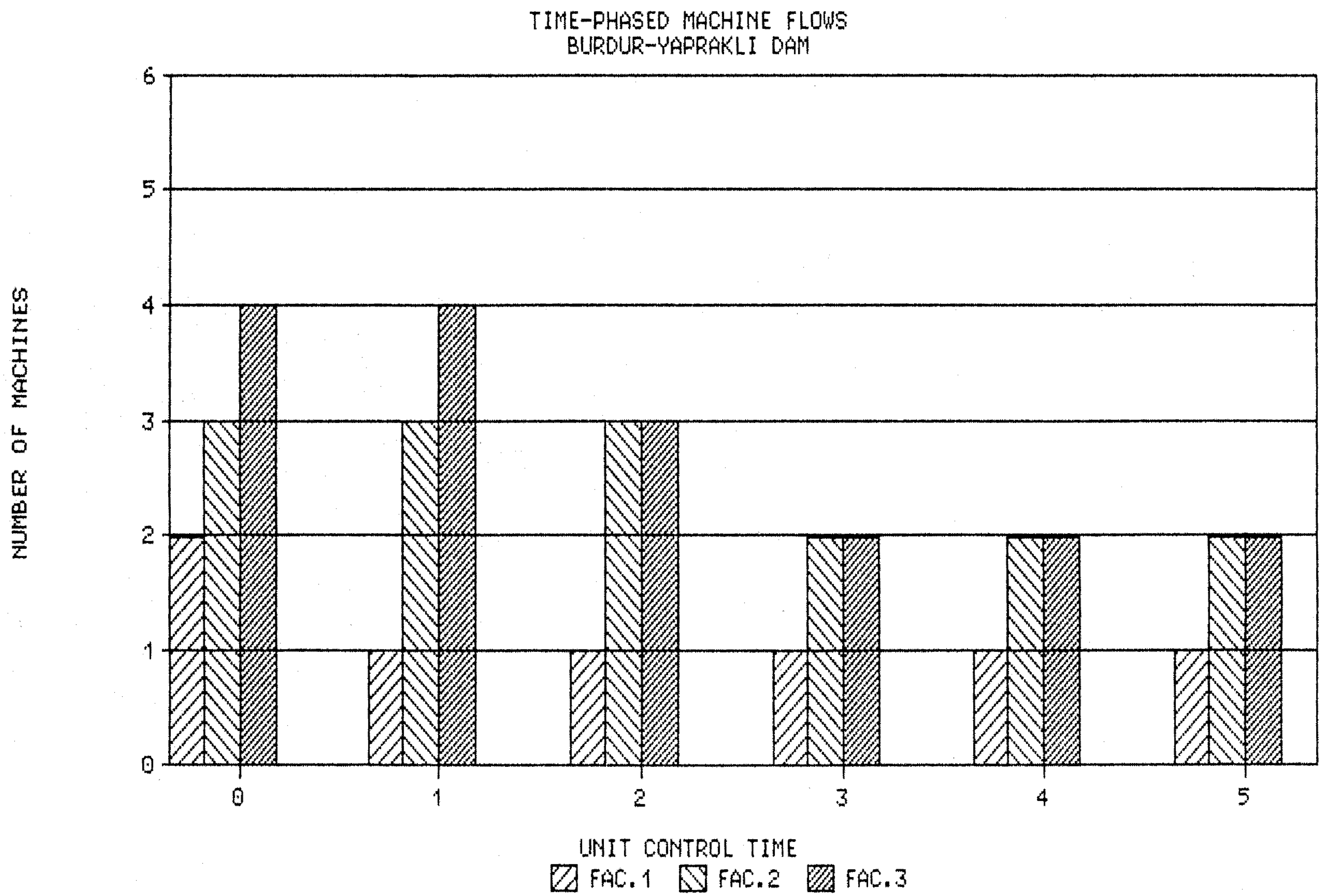
2.Model has currently 99 decision variables and 174 constraints. If the length would be halved (ie 1.5 month) number of decision variables would be doubled and constraints would more than double, ending with a very scarse matrix. But if no solution would have been found, we would shorten each period in order to achieve feasibility.

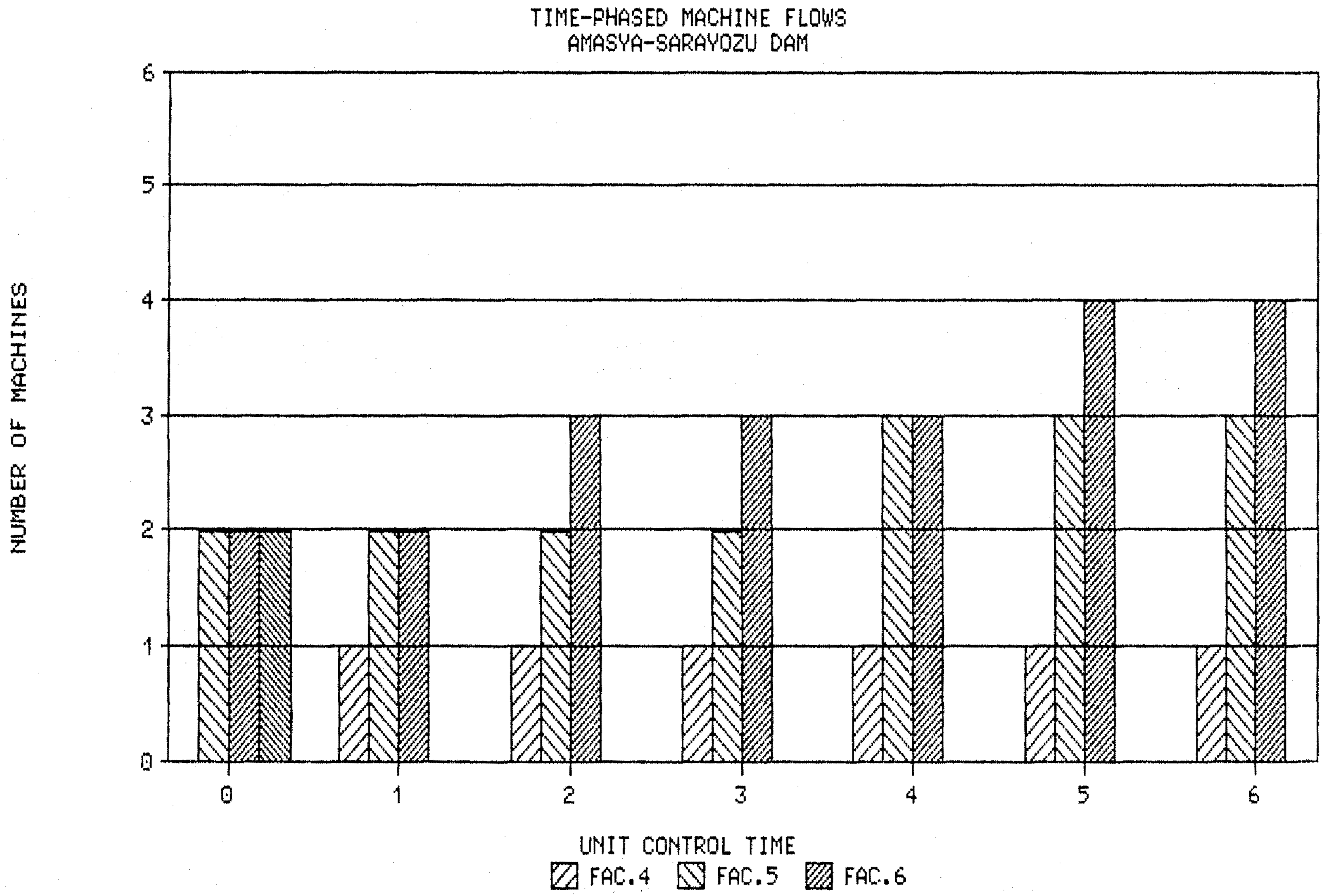
.....II.Since model is a mixed-integer LP it is not possible to make sensitivity analysis. But we played with the number of available machinery (decreasing & increasing the number of any class($i+i_c$) machinery) But the model was so tight that even decraesing the number of class(3+6) and class(2+5) machinery by one unit ended up with infeasible solution. However this might have been relaxed by decreasing the length of unit time but as mentioned above this would increase the problem size. Incraesing the number of class(2+5) and class(3+6) machinery resulted with decreased total cost if we put that machine in Amasya-Sarayozu dam at t=0. Reason is as follows. The firm has just completed another contract in Antalya-Manavgat (which is very close to Burdur -Yaprakli site,aprox.120 km) Upon the completion of that project, machinery available there has been carried to Burdur. Therefore much of the machinery now is in Burdur.(as can be seen in Appendix.H) Also if we inspect machine flow diagrams, we see that flow is from Burdur to Amasya. So locating another machine in Amasya reduces cost. But this cost decrease is very small when compared with the purchase price of that class of machinery,0.10% to 0.15% But this analysis gave us an idea that, upon completion of a project, machinery in that site must be tried to be distributed evenly among the other projects.

