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Title: Assembly Plant Operation: Labor and Inventory Optimization

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Abstract: This report studies inventory levels and the allocation of hours between temporary employees and permanent employee overtime. It uses a Linear Programming Model to optimize the inventory levels and work allocation of an assembly plant. Inventory levels are optimized on the basis of prior demand quantities, cost of air freight, carrying cost of fixed inventory, and space constraints.

ASSEMBLY PLANT OPERATION

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Good project, Good analysis

Assembly Plant Operation:
Labor and Inventory Optimization

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Inventory Strategy Model - An Assembly Plant Profit Maximization

Table of Contents

Executive Summary

This paper describes an optimization (using linear programming techniques) of the inventory levels and work allocation of an assembly plant. The plant studied is a quick turnaround facility located on the west coast. Its purpose is to provide shipment of product in 2 weeks or less. It stocks a fixed level of inventory, each part having its own fixed level each month. The plant operates using Just in Time inventory control. Since the plant is a service center, the demand for parts is not predictable since parts are used as failures are encountered or upgrades are made. Parts are allocated varying amounts of shelf space. Parts vary in weight, carrying cost and material cost. Due to weight differences, the cost of air freighting parts also varies. If the plant runs short of parts during a month, the additional parts are then air freighted in from the main plant on the east coast. Otherwise, parts are reordered as they are used.

The plant employs four permanent employees in the assembly area. The plant manager prefers to hire temporary employees rather than having the permanent employees work overtime since the employees have stated that preference.

This project looks at inventory levels and allocation of hours between temporary employees and permanent employee overtime. Inventory levels are optimized on a basis of prior demand quantities, cost of air freight, carrying cost of fixed inventory, and space constraints. A sensitivity analysis concludes the study resulting in information for use by the plant manager in making future decisions concerning inventory levels and allocation of work hours. The optimal solution supports the plant manager's practice of hiring temporaries to avoid overtime by permanent employees.

The study shows that the inventory levels set by the corporation are suboptimal. Increasing certain inventory levels will result in increased profits. It should be noted that this result will not necessarily hold true for similar facilities located nearer the main plant. The cost of air freight from the east coast to the west coast is an overriding factor in this optimization.

Several rules of thumb are developed here to aid the plant manager in making future inventory decisions.

Introduction

The manufacturing division of a large corporation has six satellite assembly plants located around the United States. The purpose of these satellite plants is to provide quick assembly - quick ship service for standard products. All major components inventoried at these quick ship facilities are ordered from the main manufacturing plant and warehouse located on the east coast.

Several years ago, a just-in-time inventory philosophy was implemented at the main manufacturing plant and the six satellite facilities. Inventory at the satellite facilities was pared down to increase the corporation's overall profitability.

The cycle time for replacing stock at the satellite facilities on the west coast is approximately four weeks. The main facility requires one to two weeks to manufacture the item and regular truck freight takes two to three weeks in addition.

Pressure to reduce inventory has led to an increase in the amount of material that must be brought in more quickly than regular freight to meet quick ship requirements. In addition to regular freight, the satellite facility used a special delivery service and air freight. The special delivery service promised one week deliveries for a cost of approximately three times regular freight. This use of the service was recently discontinued because they rarely met the one week commitment.

The cost of air freight is approximately five times the cost of regular freight. Normally, this cost cannot be passed on to the customer due to the competitive nature of the business. Thus, the amount of air freight required has a significant effect on the profitability of a satellite facility. It should be noted that these costs reflect only the costs for west coast facilities. Other facilities located near the main manufacturing facility would have entirely different costs and times associated with freight transportation.

The main purpose of this study was to explore the effect of inventory constraints and freight costs on the satellite facility's profitability. As a side issue, the plant manager requested that we explore his policy of minimizing overtime for regular employees by using temporary employees before resorting to regular employee overtime. The facility is limited by the corporation to four full-time permanent shop employees. We were able to demonstrate a relationship between sales billed and labor hours required. Using the cost of regular employees (wages and benefits) and the cost of temporary employees, we were able to show the optimum usage of employees' time.