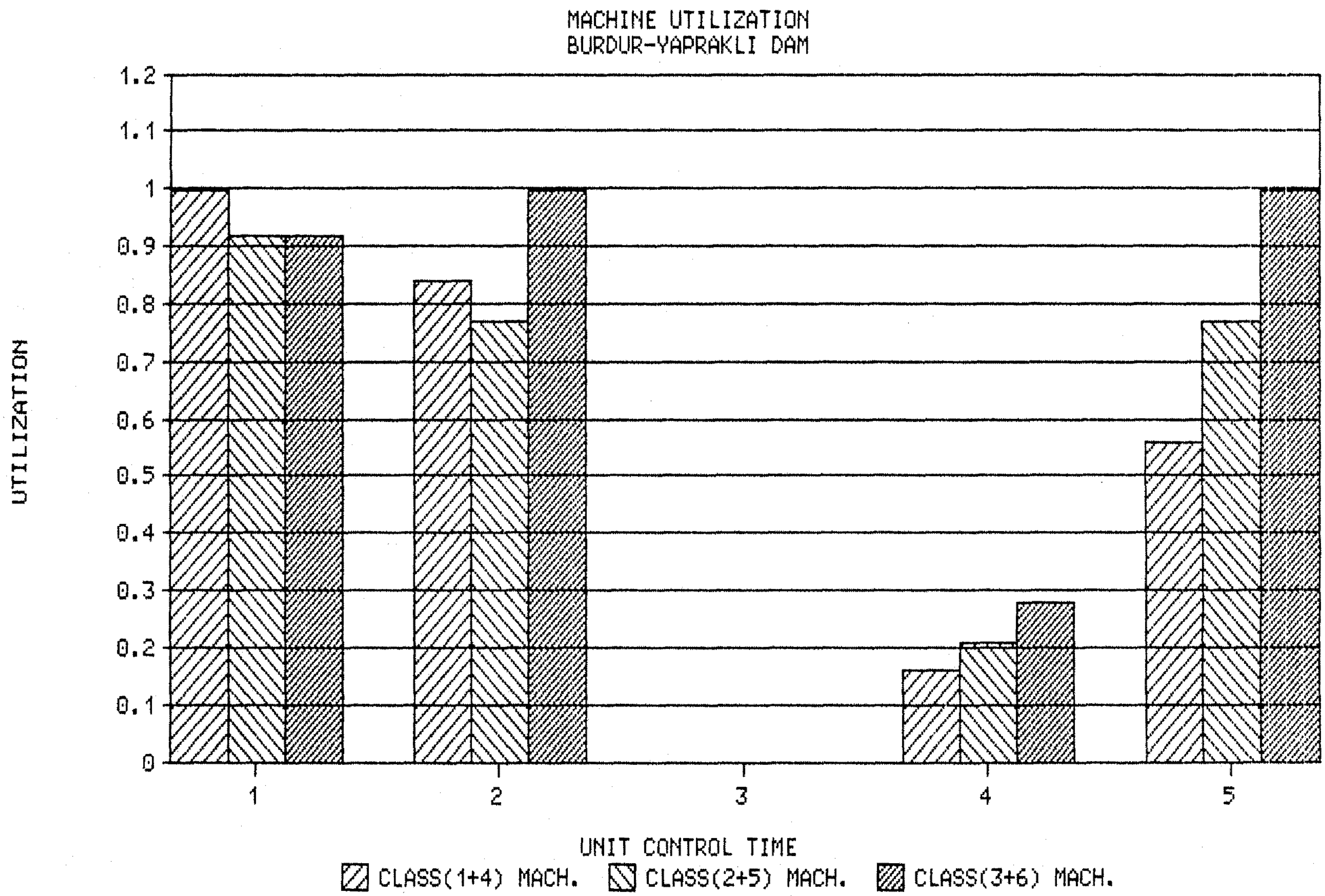
.....III.Reliability of the model is valid if and only if the firm does not undertake another contract(s) before the completion of these projects, which seems unprobable. If company takes another contract, general model of LP (which is given in chapter VI) must be solved in order to come up with new plans. However class (1+4), (2+5) and (3+6) machines are highly demanded after the 12th or 15th month from the beginning of a project. Upto that, other activities are performed in the site.