Scope of the Problem

The problem resolved into two main issues. The first was the conflict between inventory constraints and transportation costs. The second was the cost of permanent employee overtime hours versus the cost of skilled temporary employee regular time hours. These issues will be discussed in general below and in detail in the problem formulation section.

Initially, only two inventory constraints were proposed by the plant manager. The corporation required that sales over two months must be greater than or equal to the total dollars tied up in inventory. The second constraint identified the amount of warehouse shelf space available for inventory use.

The facility stocks over 350 items. In our discussions with the purchasing agent we identified several items that were candidates for additional inventory. The JIT inventory system made it fairly easy to identify which items required air freight most frequently.

In our initial run of the model, we limited the inventory which could be varied to two of the items with the highest usage. The results of the model indicated that the plant manager would be optimizing profit if he increased his inventory to the highest usage. In other words, the cost of air freight completely dominated the problem. The inventory constraints were not active in the solution.

Discussions with the plant manager revealed that, though the solution was valid, it was not reasonable within the corporate climate. Two more constraints were proposed. The first constraint imposed a carrying cost on inventory. This cost was not specifically subtracted from the manager's profitability statement. However, the plant manager suggested that it be included to enhance his chance of securing approval for increased inventory since carrying cost is a real cost to the corporation.

Good interaction w/ the decision maker

The second constraint was the maximum inventory he felt he could successfully add given the political constraints of the corporate climate. This second constraint was an obvious candidate for study in the sensitivity analysis. The cost of ignoring the corporate political climate can be quite high as any out-of-work middle manager can testify. The cost to corporate profitability is rarely so easily identified. No attempt was made to quantify this factor in the model. Rather, the approach was to observe the change in profitability that could be attributed to arbitrarily set inventory levels.

With these four constraints, the inventory problem was felt to be well defined. The model was run with several different inventory candidates. Our intention in confining the model to looking at two types of inventory candidates at a time was to give the plant manager a clear understanding of how the various constraints interacted.

For example, a low cost item is only limited by air freight cost and total inventory cost whereas a high cost item's carrying cost can become more significant than the total inventory cost.

A detailed discussion of the results is included in the conclusion of this paper. It was felt that leaving the manager with a working knowledge of the constraints and trade-offs under which he operated was more useful than a single solution.

The labor problem proved to be trivial due to the large difference in cost of overtime and cost of temporary labor. Loss of regular time productivity to train the temporaries was not considered significant enough to be included.

Problem Formulation

Objective Function Formulation

In developing the model for this plant, we needed to identify an objective function. Talking with the plant manager, we found that the performance of the plant was judged by the plant's profit level. Profit level is calculated by looking at the sales volume (in $) and subtracting out both fixed costs and variable costs.

Profit = Sales - Variable costs - Fixed costs

All of the data for the model (sales, item demands, costs) are actual figures from 1988. Our approach is to demonstrate the effects that changes in inventory and staffing strategy have on plant profitability. (Note that sales refers to actual monthly sales for the plant.)

Fixed costs include the cost of permanent employees (wages and benefits) and the cost of the facilities (leases, maintenance, utilities, custodial). This value can be characterized by a fixed dollar amount

Variable costs include the cost of fixed inventory (material cost, carrying cost, freight), air freight charges for materials in excess of the fixed inventory, cost of the excess materials, cost of temporary labor, and cost of permanent employee overtime. Our initial study looked at two parts. The air freight charge on each part was the same but the material cost differed.