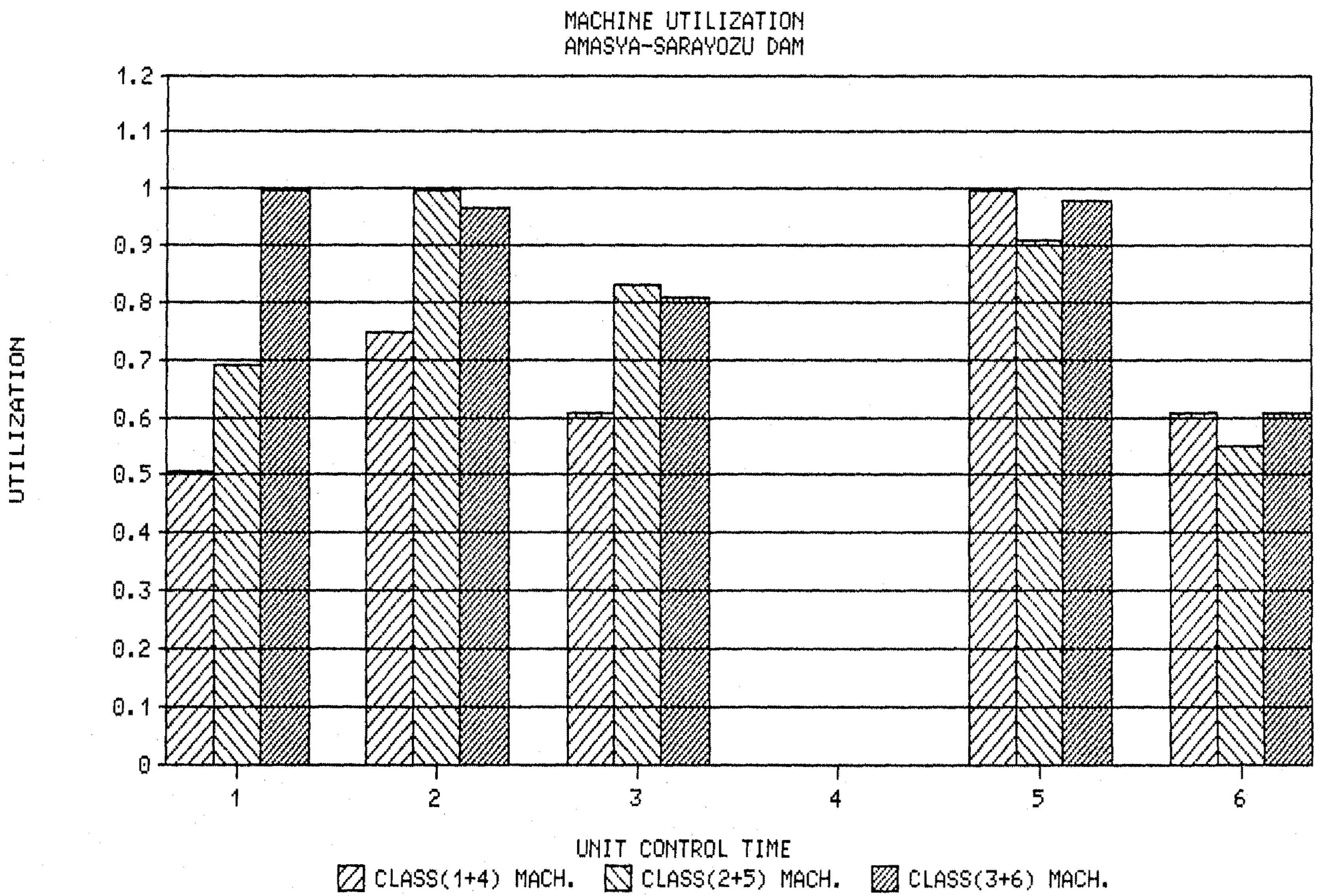
.....IV. An adaptive control model for excavation rate control is of great necessity in each sub-plan. Since probability of breakdown in heavy duty machinery is very high due to working conditions and since the excavation rate for some filler types (especially for clay) is highly dependent on season, amodel which can handle these problems must be utilized in each sub-plan (ie. in each 3-month period) in order to meet original planned excavation rates plus excess demand if any.

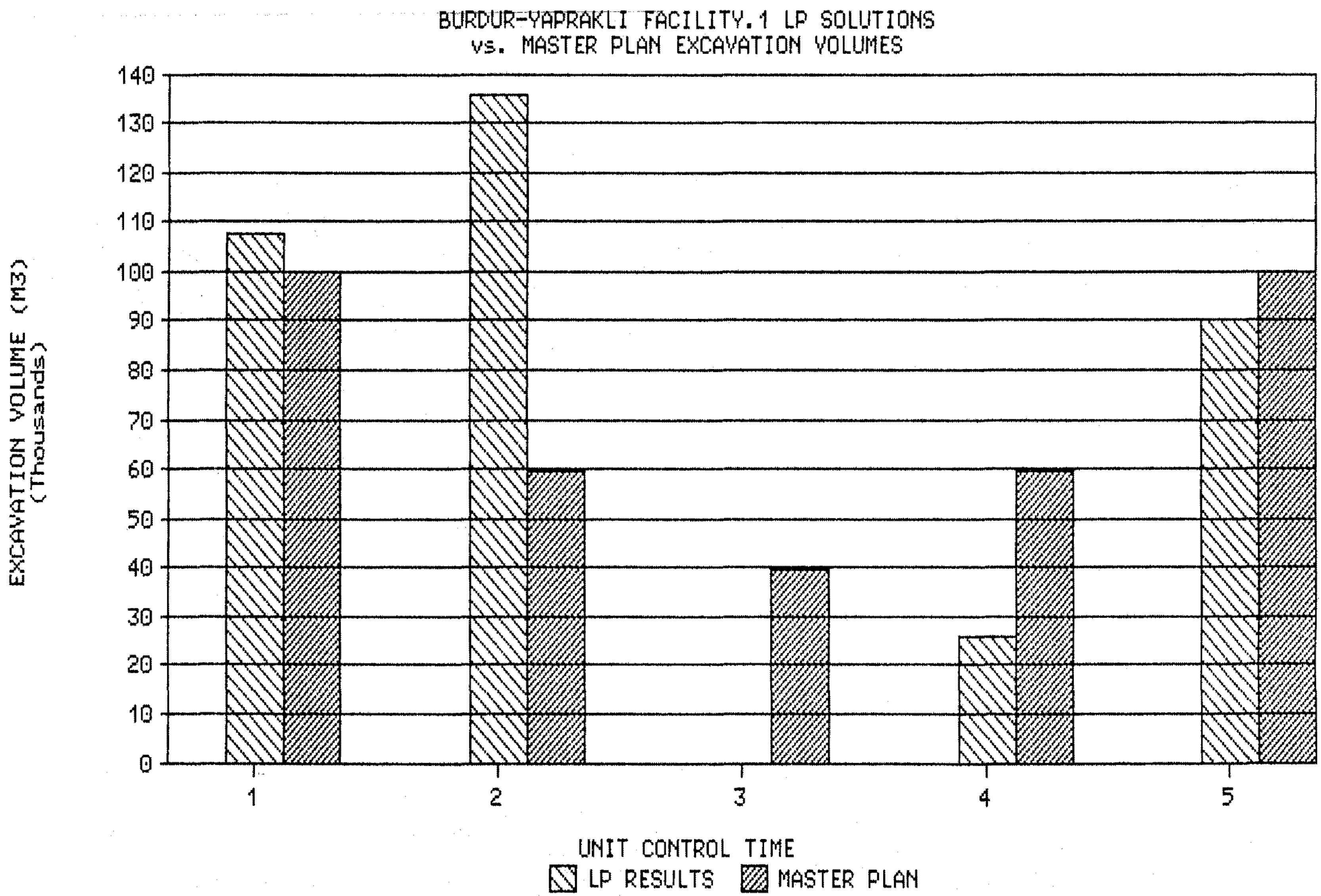
.....V. As explained in chapter.VII , α of adaptive model is of great importance. It may be changed within the period if there are really big gaps between the figures of planned & actualized excavation volumes occur. Enclosed computer disk contains the computer program which performs the methods explained in Chapter.VII for each facility, but uses a constant α for each facility. Since the season factor is the most important in keeping up with a plan, by keeping α high in rainy seasons system may be made more responsive to planning errors, resulting in greater changes in excavation rates. And it may be kept somewhat lower during dry season since excavation rates are much more predictable and realizable in these periods.