In formulating the objective function the following values were used. These values were obtained from the monthly reports of the company.

regular freight charge (same for both parts) = $7.73 per part
air freight charge (same for both parts) = $40 per part
material cost for part a = $210 per part
material cost for part b = $220 per part
hourly cost of temporary labor = $8 per hour
hourly cost of permanent employee overtime = $22 per hour

Carrying costs for the inventory were added to our model after the initial formulation. The carrying costs are based on the cost of money, calculated at 12% compounded over one year. The monthly rate was calculated then we multiplied it by the cost of one part to determine the carrying cost per part for part a and part b. The value of the remaining fixed inventory was also multiplied by the monthly cost of money to determine the carrying cost of parts other than parts a and b. This yielded the following values:

carrying cost for part a = $8.60 per part for a four month period
carrying cost for part b = $9.00 per part for a four month period
carrying cost of remaining inventory = $8520.60 for a four month period

The following formula was then developed for calculation of monthly profit:

Profit = Sales - 7.73 x (parts drawn from inventory, both a and b)
- 40 x (parts air-freighted, a and b)
- 210 x (total number of part a used)
- 220 x (total number of part b used)
- 8 x (temporary hours)
- 22 x (permanent employee overtime hours)
- 8.6 x (inventory level for part a)
- 9.0 x (inventory level for part b)
- 8520.6

The objective function sums the profit for four consecutive months (excluding fixed costs) and attempts to optimize that value. Fixed costs as defined previously and corporate overhead are omitted since they are constant and therefore have no effect on the strategy indicated by the maximization of profit.

Model Constraints

Having developed the objective function, the next step was to determine the constraints associated with management of costs at the plant. These constraints were identified during interviews with the plant manager.

We first discussed constraints associated with the hours worked at the plant. The plant employs four permanent workers for assembly of units. The plant policy is to attempt to avoid having these employees work overtime. This policy was not mandated by the company but was developed due to the employees expressing desire to minimize their overtime hours. Since regular employee overtime cost is considerably higher than temporary hourly costs, a constraint was not needed for this. The model should use the maximum number of temporary hours before allocating hours to permanent employee overtime base simply on the fact that costs are being minimized. We next looked at temporary help. Due to space constraints and assembly fixture constraints, a maximum of 3 temporary workers could be added to the regular assembly staff. A constraint was then added to our model to limit the number of temporary hours available. The value of 520 hours was used based on a 52 week year, three temporary employees and twelve equal division resulting in a monthly maximum.

Maximum temporary hours = (52 x 40 x 3) / 12

Next, we need to determine the number of hours required each month based on sales. In order to determine this value, we ran a linear regression on hours vs sales over a twelve month period for which we had records (see Figure 1). There were three months which did not map well with the remaining nine months. When these were omitted from the linear regression, the X coefficient was found to be 0.004079 hours/sales with an R squared value of 0.92.