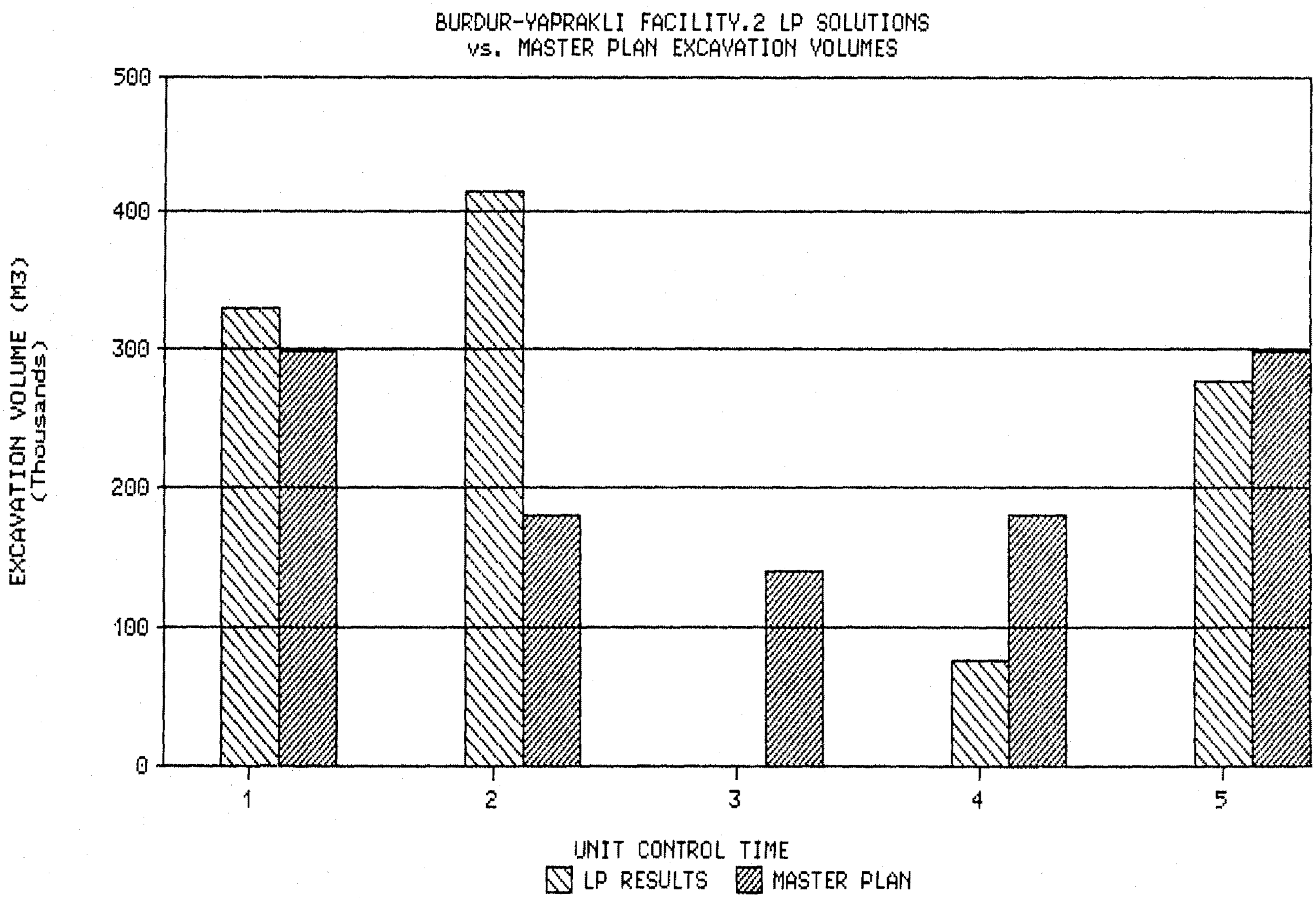

TIME-PHASED MACHINE FLOWS
BURDUR-YAPRAKLI DAM
NUMBER OF MACHINES
0
1
2
3
4
5
6
0
1
2
3
4
5
UNIT CONTROL TIME
FAC.1
FAC.2
FAC.3

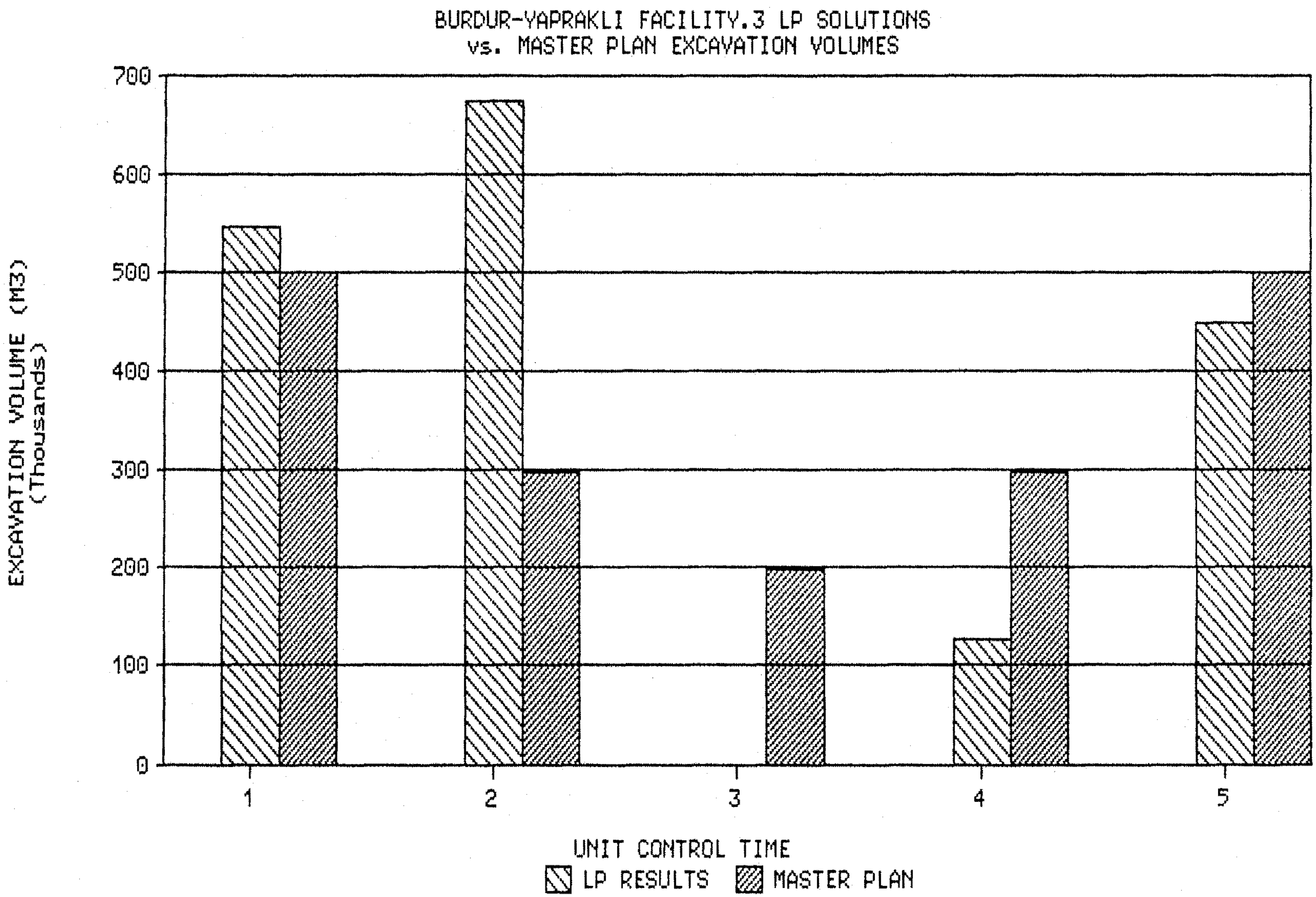

TIME-PHASED MACHINE FLOWS
AMASYA-SARAYOZU DAM
NUMBER OF MACHINES
0
1
2
3
4
5
6
0
1
2
3
4
5
6
UNIT CONTROL TIME
FAC.4
FAC.5
FAC.6

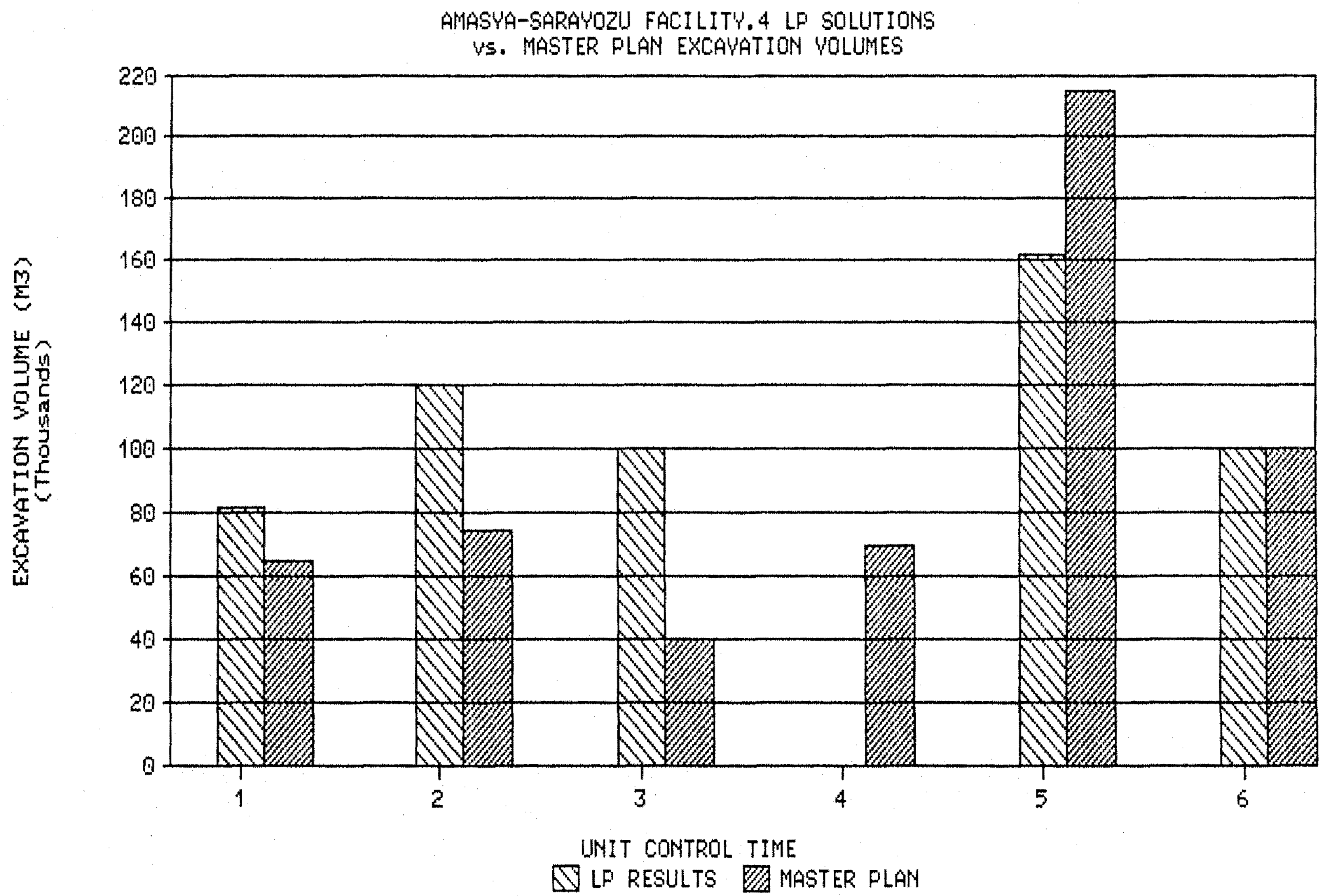

MACHINE UTILIZATION
BURDUR-YAPRAKLI DAM
UTILIZATION
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
UNIT CONTROL TIME
CLASS(1+4) MACH.
CLASS(2+5) MACH.
CLASS(3+6) MACH.


MACHINE UTILIZATION
AMASYA-SARAYOZU DAM
UTILIZATION
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
UNIT CONTROL TIME
CLASS(1+4) MACH.
CLASS(2+5) MACH.
CLASS(3+6) MACH.