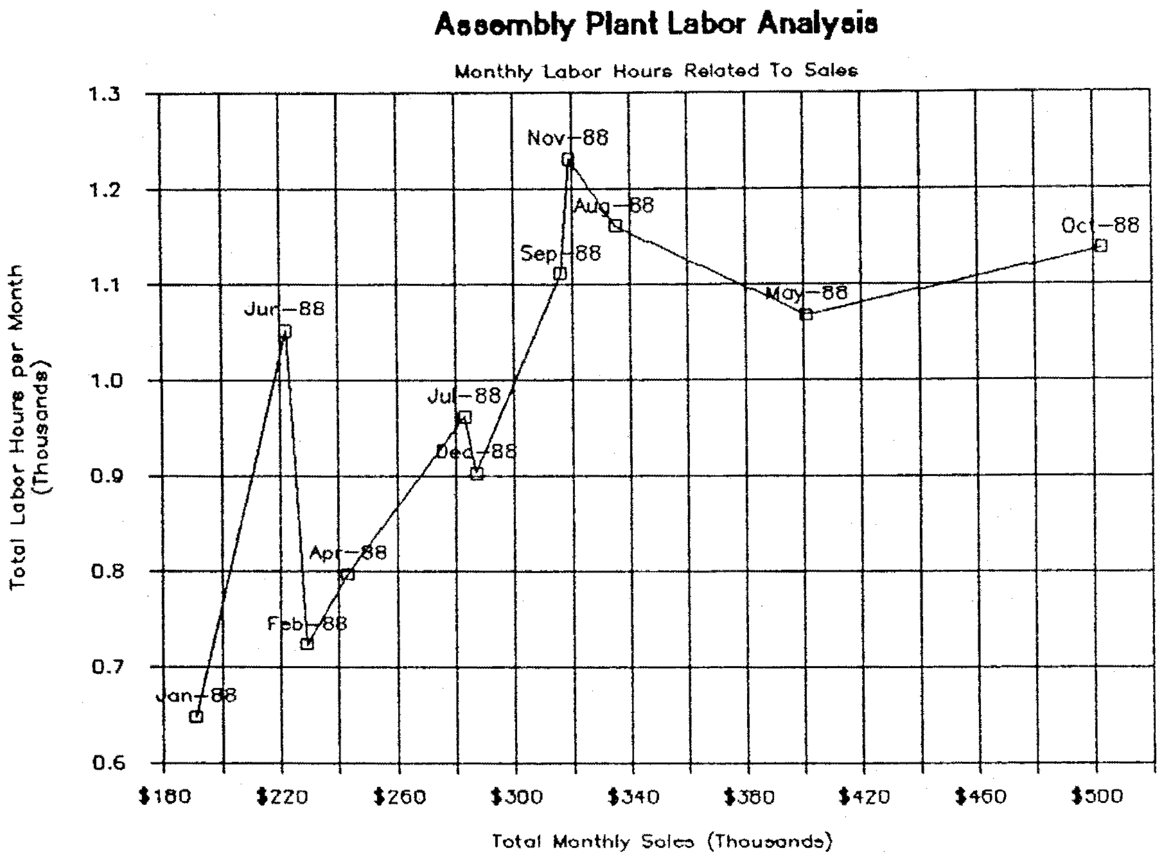

Figure 1

Looking at the three points which were omitted, two months were consecutive months and when the sales and number of hours are averaged between these two months, they actually then conformed to the equation of the line computed using the nine remaining points in the regression. The number of hours required for a month was then calculated as follows:

$$\text{hours required} = 0.00408 \times \text{sales for the month}$$

Since permanent employees work 40 hours per week regardless of demand, we needed to add a slack variable (idle time) to the constraint for number of labor hours per month. The fixed number of hours per month for four permanent employees was calculated as follows:

$$\text{Monthly permanent employee hours} = (52 \times 40 \times 4) / 12 = 693.3$$

The number of labor hours per month is then the number of hours needed due to sales demand adjusted by either idle hours (N1 values) or supplemented by temporary hours (T) and overtime hours (W).

$$0.00408 \times \text{sales} = 693.3 - N1 + T + W \quad \text{or} \quad 0.00408 \times \text{sales} + N1 - T - W = 693.3$$

In this equation, either N1 is zero and T and W may be positive; or T and W are zero and N1 may then be positive.

We next needed constraints to describe the monthly demand for each part and the sources of those parts. The plant maintains a fixed inventory level for each part stocked. Depending on demand for a month, a portion of that inventory could be used resulting in parts remaining in inventory after the demand is met, the entire inventory could be used to meet demand, or demand could exceed the inventory level resulting in the need to air freight additional parts to meet the demand. The following equation was then formulated:

$$\text{Demand} = \text{parts from inventory} + \text{air freighted parts} - \text{surplus parts}$$

$$Xa_i = PIa_i + Na_i - Pa_i \qquad Xb_i = PIb_i + Nb_i - Pb_i$$

where:

Xa_i, Xb_i = demand for part a or b in month i

PIa_i, PIb_i = inventory level for part a or b in month i

Na_i, Nb_i = quantity air freighted in for part a or b in month i

Pa_i, Pb_i = quantity remaining in inventory for part a or b in month i after demand is met

In order to insure that the draw from inventory did not exceed the fixed inventory level, an additional constraint was added to limit the inventory level plus surplus so that it did not exceed the maximum level.

$$PIa_i + Pa_i = Ia \qquad PIb_i + Pb_i = Ib$$

where: Ia, Ib are the decision variables used to determine the fixed inventory ceiling for part a or b which is held constant each month.

While interviewing the plant manager, we also found that the plant had a policy which limited the inventory levels by demanding that the value of the inventory (in dollars) must not exceed the sum of the sales for a two month period; in other words, the value of the inventory must be turned every two months. The value of the inventory is determined by summing the number of parts times the per part value of each inventory item. Since this model examines only two high demand parts, the value of the remaining inventory is a fixed value. The inventory turn constraint is then modeled as follows:

sales for month i + sales for month (i+1) >= value of fixed inventory
+ (quantity of part a) x (value of part a)
+ (quantity of part b) x (value of part b)

In addition, a constraint was needed since there are space limitations for storing inventory at the plant studied. Both parts had the same space requirements. Parts are allocated pallet shelves in the warehouse area and the plant manager provided us with data indicating the number of parts which could be stored on a pallet shelf, the number of shelves currently totalized for each part and the number of unused shelves available for expansion in the warehouse area. From these numbers, we determined that a total of 144 parts could be stored in the warehouse area. The mix of parts was left for the model to determine. The constraint is then stated as:

$$Ia + Ib <= 144$$

The only remaining constraint equations were then equations which set the values for sales for each of four months and demands for each part for each of four months. These values appear in the Monthly Sales Data and Monthly Demand tables attached. The data came from monthly reports given to us by the plant manager. Those reports are not duplicated here due to the need for confidentiality. Only the data values used in our model are listed.

Additional Constraints to Limit the Model

In the initial runs of our model, we found that the inventory levels for the two parts being modeled were simply pushed to their maximum demand since the cost of increased inventory levels was less than the cost of air freighting in parts beyond the fixed inventory level. This resulted in more discussion with the plant manager to determine any other constraints on inventory levels. We then found that there was an additional constraint which we had failed to include in our model. The plant manager was limited to a maximum dollar value for the inventory maintained in the warehouse. A dollar figure for maximum inventory value was then produced ($220,000). The value of the fixed inventory ($207,820) was then subtracted from this dollar amount to determine the maximum value of inventory for the two parts being modeled and the following constraint was added to our model:

$$210 \times Ia + 220 \times Ib = 12180$$

where the cost of part a is $210, the cost of part b is $220, Ia represents the fixed monthly inventory level for part a and Ib represents the fixed monthly inventory level for part b. These parts are then limited to a maximum monthly value of $12180. This formulation is shown in the appendix with its resulting output and is labeled as run 1a.

During the sensitivity analysis, this final constraint was modified in two ways. When an additional part was added to the model its value was removed from the value for fixed inventory and the maximum inventory value level was then increased to allow for the additional part. These formulations are shown in runs 2a, 2b, 3a, 3b, 4a, and 4b. For each run made, the maximum value was allowed to rise until it no longer actually constrained the model. In this way, the optimal profit for the plant without artificially imposed constraints could be seen. This formulation is shown in the appendix labeled as run 1b.

Monthly Sales Data - Table 1

month 1	month 2	month 3	month 4
401,000	222,000	283,000	335,000

Monthly Demand - Table 2

	month 1	month 2	month 3	month 4
part a	73	29	14	67
part b	13	11	5	27
part c	2	1	7	0
part d	2	2	1	1
part e	3	1	1	0
part f	1	9	3	1

Monthly Hours - Table 3

	month 1	month 2	month 3	month 4
Regular	504	456	432	504
Overtime	60	59	38	1
Temporary	504	537	492	656
TOTAL	1068	1052	962	1161

Part Costs - Table 4

part a	$210
part b	$220
part c	$680
part d	$680
part e	$806
part f	$812

Objective Function

Maximize: $Z = \text{sum}_{i=1 \text{ to } 4}(S_i - 7.73\ PIa_i - 7.73\ PIb_i - 40\ Na_i - 40\ Nb_i - 210\ Xa_i - 220\ Xb_i - 8\ T_i - 22\ W_i) - 8.6\ Ia - 9.0\ Ib - 8520.6$

Subject to:

Monthly Sales levels:

$S_1 = 401000$

$S_2 = 222000$

$S_3 = 283000$

$S_4 = 335000$

Limit on temporary hours:

$T_1, T_2, T_3, T_4 <= 520$

Total hours constraint (allows for idle time for permanent employees):

For i = 1 to 4: $0.00408\ S_i + N1_i - W_i - T_i = 693.3$

Sources and inventory limits for part a:

For i = 1 to 4: $PIa_i + Na_i - Pa_i - Xa_i = 0$

Sources and inventory limits for part b:

For i = 1 to 4: $PIb_i + Nb_i - Pb_i - Xb_i = 0$

Limit sum of parts drawn plus surplus to fixed inventory level:

For i = 1 to 4: $PIa_i + Pa_i - Ia = 0$

For i = 1 to 4: $PIb_i + Pb_i - Ib = 0$

Demand for parts a and b (monthly - from monthly usage report)

$Xa_1 = 73$

$Xa_2 = 29$

$Xa_3 = 14$

$Xa_4 = 67$

$Xb_1 = 13$

$Xb_2 = 11$

$Xb_3 = 5$

$Xb_4 = 27$

Inventory must turn every 2 months:

For i = 1 to 4: $-210\ Ia - 220\ Ib + S_i + S_{i+1} >= 112500$

$S_5 = 316000$ (extra month needed for inventory turn equation)

Space constraint:

$Ia + Ib <= 144$

Limit to dollars tied up in inventory

$210\ Ia + 220\ Ib <= 12180$

Solution

The formulation of the problem presented to this point demonstrates several key behavioral patterns of the model:

1. The inventory turns constraint is never an active constraint. This plant always beats the "turns objective" of once every 2 months easily.
2. The use of temporary labor will always be chosen over permanent employee overtime because of the large cost difference.
3. The "political" inventory value constraint clearly reduces profitability by a significant amount. The increased profit potential attained by investing in more inventory is attractive despite the added carrying costs.
4. Space limitations did not become active. There is enough room to store all material that might be suggested by the model.
5. The cost of air freight, being so much larger than carrying cost plus surface freight, dominates the decision variable of how much inventory should be stocked. As formulated, the model suggests stocking items at the maximum monthly demand level.

Based on behavior observed in the model thus far, we were interested in how sensitive the objective function and inventory levels would be to changes in the allowed inventory level and the addition to the model of items that had a closer match between the cost of air freight vs the cost of carrying and surface freight. The sensitivity analysis focuses on these issues.

Sensitivity Analysis

The computer runs labeled 1a through 4b (included in the appendix) are not alternatives. Each one represents a more complete model of the facility. The objective function should be viewed as an indication of change in profitability not an indication of actual profit. Table 5 summarizes key data changes and resulting decision variable changes.

The most simple form of the model is shown in run 1a and 1b. This model does not include the costs associated with freight and carrying costs for any but the fixed inventory and items a and b. Run 1a constrains total inventory dollars to $220,000. Run 1b raises this value until it no longer actively constrains the model. The increase in the objective function demonstrates that lifting the arbitrary inventory limits allows a significant increase in profit (an increase of four month profit of $1331 for an investment of $9090).

Computer Run	Maximum Allowed Inventory	Air Freight Cost	Carrying Cost plus surface freight (1)	Run a and b Change	
				Inventory	4 mo. profit
1. Items a and b only are included. This simplification causes the cost of air freight and changes in carrying costs for parts c through f to have been excluded from the objective function.					
1a.	$220,000	a=40 b=40	16.33 16.73		
1b.	Unlimited ($229,090) (2)			$9090	$1331
2. Items a and b have been combined into the model as part a. Item c has been added as modeled part b.					
2a.	$220,000	a=40 b=45	16.44 36.58		
2b.	Unlimited ($229,100)			$9100	$1278
3. Same as run 2 except items c and d are modeled as part b.					
3a.	$220,000	a=40 b=45	16.44 36.58		
3b.	Unlimited ($227,780)			$7780	$1164
4. Same as run 3 except items c,d,e,f are modeled as part b.					
4a.	$220,000	a=40 b=45	16.44 39.47		
4b.	Unlimited ($229,207)			$9207	$ 672

(1) Carrying costs plus surface freight costs for the modeled items a and b are computed by volume, weighting the actual costs for each part a through f appropriately.