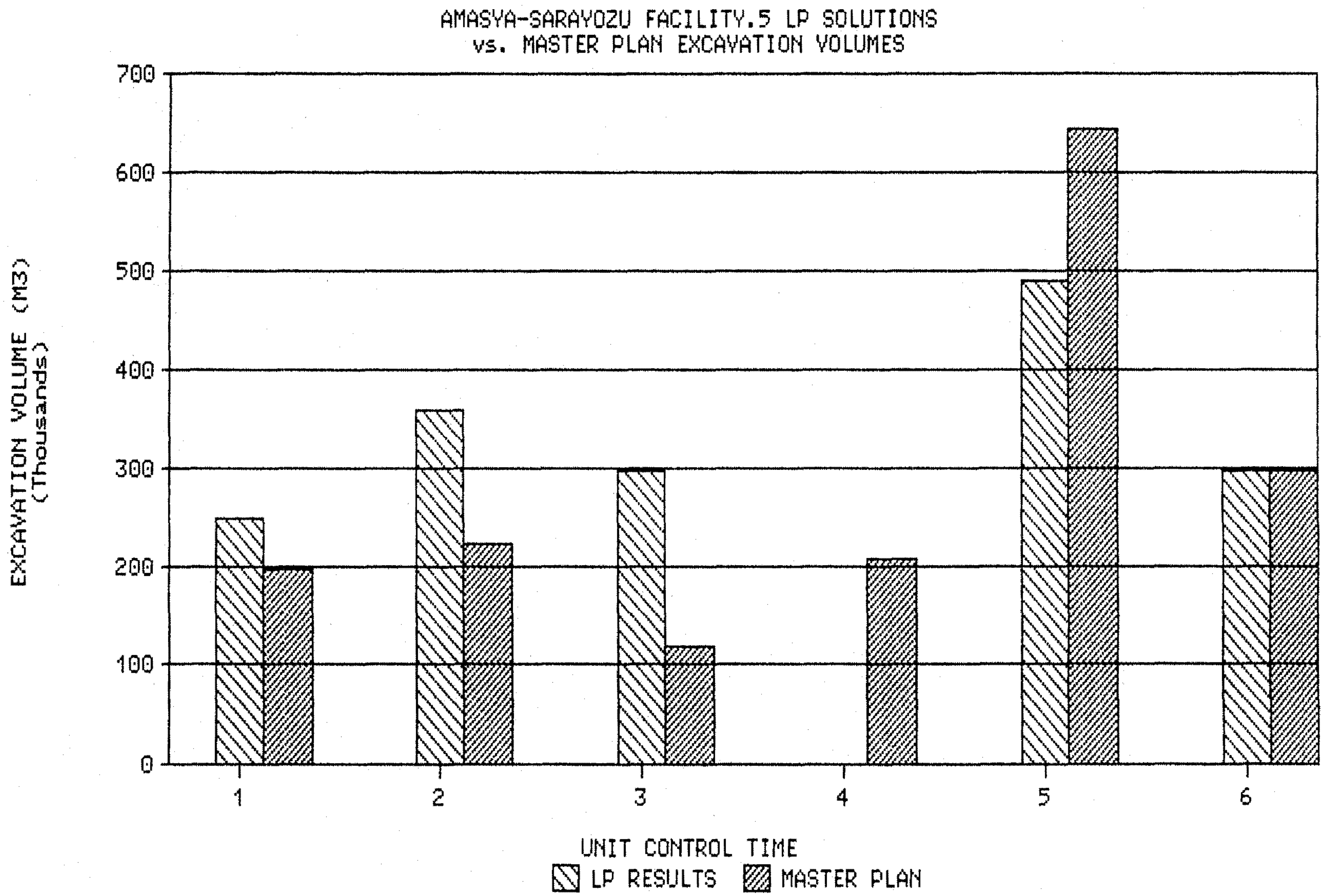

BURDUR-YAPRAKLI FACILITY.1 LP SOLUTIONS
vs. MASTER PLAN EXCAVATION VOLUMES
EXCAVATION VOLUME (M3)
(Thousands)
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
UNIT CONTROL TIME
LP RESULTS
MASTER PLAN


BURDUR-YAPRAKLI FACILITY.2 LP SOLUTIONS
vs. MASTER PLAN EXCAVATION VOLUMES
EXCAVATION VOLUME (M3)
(Thousands)
500
400
300
200
100
0
1
2
3
4
5
UNIT CONTROL TIME
LP RESULTS
MASTER PLAN

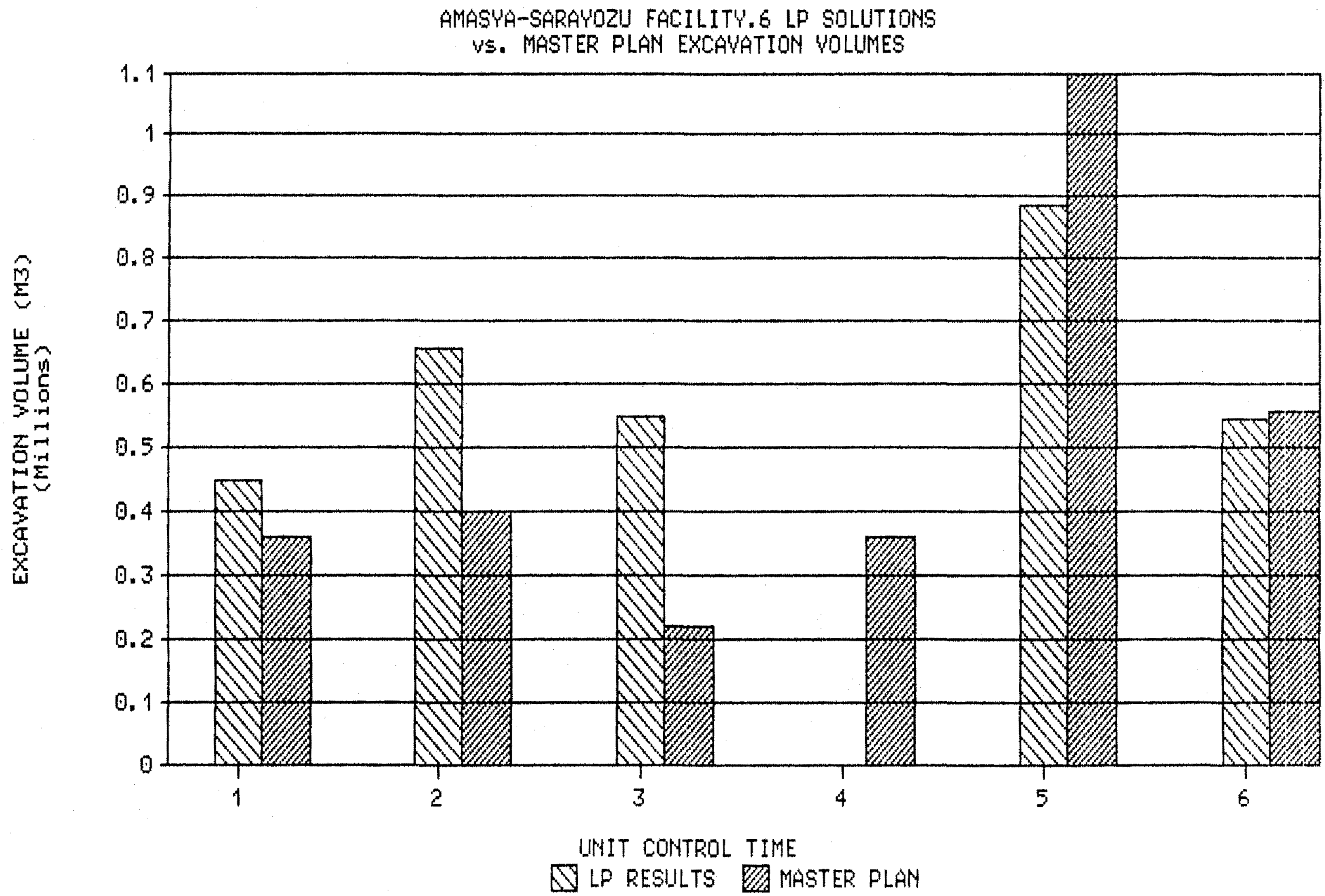

BURDUR-YAPRAKLI FACILITY.3 LP SOLUTIONS
vs. MASTER PLAN EXCAVATION VOLUMES
EXCAVATION VOLUME (M3)
(Thousands)
700
600
500
400
300
200
100
0
1
2
3
4
5
UNIT CONTROL TIME
LP RESULTS
MASTER PLAN



AMASYA-SARAYOZU FACILITY.4 LP SOLUTIONS
vs. MASTER PLAN EXCAVATION VOLUMES
EXCAVATION VOLUME (M3)
(Thousands)
220
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
6
UNIT CONTROL TIME
LP RESULTS
MASTER PLAN



AMASYA-SARAYOZU FACILITY.5 LP SOLUTIONS
vs. MASTER PLAN EXCAVATION VOLUMES
EXCAVATION VOLUME (M3)
(Thousands)
700
600
500
400
300
200
100
0
1
2
3
4
5
6
UNIT CONTROL TIME
LP RESULTS
MASTER PLAN


AMASYA-SARAYOZU FACILITY.6 LP SOLUTIONS
vs. MASTER PLAN EXCAVATION VOLUMES
EXCAVATION VOLUME (M3)
(Millions)
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
UNIT CONTROL TIME
LP RESULTS
MASTER PLAN

FORMATION OF A GENERAL MODEL

In a more general problem , firm may be undertaking more than 2 constructions at the same time,say m. Then there will be m facilities for rock, clay and filler no.3 Model defined at the next page enables to make excavation volume schedules for this case. In addition to the previous one , this model counts for the possibility of bringing amachine to a facility from the remaining other (m-1) facilities.

Variables differing from the previous model are explained below:

W^{+}_{ijt} :number of machines transported to facility i from facility j during the t^{th} 3-month period and λ_{ijt} is the associated tranportation cost.

W^{-}_{ijt} :number of machines transported to facility i from facility j at the beginning of t^{th} 3-month period and $\lambda^{\bullet}_{ijt}$ is the associated cost.

GENERAL MODEL:

min: $$Z = \sum_{t=1}^{T} \sum_{i=1}^{n} \left[c_{it} H_{it} + \sum_{\substack{j=1 \\ i \neq j}}^{m} \left(\lambda_{ijt} W_{ijt}^{+} + \lambda_{ijt}^{\bullet} W_{ijt}^{-} \right) \right]$$

subject to:

$$D_{i1} \leq \alpha_i H_{i1}$$

$$\sum_{t=1}^{k} \alpha_i H_{it} \geq \sum_{t=1}^{k} D_{it} \quad ; \forall i \; \forall k; \quad k = 2, 3, \ldots, T-1$$

$$\sum_{t=1}^{T} \alpha_i H_{it} \geq \sum_{t=1}^{T} D_{it}$$

$$\sum_{t=1}^{T} \alpha_i H_{it} \leq (1.04) \sum_{t=1}^{T} D_{it}$$

$$H_{it} \leq \mu_{it} W_{it} \quad ; \forall i, \forall t$$

$$W_{it} = W_{i,t-1} + \left(\sum_{\substack{j=1 \\ i \neq j}}^{m} W_{ijt}^{+} \right) - \left(\sum_{\substack{j=1 \\ i \neq j}}^{m} W_{ijt}^{-} \right) \quad ; \forall i, \forall t$$

$$\left(\sum_{\substack{j=1 \\ i \neq j}}^{m} W_{ijt} \right) \leq M_{ii,t} \quad \forall i, t$$

$$(0.995)(\alpha_{i+1}) H_{i+1,t} \leq (G_{i,i+1})(\alpha_i) H_{it} \leq (1.02)(\alpha_{i+1}) H_{i+1,t} \quad ; \forall i, \forall t$$

$$(0.995)(\alpha_{i+2}) H_{i+2,t} \leq (G_{i,i+2})(\alpha_i) H_{it} \leq (1.02)(\alpha_{i+2}) H_{i+2,t} \quad ; \forall i, \forall t$$

$$H_{it} \geq 0$$

$$W_{it}, W_{ijt}^{+}, W_{ijt}^{-} = 0, 1, 2, \ldots$$

FORMATION OF AN ADAPTIVE MODEL FOR

EXCAVATION VOLUME CONTROL

i. Model

ii. Computer Program Sample Outputs

iii. Stochastic Analysis of Model & Comments

Now we have solved the mixed integer LP and come up with the excavation schedule over the planning horizon. This plan gives for each facility an excavation rate X^* for each period in the interval. Assume that the plan is considered a 'preliminary' one, which at time zero establishes targets for excavation rates. However as time passes we may wish to adjust excavation rates from this originally established levels, because actual demand and actual excavation volume differ from planned at time zero. Adaptive model which is developed here works in sub-plans. In other words an adaptive model is developed for the control of excavation rate in any t^{th} 3-month period. Each 3-month period is subdivided into 30 time units each being 3-day in length.

Lets say that total excavationvolume planned for Burdur-Yaprakli clay facilities in 2nd 3-month period is 100.000 m3. Then planned excavation volume for every 3-day period in 2nd 3-month period will be (100.000/30) m3.

Define:

X_t^*:excavation rate originally planned for a 3-day period under a preliminary plan made at time zero. Will change due to changing environmental conditions as time passes.

X_t:actual excavation rate realized during t

I_t:Actual inventory level at the end of t

T :number of periods in the sub-plan (ie.30)

τ:order lead time

Order lead time is the number of periods between a decision to change the excavation rate & the time the change becomes effective. For our model this will be equal to one time unit (ie.3 days). But the adaptive model will be developed for more general form therefore we leave it as τ

After now on due to increasing mathematical symbols, we will continue with hand writing.(Sorry for the inconvenience.)

At the end of period t, we must specify $X^*_{t+\tau+1}$, the excavation rate in $t+\tau+1$ (which is assumed to be available for use in period $t+\tau+1$) We already have decided on excavation becoming available in periods $t+1, t+2, \ldots, t+\tau$ at values of $X^*_{t+1}, X^*_{t+2}, \ldots X^*_{t+\tau}$ which might differ from the actualized amounts X_{t+1}, $X_{t+2}, \ldots, X_{t+\tau}$.

Define the correction of excavation in period t as:

$$\Delta_t = X_t - X^*_t$$

if D_t is the actual demand in period t then;

$$I_t = I_{t-1} + X_t - D_t$$

Our adaptive control rule is to schedule excavation rate $X^*_{t+\tau+1 \atop new}$ equal to a fraction α of the inventory discrepancy projected at the end of period $t+\tau$. That is:

$$\left.\begin{aligned} X^*_{t+\tau+1 \atop new} &= X^*_{t+\tau+1 \atop old} + \alpha\left[-I_t - \sum_{j=1}^{\tau}\left(X_{t+j} - X^*_{t+j}\right)\right] \\ &= X^*_{t+\tau+1 \atop old} + \alpha\left[-I_t - \sum_{j=1}^{\tau}\Delta_{t+j}\right] \quad \text{where } 0 \le \alpha \le 1 \end{aligned}\right] \quad \ldots\ldots (A)$$

STOCHASTIC ANALYSIS.

In this part, we will analyze the control rule defined above for the case of stochastic demand having a stationary probability distribution.