(2) Inventory value at which the inventory value constraint no longer constrains profit.

Table 5

Once the inventory constraint was lifted, the inventory level was driven by the demand for the item. All other inventory constraints (carrying cost, turns, and space) are inactive.

The cost of overtime permanent employee hours is so much larger than the cost of temporary employee regular time hours that the model will supply temporary hours until that resource is exhausted prior to supplying overtime.

The next set of computer runs, 2a and 2b, add item c to the model. Item c has a much lower usage than items a and b. The cost of item c is approximately three times the cost of items a or b. The cost to air freight item c is not significantly greater than items a or b. The model suggests that the less expensive items a and b be inventoried prior to considering stocking item c. The inventory limit is the active constraint in run 2a. The space constraint was restated since parts a and b do not take the same space on a shelf. Eighteen of part a fill a shelf. Six of part b fill a shelf. The total available shelves are 8.5.

In run 2b the total inventory limit is raised until it is no longer an active constraint. Part a (a weighted average of items a and b) is limited only by usage as was seen in run 1b. Notice that item c is limited by the carrying cost and regular freight cost constraint. The model shows an increase in profit of $1278 for a $9100 investment. As we added the more expensive items to the model we saw that the carrying cost plus regular freight cost began to approach the cost of air freight. When this occurred, the usage constraints became active in a different mode. If usage is relatively constant over the four month period then items will be stocked so long as carrying cost plus regular freight cost are less than air freight.

An occasional spike in usage over a generally constant usage pattern may not result in increased inventory levels since the air freight cost is a one time cost and carrying costs are charged for all four months. Should carrying costs plus regular freight costs ever exceed the cost of air freight, the model would not inventory that item. The model does not take into account the loss of good will or business due to unavailability of a part.

Run 3a expands the model to include item d. Part a is still the weighted average of items a and b. Part b is the weighted average of items c and d. The inventory limit of $220,000 limits the parts inventoried to part a. Since the maximum usage is not inventoried, none of part b is inventoried.

In run 3b, the inventory dollar constraint is raised until it no longer restricts the model. Once again the maximum demand for part a having been met, the model optimizes the amount of part b inventoried by balancing the carrying cost plus regular freight cost and the air freight cost over the usage pattern. The turns and space constraints are not active. The mode shows an increase in profit of $1164 for an investment of $7780.

Run 4a adds items e and f to the model. Part b is now the weighted average of items c, d, e, and f. The main reason for running this was to explore the total increase in inventory levels required by the model. As was expected, run 4a inventoried part a only. Run 4b inventoried nearly all the items required by usage but at a very small increase in profit. An investment of $9207 causes an increase in profit of $672.

At no time did the space constraints or inventory turns constraints become active.

Conclusions

Based on the results of the sensitivity analysis, it is clear that a change in inventory stocking policy at the plant would result in increased profitability. Labor costs are already being optimized, space is not an active constraint and the desired inventory turn ratio is easily exceeded.

The following rules of thumb detail the conditions and prescribed actions that will result in attractive increases in profitability with reasonable investment to pay back ratios:

1. If air freight costs for an item greatly exceed (more than 150%) the costs of surface freight plus carrying costs, this item's inventory level should be raised to its highest level of usage. This condition may be more of a factor on the west coast since transcontinental air freight is expensive.
2. As an item's air freight cost approaches its cost of surface freight plus carrying cost, the recommended inventory level is the most frequent monthly demand (not the average).
3. Should the cost of surface freight plus carrying cost exceed the cost of air freight (a light but expensive part such as a circuit board for example), the part should not be stocked at all. This assumes that the part is available upon demand from the manufacturing plant on the east coast. If availability is questionable, the cost of lost sales and customer goodwill must be weighed against the cost of inventory.

In summary, additional profit is available through the close management of the air freight vs carrying cost trade off. This is affected both on a cost per unit and volume weighted basis.