For the above generated rule, inventory level at the end of period $t+\tau$ can be represented as a linear combination of planning errors for the periods $1, 2, 3, \ldots, t+\tau$.

$$I_{t+\tau} = I_t + \sum_{j=1}^{\tau} \left(X_{t+j} - X^*_{t+j} \right) - \sum_{j=1}^{\tau} \left(D_{t+j} - X^*_{t+j} \right)$$

$$= I_t + \sum_{j=1}^{\tau} \Delta_{t+j} - \sum_{j=1}^{\tau} e_{t+j}$$

Define: $E_{t+\tau}$ as the excess inventory at time $t+\tau$ projected at time t and $e_t = D_t - X^*_t$ is the planning error for period t. Then:

$$I_{t+\tau} = E_{t+\tau} - \sum_{j=1}^{\tau} e_{t+j} \quad \ldots\ldots\ldots\ldots\ldots \text{(B)}$$

and since;

$$\Delta_{t+\tau+1} = \alpha \left[- I_t - \sum_{j=1}^{\tau} \Delta_{t+j} \right] \ldots\ldots\ldots\ldots\ldots \text{(C)}$$

$$E_{t+\tau} = I_t + \sum_{j=1}^{\tau} \Delta_{t+j} \quad \text{and} \quad \Delta_{t+\tau+1} = -\alpha \cdot E_{t+\tau}$$

By resurgive relation we can show that;

$$E_{t+\tau} = (1-\alpha)\, E_{t+\tau-1} - e_t$$

Substituting successively for $E_{t+\tau-1}$, $E_{t+\tau-2}$, ... and so on we obtain

$$E_{t+\tau} = (1-\alpha)^{t+\tau} E_0 - \sum_{n=0}^{t+\tau} (1-\alpha)^n e_{t-n}$$

We define $e_t = 0$ for $t \leq 0$ and since there is no initial inventory discrepancy; E_0 is zero. Then:

$$E_{t+\tau} = -\sum_{n=0}^{t-1} (1-\alpha)^n e_{t-n}$$

If we substitute this equation in (B)

$$I_{t+\tau} = -\sum_{n=0}^{t-1} (1-\alpha)^n * e_{t-n} - \sum_{n=1}^{t} e_{t+n}$$

Here we make the following assumption where the reliability of the model mainly depends on the validity of this assumption.

"Demand process and the planning procedure result in forecast errors that are independent, identically distributed Random Variables with mean zero and variance σ_e^2."

Then inventory discrepancy, $\delta_{t+\tau}$, will be a Random Variable having zero mean and variance:

$$\sigma_\delta^2(t) = \sum_{n=0}^{t-1} (1-\alpha)^{2n} \sigma_e^2 + \tau \cdot \sigma_e^2$$

$$\sigma_\delta^2(t) = \left[\frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2} + 1 \right] \sigma_e^2$$

So $\delta(t)$ (inventory discrepancy) will have a variable variance by time. But as t gets larger; then:

$$\lim_{t\to\infty} \sigma_\delta^2(t) = \lim_{t\to\infty} \left(\frac{1-(1-\alpha)^{2t}}{2\alpha-\alpha^2} + 1 \right) \sigma_e^2 \cong \left(\frac{1+2\alpha-\alpha^2}{2\alpha-\alpha^2} \right) \sigma_e^2$$

If we can find σ_δ, it may be useful in establishing safety stock levels (especially during heavy rainy season).

Deviation of planned excavation from actual is;

$$\Delta_{t+\tau+1} = -\alpha \cdot E_{t+\tau}$$

$$= \alpha \sum_{n=0}^{t-1} (1-\alpha)^n e_{t-n}$$

Then we can find the variance of Random Variable $\Delta_{t+\tau+1}$ which has a zero mean:

$$\sigma_\Delta^2(t) = \alpha^2 \left[\frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2} \right] \sigma_e^2$$

We again see that σ_Δ^2 changes by time. But as t gets larger;

$$\lim_{t\to\infty} \sigma_\Delta(t) = \lim_{t\to\infty} \sqrt{\alpha^2 \frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2} \sigma_e^2} \cong \sqrt{\frac{\alpha}{2-\alpha}}\ \sigma_e$$

The change in excavation rate from period to period is:

$$X_t^* - X_{t-1}^* = (X_t^* - X_{t-1}^*) + (\Delta_t - \Delta_{t-1})$$

(again note that "=" is not an "equal" sign.)

Above formula can be thought of as planned change (deterministic) plus an adjustment (stochastic) to the planned change.

$$\Delta_t = -\alpha . E_{t-1} = \alpha \sum_{n=0}^{t-3} (1-\alpha)^n e_{t-n} \; ; \quad \sigma^2_{\Delta_t} = \alpha^2 \left[\frac{1-(1-\alpha)^{2(t-2)}}{1-(1-\alpha)^2} \right] \sigma_e^2$$

$$\Delta_{t-1} = -\alpha E_{t-2} = \alpha \sum_{n=0}^{t-4} (1-\alpha)^n e_{t-n} \; ; \quad \sigma^2_{\Delta_{t-1}} = \alpha^2 . \left[\frac{1-(1-\alpha)^{2(t-3)}}{1-(1-\alpha)^2} \right] \sigma_e^2$$

Then:

$$\sigma^2_{(\Delta_t - \Delta_{t-1})} = \alpha^2 . \sigma_e^2 \left[\frac{1-(1-\alpha)^{2(t-2)} + 1-(1-\alpha)^{2(t-3)}}{1-(1-\alpha)^2} \right]$$

(as t gets larger)

$$\sigma^2_{(\Delta_t - \Delta_{t-1})} = \frac{2 . \alpha^2}{1-(1-\alpha)^2} . \sigma_e^2 = 2 . \frac{\alpha . \sigma_e^2}{2-\alpha}$$

Note that $\frac{\alpha}{2-\alpha} . \sigma_e^2 = \sigma_\Delta^2$ then:

$$\sigma_{(\Delta_t - \Delta_{t-1})} = \sigma_\Delta \sqrt{2}$$

Then the change in excavation rate $(X_t^* - X_{t-1}^*)_{new}$ has mean $(X_t^* - X_{t-1}^*)_{old}$ and standard deviation $\sigma_\Delta \sqrt{2}$ so that

$$\sigma_{(X_t^* - X_{t-1}^*)} \cong \sqrt{\frac{2\alpha}{2-\alpha}} \cdot \sigma_e$$

To operate this rule we must decide a value for α. From the analysis, we can deduct that, as α increases standard deviation of inventory fluctuations about planned rates decreases but standard deviation of excavation rate fluctuations about previous planned levels increases. Larger values of α seems to make the system more responsive to planning errors which will result in greater changes in excavation rates but with lower inventory requirements. And small values of α will have the opposite effect.

Enclosed output samples of the program are carried 8 time units only for Burdur-Yaprakli Rock Facilities.

In order to activate computer program, type EAS upon DOS prompt.

A>EAS (RETURN)

note:Attached disk contains Turbo.Pascal vs.3 compiler, program (Adaptive Control Model) source code and program binary code. Binary code is enclosed in order not to bother the user with compilation and Turbo.Pascal access.